Perturbative and Non Perturbative Aspects of Lattice Quantum Chromodynamics

By

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STATEMENT BY AUTHOR

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree/diploma at this or any other Institution/University.

Abhishek Chowdhury

Abhishek Chowdhury

DEDICATIONS

Dedicated to my mother.

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