87. DETERMINATION OF THE ELECTRON CONCENTRATION AND THE COLLISION FREQUENCY IN THE IONOSPHERE LAYERS OF THE O AND X WAVES*

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Abstract

It is well-known that according to the ray-theory, the O and X waves reflected from different layers of the ionosphere would be ellipses, with their major axes tilted at an angle ψ to the geomagnetic N-S axis, and the ratio of the axes (b/a) varying from one at the GM poles to zero at the GM equator. These deductions are also confirmed from the wave theory, particularly for stations of low latitudes; for higher latitudes, the wave coupling factor may not be negligible. Experimental studies of the polarization characteristics of the downcoming waves have been so far mostly confined to high latitudes. The patterns are found to be nearly circular $(b/a \simeq 1)$, and it is difficult to measure the tilt angle accurately. There are in fact very few measurements for intermediate latitudes, where there is possibility of accurate measurement of the tilt angle and of getting a value of (b/a) different from one and zero.

This subject has been discussed by R. Roy and J. K. D. Verma from the wave theory of propagation in the form given by Saha, Banerjee and Guha (1951), and it has been shown that it is possible to deduce from the values of ψ and b/a the electron concentration and the collision frequency in the ionospheric regions from which the waves are reflected.

The mathematical work needed for these calculations may be briefly given here. We introduce two quantities:

$$\xi = \nu/\nu_c, \qquad \eta = (1-r)p/\nu_c$$

where ν_c is the critical collision frequency of Appleton, viz., $\nu_c = -\frac{|p_n|}{2\cos\Theta}\cdot\sin^2\Theta$, Θ being the angle of propagation varying from zero in the south G. M. pole to π in the north G. M. pole, ν is the collision frequency, $r = \frac{4\pi Ne^2}{mp^2}$, N being the electron concentration, and p the pulsatance of the exploring wave.

It has been shown (Saha *et al*, 1951) that the electric field intensity of the exploring wave is split up into two given by:

$$\frac{E_x + i\rho_1 E_y}{\sqrt{1+\rho_1^2}} \quad\text{and}\quad \frac{E_x + i\rho_2 E_y}{\sqrt{1+\rho_2^2}}$$

where:

$$\rho_1 = G - \sqrt{1+G^2}, \qquad \rho_2 = G + \sqrt{1+G^2}, \qquad G = 1/(-\eta + i\xi)$$

the waves being propagated with the velocities cq_0 and cq_x where q_0 and q_x are the refractive indices of the O- and X-waves given by the ray theory.

We can put:

$$i\rho_1 = Re^{i\alpha}, \qquad i\rho_2 = R^{-1}e^{-i\alpha}.$$

*This reports the work done by Mr. B. M. Banerjee, Mr. R. Roy and Mr. J. K. D. Verma at the Institute of Nuclear Physics, Calcutta.

To obtain the values of R and α, we make use of a ξ-η plane, covering all the characteristics of the ionosphere. It may be called the characteristic plane.

The values of R and α may be obtained with the aid of the following formulae:

$$\left(\frac{\xi}{\zeta^2}\right)^2 \cdot \left(\frac{2R}{1+R^2}\right)^2 + \left(\frac{\eta}{\zeta^2}\right)^2 \cdot \left(\frac{2R}{1-R^2}\right)^2 = 1$$

$$\frac{(\xi/\zeta^2)^2}{\cos^2\alpha} - \frac{(\eta/\zeta^2)^2}{\sin^2\alpha} = 1$$

or,

$$\frac{\zeta^2}{2}\sin^2 2\alpha + \cos 2\nu + \cos 2\alpha = 0$$

where:

$$\zeta^2 = \xi^2 + \eta^2, \text{ and } \nu = \tan^{-1}(\xi/\eta).$$

The reader can verify that the above relations hold even if R is changed to 1/R, or α changed to $-\alpha$. The R=constant and α=constant curves in the $\xi-\eta$ plane have been shown in fig. 1. Though only the values of $R<1$, and positive values of α have been shown, the R=constant, and α= constant curves hold for R^{-1} and $-\alpha$ as well.

These ambiguities have been cleared by detailed discussions on the polarization factors:

$$\rho_1 = G\{1 - \sqrt{1+1/G^2}\}, \quad \rho_2 = G\{1 + \sqrt{1+1/G^2}\}$$

in which $\sqrt{1+1/G^2} = \{(1-\xi^2+\eta^2) - 2i\,\xi\eta\}^{\frac{1}{2}}$ can have two signs. For the case of the northern hemisphere where $\xi > O$, the values of R and α are shown in figure 2. It is seen that the ξ-η plane is divided into four sections by the lines $\eta = O$, $\xi = 1$. For a place like Calcutta where $\nu_c = 3.66\times10^6$, ξ is less than 1, the wave coupling factor may be neglected, and the split waves are confined to the region I for ν as high as 10^6, It is found that for the O and X waves, the electrical field intensities satisfy the relations:

O wave→ $E_x^2+2R\cos\alpha\, E_x E_y+R^2 E_y^2=C_0^2$

X wave→ $E_x^2+2R^{-1}\cos\alpha\, E_x E_y+R^{-2} E_y^2=C_x^2$

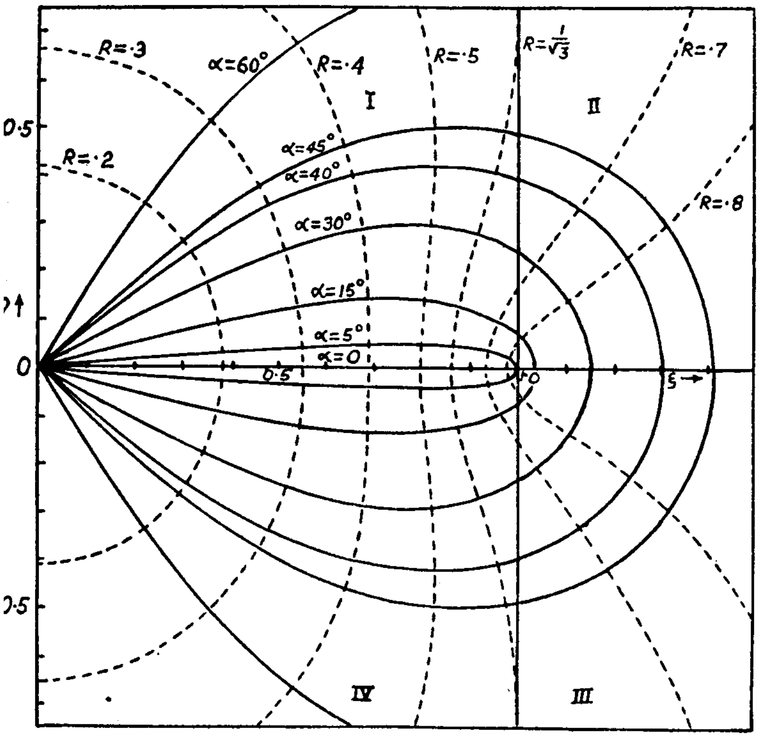


Fig. 1

From these relations, it is easily deduced that the polarization patterns would be ellipses tilted with respect to the x axis (N-S) through the angles ψ given by:

$$\tan 2\psi_0=\frac{2R\cos\alpha}{1-R^2},\quad \tan 2\psi_x=-\frac{2R\cos\alpha}{1-R^2}$$

It has not been found possible to determine from theoretical considerations, the level to which the limiting polarization of the downcoming waves should correspond. It may be noted that the downcoming waves would appear with the characteristic polarization corresponding to some level in the ionosphere of particular electron density and collision frequency. If after reflection, the waves travel without any change of phase velocity, the polarization factors and hence the values of R and α will correspond to the levels of ionization in the layer from which the waves had been reflected. The values of R and α are obtained from the experimental results as follows: it can be shown that:

$$x=\frac{\text{minor axis}}{\text{major axis}}=\frac{b}{a}=\left\{\frac{1+R^2-\sqrt{1+2R^2\cos 2\alpha+R^4}}{1+R^2+\sqrt{1+2R^2\cos 2\alpha+R^4}}\right\}^{\frac{1}{2}}$$

$$y=\tan 2\psi_0=\frac{2R\cos\alpha}{1-R^2}.$$

from which the following relations can be deduced:

$$R=\left\{\frac{\sqrt{1+y^2}\,(1+x^2)-(1-x^2)}{\sqrt{1+y^2}\,(1+x^2)+(1-x^2)}\right\}^{\frac{1}{2}}$$

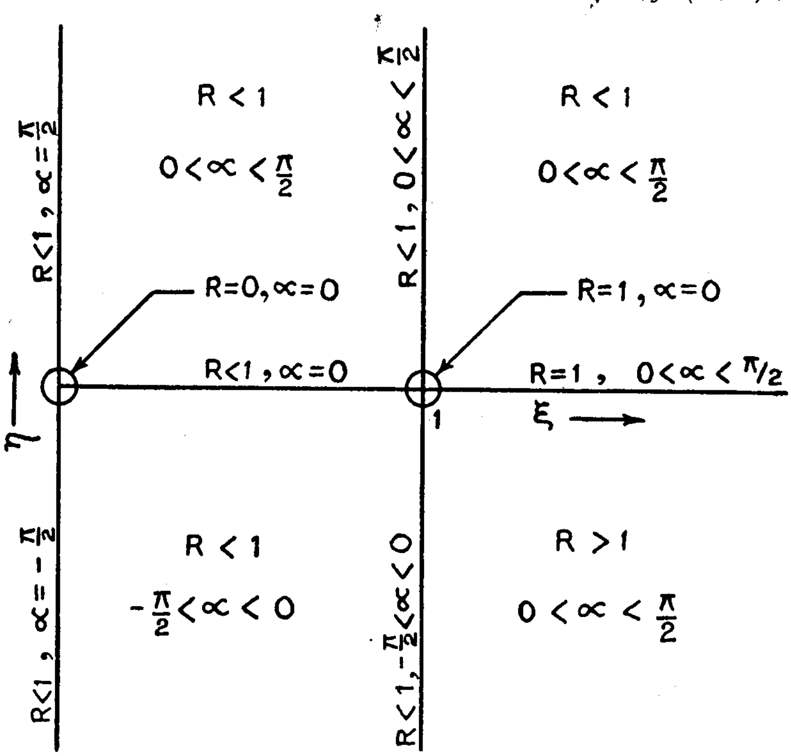


Fig. 2

$$\cos \alpha = \frac{y(1-x^2)}{[(1+y^2)(1+x^2)^2-(1-x^2)^2]^{\frac{1}{2}}}$$

From the values of R and α, we deduce:

$$\tan \nu = \xi/\eta = \frac{1+R^2}{1-R^2} \cot \alpha,$$

$$\eta^2+\xi^2 = -\frac{2(\cos 2\alpha + \cos 2\nu)}{\sin^2 2\alpha},$$

then we have:

$$\eta = \frac{(1-r)\,p}{\nu_c} = (\xi^2+\eta^2)^{\frac{1}{2}} \cos \nu,$$

$$\xi = \frac{\nu}{\nu_c} = \eta \tan \nu.$$

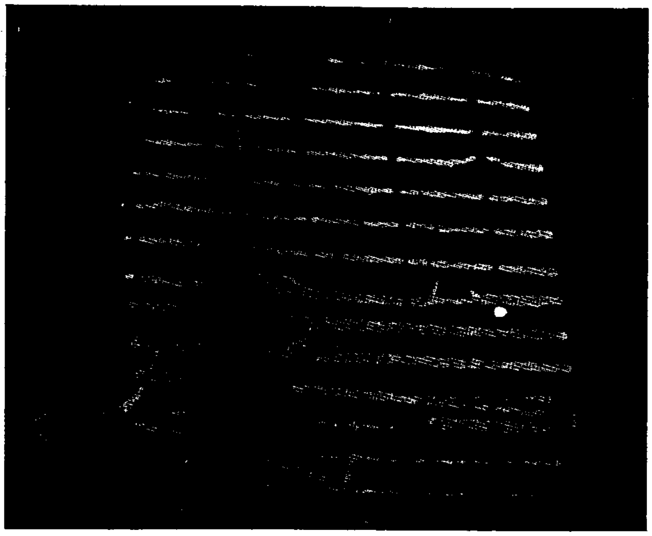


Plate I

Type A display of echoes from the ionosphere at 2·9 Mc/s (recorded at 1652 hours IST on 1-1-53). The raster sweep of twelve lines are executed from right to left (the faint lines are due to the other beam of the CRO), and each line corresponds to 50 kms. of equivalent height and there are markers (bright and dark spots) at intervals of 5 kms.

Line	Height km	Remarks
1	—	Ground Pulse
3	108·5	E*s* Cloud
3	111·5	E*s* Thin layer type
5	197	F*o*
5	212	F*s* Cloud
5	215	F*s* Cloud
5	217·5	F*x*
7	323·5	(E*s*+F*s*)
9	388·5	F*s* Cloud

The echoes due to scattering from electron-clouds embedded in E and F layers are transitory and rarely persist for more than two minutes.

These results will be useful, if good experimental methods for the accurate measurement of ψ_0 and b/a, can be found.

So far the experimental studies have suffered from two handicaps, viz.,

(*a*) On account of insufficient resolution, it is not often possible to separate the O- and X-waves sufficiently,

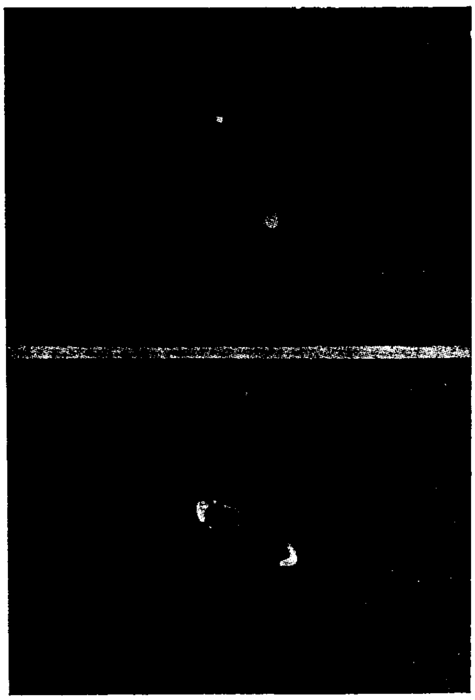
(*b*) The intensities of the reflected beams undergo rapid fluctuations due to the superposition of waves scattered at small angles by moving electron clouds.

The difficulties have been overcome by the design of a new type of ionospheric recorder (Banerjee and Roy, 1950, 1952) using two principles viz.,

(*a*) Use of a short duration exploring pulse 6-30 μseconds in place of the usual pulse of 100-200 μseconds, with suitably designed receiving systems,

(*b*) Use of a raster-sweep time base of twelve lines, each corresponding to 50 km of equivalent height. It is then possible to obtain a resolving limit of 2 km.

Experiments with this recorder have shown that it is possible to separate the reflected O- and X-waves (Plate I) completely from the scattered waves due to irregularities like moving electron clouds (Roy and Verma 1953). Distinct pips corresponding to all these waves appear on the CRO screen, but while the O- and *x*-wave pips are fairly steady in position and intensity, the pips owing their origin to scattered waves fluctuate in position and intensity. The echoes for each type of wave, can therefore be picked up and studied separately.



(A)

(B)

Plate II

(A) Polarization pattern of the *o*—wave.
(B) Polarization pattern of the *x*—wave.
Recorded at 12 noon IST on 9.3.54 at 5.45 Mc/s.

The state of polarization of these resolved echoes were studied by Roy and Verma with the help of a radio polarimeter following Eckersley's method of echo-selection. The O– and X–waves always come as tilted ellipses and with an axis ratio of approximately 2 to 4, and the sum of their tilt angles is 270°. It is difficult to obtain the polarization patterns of scattered waves. They come sometime as circles, sometime as tilted lines and sometime as ellipses. Not much study has been made of them yet.

To elucidate how N and ν are determined from the polarization date, the results of a typical observation for the F2-layer echoes are given in (Plate II).

O– ellipse: $x=\cdot 2,\ \psi_o=119°$

X– ellipse: $x=\cdot 32,\ \psi_x=150°$

So $\psi_o+\psi_x=269° \simeq 270°$ as theoretically expected. We obtain by the use of the formulae:

$$R = \cdot 5856,\ \alpha = 26°3' \text{ for the O– wave.}$$

$$\tan\nu = 4\cdot 182,\ \xi^2+\eta^2=\cdot 891,\ \eta=\cdot 2196.\ \xi=\cdot 9183$$

from which we get $r=\cdot 98$, $\nu=3\cdot 07\times 10^6$, and for the x-wave similarly r is found to be ·95 and $\nu=3\cdot 55\times 10^6$. Polarization patterns of the echoes from the E-layer show that they correspond to a value $r=1$, and $\nu=1\cdot 72\times 10^6$. It is to be noted that the limiting polarization of the waves correspond to the reflection level which leads to the conclusion that after reflection the waves are propagated without any change of phase velocity.

The experiments are still in the preliminary stage, but sufficient results have been obtained which convince us of the great potentialities of the method.

References

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2. B. M. Banerjee and R. Roy.—*Ind. J. Phys.*, **24**, 411, 1940.
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88. ON ELECTRON-CHEMISTRY AND ITS APPLICATION TO PROBLEMS OF RADIATION AND ASTROPHYSICS*

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Read on the 6th July, 1920.

Both classical and modern Thermodynamics have hitherto been confined to the treatment of the influence of heat on material substances up to the stage of chemical decomposition and vaporisation (or atomisation). The successive stages which come up for treatment can be thus schematically written:-

Phase	Phenomenon	Example
Solid	} Liquefaction	Ice
Liquid	} Vaporisation	Water
Gas (Consisting of molecules)	} Decomposition	H_2O (Steam)
Gas (Consisting of constituent atoms).	} Atomisation	H_2O_2
Gas (Elementary)		H, O

What happens when the gaseous mass consisting purely of atoms is further heated ? The problem has not merely an academic interest, for though the temperatures we are considering may not be commanded in the Laboratory, such is usually the case in the Stellar universes with which we are acquainted through their spectra.

The answer to the problem raised easily follows from considerations of the Rutherford-Bohr theory of the atom viz., further heating of the gaseous mass will cause ionization *i.e.* some of the atoms will lose one electron, and under particular conditions of temperature and pressure, a definite chemical equilibrium will be established between the neutral atoms, the atoms which have lost one electron and the electrons split of, according to the scheme

$$Ca \rightleftarrows Ca^+ + e \tag{1}$$

That this will be the case may be seen from the fact that at high temperatures, many metals throw off copious quantities of electrons in these substances (Tungsten for example), ionization precedes liquefaction, just as in camphor, carbon and other volatile substances, vaporization (sublimation) precedes liquefaction.

These problems were foreshadowed by Nernst in his book

* An introduction and synopsis of the following four papers communicated to the Phil. Mag.
Paper A. Ionisation in the Solar Chromosphere.
B. On the Problems of Temperature-Radiation of Gases.
C. On Elements in the Sun.
D. On the Harvard Classification of Stellar Spectra.