

LENSE-THIRRING PRECESSION IN STRONG GRAVITATIONAL FIELDS

By

CHANDRACHUR CHAKRABORTY
(PHYS05201004002)

*Saha Institute of Nuclear Physics, Kolkata
India*

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

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List of Publications arising from the thesis

Journal

1. **Dragging of inertial frames inside the rotating neutron stars**
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3. **Strong gravity Lense-Thirring Precession in Kerr and Kerr-Taub-NUT spacetimes**
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4. **Lense-Thirring Precession in Plebański-Demiański spacetimes**
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5. **Anomalous Lense-Thirring precession in Kerr-Taub-NUT spacetimes**
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Chandrachur Chakraborty

“जननी जन्मभूमिश्च स्वर्गादपि गरीयसी....”

(Mother and Motherland are greater than even 'Heaven')

Dedicated To

My Mother and Motherland

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Here are my traveller’s greetings to thee...”*

— *Rabindranath Tagore*

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