

**Statistical perspectives on soft matter and
biologically motivated systems**

Thesis submitted for the degree of
Doctor of Philosophy (Sc.)
in
Physics

by

Niladri Sarkar

University of Calcutta

2014

Declaration

I hereby declare that this dissertation represents my works in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic integrity and honesty and have not misrepresented or fabricated or falsified any idea/data/fact/source in my thesis.

*Niladri Sarkar
Kolkata, India.*

Abstract

We elucidate the statistical physics descriptions for a class of soft and nonequilibrium systems in this thesis. In particular, we discuss fluid membranes in thermal equilibrium as well as driven out of equilibrium being in contact with an active fluid layer, and asymmetric exclusion processes on a ring with a pair of bottlenecks. We first discuss the equilibrium phase transitions in model two-component asymmetric inhomogeneous fluid membranes. To this end, we propose two coarse-grained models belonging to two different symmetries in the spirit of the Ginzburg-Landau approaches and analyse them. The models display miscibility phase transitions for the order parameter that describe the membrane composition. These transitions may be both first and second order through critical and tricritical points. In addition, the membrane fluctuations near the phase transitions are either enhanced or suppressed, depending upon the symmetry of the models and the nature of the miscibility phase transitions.

In the next chapter, we set up the effective two-dimensional dynamics of a fluid membrane with an ordered active polar fluid layer anchored to it in the hydrodynamic limit. We consider a free standing system and a system resting on a solid surface. We analyse the long wavelength linear stabilities in the models. The instabilities can be either static or moving, and may exist for both signs of a parameter that characterises the strength of the nonequilibrium active stresses in the models. We illustrate their dependences on the boundary conditions and the ordering directions of the active fluid layer.

In the last chapter, we consider asymmetric exclusion processes on a ring with a pair of point bottlenecks. The resulting steady state densities display a variety of profiles, including localised domain walls and their delocalisation, and confinement of delocalised domain walls, depending upon the microscopic bottleneck configurations. We argue that the steady state density profiles cannot be reliably used to gain information about the microscopic bottlenecks, due to screening of the weaker bottleneck by the stronger one. This reveals a notion of an underlying universality in the system.

Acknowledgements

To begin with I would like to acknowledge my supervisor Dr. Abhik Basu, who had been a constant source of inspiration, not only in physics but also in life. Throughout my years as a Ph.D student, he guided me with a lively spirit and had been tolerant when I asked him questions, any number of times, some of which were really silly. His impeccably sharp insight in the subject, conceptual perfection and the simple way in which he conveys them is infectious to say the least, and has inspired me to give my best. An affectionate person, enthusiastic and ebullient in physics, he had been a friend, philosopher and guide in the truest sense. It is his constant counselling and indomitable spirit, which has turned me into a person that I am today, with a purpose in life. Without him, his constant support and belief in me, this thesis would not have been possible.

I would like to thank the Director of Saha Institute of Nuclear Physics, Prof. B. K. Chakrabarti, a great physicist and a constant source of inspiration, and Prof. R. Ranganathan, the Divisional Head of the Condensed Matter Physics Division, SINP for providing me with the facilities required to carry out my thesis studies. A lot of thanks to Kausik Das, our system administrator, who helped me a lot with my desktops and laptops and computer servers, needed for my Ph.D studies. He was always available whenever we needed him. Also I would like to thank the staff of the Condensed Matter Physics division office, who took care of every administrative matters and made dealings with the official proceedings easy.

The Physics department of Serampore College, and Indian Institute of Technology Kharagpur deserves a lot of credit for the way they guided me through my B.Sc and M.Sc years. I learnt a lot in those years and was encouraged to aim higher.

A lot of friends and well wishers deserves a space in this acknowledgements column. To begin with, my heartiest reagrds to Ritabrata, Dipanjan, Tanmoy and Partha, who had been my best buddies in school. I grew up playing with them and we keep in touch even now. I would like to appreciate Arnab, Promit, Ram, Dipayan, Sanchari and Sangita for being my best friends during my B.Sc years. We studied together and roamed around a

lot keeping each other company. I cannot forget the frequent visits to Arnab's house and Promit's house and the good times we had, sharing jokes. I really miss those days. My days in IIT KGP would have been boring had I not come across friends like Arunabha, Rajib, Somnath, Abhishek, Subhasis, Somdeb, Chancal and all others of lateral M.Sc batch 2006 – 2008. They ensured every day I stayed in the IIT hostel was as lively as possible, discussing physics, playing cricket, football and even pranks with each other. The frequent visits to the night canteen, staying awake together during exams, watching movies religiously and exploring places outside the campus will remain etched in my memory for a very long time. An extremely talented set of individuals, they inspired me academically to achieve higher and I think a certain amount of credit should also go to them for the position in which I am today, writing this thesis. Life in Saha Institute of Nuclear Physics was not easy to say the least, but for the friends I had at this crucial juncture of life. Najmul, Ajanta, Saptaparni, Satyajit, and Manas had been very supportive during these 6 years and made my stay at the hostel entertaining and comfortable. I cannot imagine life without them. I cannot thank Dr. Swapan Majhi enough for the 3 years he stayed at SINP as a post doctorate fellow. He helped us whenever we faced any problems, be it academic or otherwise. Being a very active individual, he inspired us to take part in all kinds of hostel activities, uniting us like a family. My heartiest thanks to Atanu, Rajani, Sudip, Souvik, Parikshit and Kalyanmoy who had been exemplary juniors and like brothers to me. They did everything I asked for or not, giving me company when I needed them the most. The badminton games we played, shopping at the malls, watching movies at the multiplexes and the time we spent together will be missed dearly when I leave this place. I will be forever be indebted to Soumyajyoti, Sreemoyee and Baishali for making my life at the institute as entertaining as possible. The lively discussions at the institute canteen, the eating out at the restaurants and pandal hopping during pujas will never be forgotten. One cannot ask for better friends than what you have been to me.

On a serious note I would like to thank Asim Ghosh and Dr. Anjan Chandra for helping me academically, specially with programming. Also thanks to Tirthankar, for being a great junior. The stimulating discussions we had pushed me to delve deeper into the subject.

I would take this opportunity to thank my uncle Satyajit who taught me maths and physics when I was at my senior years at school. His teachings were really the best and helped me to overcome my fear of the subjects I had at that time. I would also like to thank my aunt Susmita, who looked after me when I was young during my school years in the absence of my mother. I am also grateful to my grandparents with whom I spent a lot of time during childhood and had some of the best years. At 80, my grandfather can still run a marathon, and he was my first childhood idol.

Lastly I would like to thank my mother and father, the two souls who sacrificed everything

to watch me grow. They just wanted the best for me and motivated me immensely to achieve greater goals in life. Their love and affection can never be repayed. During my M.Sc and Ph.D years they were patient and didn't complain even if I stayed out of home for days, even months at a time. This thesis is as much theirs as it is mine.

*Niladri Sarkar
Kolkata, India.*

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