83. ON THE CONDITIONS OF ESCAPE OF MICROWAVES OF RADIO-FREQUENCY RANGE FROM THE SUN

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ABSTRACT

In this paper, we have discussed the conditions of escape of radio-frequency waves from the solar atmosphere with the aid of magneto-ionic theories of propagation of radio-waves through an ionised atmosphere traversed by a magnetic field. It has been shown from these theories that the magnetic field of the spots actually enables the e-component of the waves to escape from deeper layers of the solar atmosphere and thus provides an explanation of the observational fact that radio-waves are actually emitted by the spot regions themselves. It is shown that the same theories give a general and satisfactory explanation of all the facts observed so far, *e.g.*, the circular polarisation, and sudden intensification of emission with the onset of radio flares. Programmes for further work are indicated.

1. INTRODUCTION

In several communications to *Nature* and elsewhere, various British, Australian and New Zealander workers have described experiments which prove conclusively that during times of solar disturbance, there are large outbursts of radio-frequency energy from the sun. The wave-lengths of the radio waves, as observed by these workers, vary from 1·5 to 30 metres (200 Mc/sec to 10 Mc/sec) and there is short account of a solitary work in the centimetre region (Dicke and Beringer, 1946), in which emission has been measured by a different technique.

The communications are mostly short. The most complete account has so far been given by Appleton and Hey (1946). Relevant points from their account are given below.

The observations were made when there was outburst of sun-spot activity which started on 22nd February, 1946. According to their statement the sun-spot crossed the solar disc at a heliographic latitude 22°N, central meridian passage occurring on 28th February, 1946. The maximum size of the spot was some two thousand millionth part of the sun's hemisphere. A very extensive and brilliant solar flare occurred from approximately 12 hours to 15-03 hours on 28th February, and the accompanying radio fadeout which began just after 12 hours lasted till 20-0-0 hours. A great magnetic storm broke out with great activity at 7-22 hours on 2nd March. The days of greatest intensity of solar noise, *i.e.*, 27th and 28th February, occurred when the sun-spot was near the central meridian passage and at the time of intense sun-spot activity several solar flares were observed in addition to the most brilliant one to which reference has already been made. The sun-spot decreased in size after the 28th February and at the same time solar noise subsided.

The reports so far published make it clear that the metre range microwave radiations are emitted only during times of solar activity. In fact a close statistical correspondence between the sun-spot activity and metre range micro-wave emission has been established by Pawsey, Payne-Scott and MacReady (1947) and others.

This close correspondence is further confirmed by the experiments of Ryle and Vonberg (1946) who, by an ingenious adaptation of the famous Michelson-Pease method of measuring stellar diameters, showed that the actual region from which these radio-waves are coming subtends an angle not greater than 10′ of arc, the solar diameter being 32′ of arc. The size of the radio-wave generating regions, thus determined, is of the same order of, though somewhat larger than, the size of the large sun-spot groups (3′ of an arc). Pawsey *et al* find by a different method that the diameter is about 6′.

The main characteristics of these solar radio noise, as determined by the various workers, can be summarised as follows:—

(*i*) The radiation is closely connected with sun-spots, appearing and disappearing with the latter.

(*ii*) The radiation has the characteristics of random noise, the intensity of the spectrum is neither steady with time nor continuous in wave-length, nor monochromatic.

(*iii*) The intensity of radio-emission is extraordinarily high.—Appleton and Hey working on 4·7 metres, found that the radio flux from the active area was 10^8 times that associated with black body radiation from the disc as a whole. Supposing that only 1/200th part of the sun's

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hemisphere is active, the intensity increases to 2.10^5 times the black body radiation from the sun's disc.

(*iv*) The intensity is subject to sudden fluctuations occurring generally with the onset of flares or other disturbances.

(*v*) Polarisation.—Experiments on the polarisation of the radio noise have been carried out by Appleton and Hey (1946) in London, by Ryle and Vonberg (1946) in Cambridge and Martyn (1946) in Canberra. They have all found that the radio noise is invariably circularly polarised, but otherwise the descriptions are confusing. Describing the polarisation of waves from the sun-spot groups of July 27 to August 3, 1946, Ryle and Vonberg say:—

"Measurements taken over the period July 27 to August 3rd, showed the polarisation to be anticlockwise, viewed along the positive direction of propagation (left handed). Between August 3 and August 7, the degree of polarisation diminished, being virtually completely random on August 7. On August 8, 40% polarisation was observed again but with right-handed polarity, the result presumably of increased activity in a subsidiary sun-spot."

Martyn (1946) working in the southern latitudes (35°S Canberra) while observing the same sun-spots occurring in the last week of July 1946 noted:—

"It was found that the right-handed circularly polarised power received was some seven times greater than that received when the system accepted only left-handed polarised radiation. Three days later, when this sun-spot group crossed the meridian these conditions were reversed, five times more power being then received on the left-handed than on the right-handed system".

Thus simultaneously observing the same sun-spot groups the sense of rotation of the polarised signal in the two hemispheres were found by the observers to be opposite.

As will be made clear later, it will be helpful to the understanding of the phenomenon if, simultaneously with the recording of the polarisation, all the characteristics of sun-spots (heliographic latitude and longitude, distance from the centre of the disc, magnetic field strength, classification, etc.) are given.

Measurements for radiation below metre range

Appleton and Hey state that they could not detect any enhancement of solar noise on wave-lengths as short as 1/10 metre. Similar observations by the T.R.E. establishment on a 3/10 metre yielded a negative result. The radio noise associated with the sun-spot activity becomes significant, without very special technique, when the wave-lengths approach 1·5 metres. The results are in accordance with the observations of MacReady, Pawsey, Payne-Scott (1946). On the other hand Dicke and Beringer (1945) working in the centimetre range and using special technique found that microwaves of 1·25 cm. length are emitted during times of solar activity, the corresponding black body temperature being 11000°K.

2. Preliminary Attempts at a Theory of the Phenomenon

The problem is to find out the physical mechanism which gives rise to the radio-frequency waves in the sun and also to discuss how these waves are propagated through the solar atmosphere. Let us first confine ourselves to the second aspect of the problem, for whatever may be the physical mechanism giving rise to the radio-waves, it is clear that their passage through the various layers of the solar atmosphere would be regulated by the laws of electromagnetism. As is well-known, the various layers of the solar atmosphere are highly ionised, the electron-density, according to well-tested astrophysical theories, being 10^{14}/c.c. for the photospheric level, 10^{11}/c.c. for the base of the chromosphere (500 km above photosphere) which fall to $3{\cdot}5\times10^8$/c.c. at the top of the chromosphere (14500 km). The density thereafter falls slowly throughout the whole corona, but is $\simeq 10^3$/c.c. even at distance of 10 solar diameters.

The quiescent sun has, like the Earth, a permanent magnetic field, the existence of which was first indicated by the investigations of G.E. Hale. On account of its small value, the reality of the effect has been sometimes called into question, but recent investigations by Thiessen (1946) appear to have satisfied the astrophysicists that the field is real. The value is 25 gauss at the solar magnetic equator, and 50 gauss at the magnetic poles. According to Hale and Thiessen the magnetic axis is inclined at an angle of 6° to the sun's axis, but for our purpose we can take the two axes to be identical. Besides this small permanent magnetic field, magnetic fields of a higher order are developed in spot regions during times of solar activity.

The conditions in the sun are, therefore, somewhat similar to those prevailing in the Earth's atmosphere (ionosphere) for transmission of radio-waves through it, and the same mathematical methods which have been developed by Appleton, Hartree and others, can be used in the present case.

The authors (Saha, Banerjea and Guha, 1947) have, however, completely recast the mathematical treatment of propagation of radio-waves through an ionised atmosphere which is traversed by a magnetic field, and results from this paper are freely used in the present one. The main conclusions of Appleton are, however, quite sufficient for a preliminary survey of the problem.

According to these workers, a beam of unpolarized radio-waves on entry into the ionosphere is split up into two waves, which are styled ordinary (shortly called o-wave), and extraordinary (shortly called e-wave), They travel with different velocities, *i.e.*, refractive indices, have different states of polarisation and are absorbed to different degrees. These quantities, *viz.*, the refractive indices, polarisation and absorption are functions of electron density N, damping, field-strength H, and θ, the angle of propagation which is the angle between the magnetic field and the direction of propagation. In the case when damping can be neglected the refractive indices for the two waves are functions of N, H, and θ and decrease steadily as N increases. When μ becomes zero, the wave can no longer proceed forward, but is reflected back. The limiting conditions (Appleton-conditions) for penetration are for the

$$\text{o-wave, } N < \frac{\pi m}{e^2} f^2 < 1{\cdot}21 \times 10^{-8} f^2 \qquad (2{\cdot}1)$$

$$\text{e-wave, } N < \frac{\pi m}{e^2} f(f \pm f_h) < 1{\cdot}21 \times 10^{-8} f(f \pm f_h). \qquad (2{\cdot}2)$$

where f = wave frequency = c/λ

f_h = gyromagnetic frequency for a field H

$$= \frac{eH}{2\pi mc}$$

$$= 2{\cdot}8 \times 10^6 \, H$$

These conditions are independent of θ.

Let us apply these conditions to the solar atmosphere. For the o-wave,

$$N < 1{\cdot}25 \times 10^{-8} \times f^2$$

$$\left.\begin{array}{lll} < 1{\cdot}25 \times 10^6 \text{ for} & f = 10 \text{ Mc/sec}, & \lambda = 30 \text{ m} \\ < 5 \times 10^8 \text{ for} & f = 200 \text{ Mc/sec}, & \lambda = 1{\cdot}5\text{m} \end{array}\right\}. \qquad (2{\cdot}1a)$$

This shows that the o-component of the metre waves cannot escape from the sun unless they originate in the corona, and that so progressively in the upper layers, as we take larger waves. The o-component of 30 metre waves can come only from beyond a distance of several diameters of the sun.

The e-wave should satisfy two conditions:

(*a*) f should be $> f_h$

(*b*) N should be $< 1{\cdot}25 \times 10^{-8} \times f(f - f_h)$

(*a*) shows that the e-component of microwaves cannot escape from the quiescent sun unless $f > 100$ Mc/sec, $\lambda < 3$ metres, but microwaves of longer wave-length have been detected. (*b*) states that e-microwaves satisfying condition (*a*) would have to originate even in higher layers, *i.e.*, their probability of escape from deeper layers is much less than that of o-waves.

These difficulties have been sought to be explained with the aid of the hypothesis that the microwaves actually originate in the higher corona to which a temperature of the order of a million degree is ascribed. But the hypothesis does not explain why the microwaves are so copiously emitted during times of solar spottedness and are actually emitted, as now appears almost certain, from the spots themselves and not from the quiescent regions, unless we make the very improbable assumption that it is only the spots which develop the million degree temperature. The assumption is extremely improbable in view of the fact that spectroscopic evidence shows that spots are regions of much lower temperature than even the photosphere. This necessitates the investigation of the problem from an altogether different point of view.

Sun-spots and microwave emission

The sun-spots, according to the classical investigations of Hale and Nicholson (1925), are found to show magnetic fields of entirely different order of magnitude than the quiescent sun, from a few hundred gauss for tiny spots to about 4000 gauss for the largest ones. Their direction is normal to the solar surface at the centre of the spot, but becomes inclined to the surface as we go outwards to the penumbral regions, as shown in Fig. 1. The largest number of spots occur in close pairs, showing opposite polarity, and and even those which are apparently single, appear to have an invisible companion, having opposite polarity somewhere inside the surface. In fact, the spots are known to be hydrodynamical vortices, passing underneath the apparent surface, the two ends of the vortices being the two components of the bifocal spots. Spot groups show very complicated forms for which reference may be made to original sources (Nicholson, 1938) and reports (Nicolet, 1942).

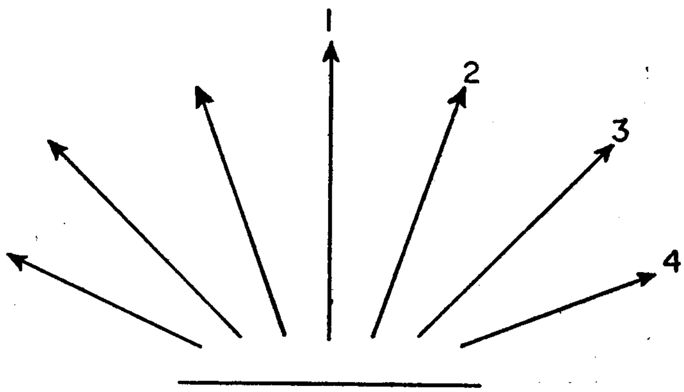

Fig. 1
Schematic representation of lines of force of the magnetic field in the region above a sun-spot (Nicholson).

Let us now apply the Appleton-conditions for escape of radio-waves from the spots, supposing that they originate somewhere within the spots.

For the o-component, since its condition of escape does not depend on the magnetic field, the conditions (2·1) will continue to apply, *i.e.*, the o-component of microwaves cannot escape from the sun, unless they originate in the corona over the spots.

We observe that $f_h = 2{\cdot}8 \times 10^9 k,$

where k=value of the magnetic field in kilogausses: so that $\lambda_h = \frac{10\cdot7}{k}$ cms. It is well known from Appleton's theory that the polarisation and refractive index of the e-component depend very largely on the quantity $\omega = f_h/f = \lambda/\lambda_h$. We have for the sunspots $\omega = 9\cdot3km$, where m=length of the microwave emitted in metres. Taking $k=1$, $m=2\cdot1$, $\omega \simeq 20$, but the value may be much larger for longer waves and larger fields. Taking an average value of 20 for ω, we can easily draw the following conclusions:

Since $\omega >> 1$, the condition that the square of the refractive index vanishes at the point where the electron concentration is given by the formula $4\pi N e^2/m < p(p-p_h)$ is absent. If we plot the value of the refractive index for the e-wave in the case $\omega > 1$ as a function of electron concentration, we find that, when damping is nelgected, the $(\mu_e^2 - N)$ curve is a smooth one, decreasing gradually to zero at $N = \frac{e^2}{\pi m} f(f+f_h)$ (*vide* Fig. 2), while μ_o^2 reaches the zero value at $N = \frac{e^2}{\pi m} f^2$.

Since the wave can proceed till this condition is satisfied the limiting concentration for escape of the e-wave is given by

$$N < 1\cdot25 \times 10^{-8} f(f+f_h) \quad \ldots \quad \ldots \quad (2\cdot3)$$

Taking $\qquad f = 1\cdot5 \times 10^8 (\lambda = 2 \text{ metres})$,

we should have $N \gtrless 1\cdot25 \times 10^{-8} \times 1\cdot5 \times 10^8 \times 2\cdot8 \times 10^9 k$

$$\gtrless 6\cdot6 \times 10^9 k.$$

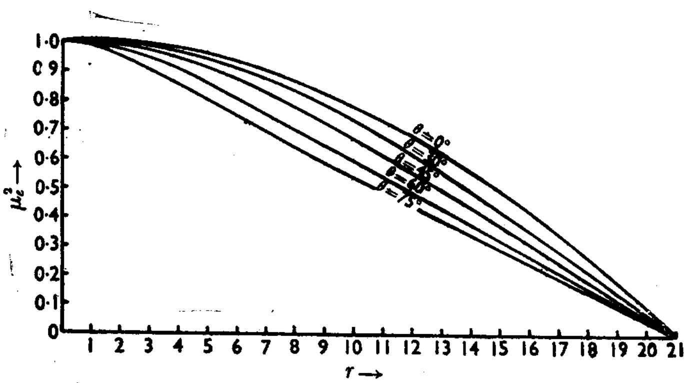

Fig. 2

Variation of the square of the refractive index of the extraordinary wave with electron concentration for different angles of propagation (for $\omega = 20$).

The calculation shows that the e-component can come from far deeper layers than the o-component, in fact from deep chromospheric layers, where the electron density is $(1+\omega) \simeq 21$ times the limiting density for o-waves.

The magnetic field measured for the spots refer to the lowest layers—in fact, as will be shown later for the reversing layer of the spots. The field above and below must be very different. We have given in §3, plausible formulae for the variation of the field above the reversing layer, and these formulae may probably be experimentally verified as suggested by Hale long ago (*vide* appendix).

But these methods are not applicable for finding out the value of the magnetic field below the reversing layer, for which we have to fall back upon some speculative theories. If the spots are hydrodynamical vortices, they should funnel out on reaching the chromosphere, but their cross-sections must be very small just below the reversing layer, and according to Chapman (1944), fields may reach enormous values of the order of a million gauss. If this be correct the e-component can escape from far deeper layers.

3. Characteristics of the Spot Atmosphere

Though the ideas outlined in the previous paragraphs make it clear that it is the strong magnetic field of the sun-spots, which allows only one component of microwave beams to come out from very deep layers, the application of the above ideas to the actual problem of emission requires a more detailed knowledge of the physical conditions prevailing in various layers of the spot-region.

The physical quantities of which knowledge is particularly needed in this connection, are:

(1) The concentration of different kinds of atoms and molecules at different levels.

(2) The electron concentration at different levels.

(3) The magnetic field and its variation with height.

Methods, based on sound physical theories, have been developed for determining the various physical characteristics of the normal solar atmosphere, with the aid of data obtained from astrophysical observations, account of which will be found in Unsöld's *Sternatmosphäre* and Stromgren's analysis of the composition of the solar atmosphere. The term "Solar atmosphere" is used in a comprehensive sense after Rosseland to denote the totality of the phenomena known under the terms: the Reversing Layer, the Chromosphere and the Corona, and they are treated together, because, inspite of the fact that the nomenclature had their origin in different groups of observational data, the problems of the three layers easily pass into one another.

It is well-known that the sun-spot and its neighbourhood show very great deviations from the normal solar atmosphere, marked by fall of temperature, development of magnetic fields, and radial, transverse and vertical motion of the solar gases, giving rise to difficult hydrodynamical problems. A comprehensive theory, explaining the whole history and physical characteristics of spots, is still wanting, but our requirements, as given above, are limited. But even here, information is very scanty. The elaborate methods, which have been used for finding out the concentration of various types of atoms and electron density of the normal solar atmosphere in different layers, can be applied also to spots, but this has not yet been done. Only indirect methods of comparison with the normal atmosphere are available (Moore 1931, Richardson 1931). For these reasons, it is necessary to review critically the methods used for the normal solar atmosphere.

The normal reversing layer is a region extending from the level of the photosphere to the base of the chromosphere, a distance of 300-500 km. The Fraunhofer lines mostly originate from absorption of continuous photospheric light by the atoms and molecules contained in this layer.

Several methods are available for finding out the composition of the reversing layer, but the results, though of the same order, do not tally with each other. According to Russell (1928) the composition (percentege of atoms) is 91% H, probably 3% He, 3% O, 1·5% other elements of which Fe forms the preponderating part, and 1·5% electrons. The atmosphere is nearly purely hydrogen, and becomes more so as we ascend upwards. The total number of H-atoms over 1 c.c. of photosphere is given by Russell as $1{\cdot}8\times10^{22}$/c.c.

The electrons in the reversing layers and the chromosphere are mostly derived from the thermal ionisation of metals.

For the chromosphere very extensive investigation by Wildt (1947), based on analysis of flash spectrum data, is available, from which Table I has been compiled:

TABLE I

Locality	Level above Photosphere	Hydrogen density	Electron density
Reversing layer	0 to 500 km	$6{\cdot}76\times10^{15}$	10^{14}
Base of chromosphere	500 km	4×10^{15}	$1{\cdot}6\times10^{11}$
Top of chromosphere	14,500 km	2×10^{10}	$3{\cdot}5\times10^{8}$
Corona	>20,000 km		3×10^{8} to 10^{4}

According to Wildt (1947) the concentration of H-atoms in the chromsophere is given by the formula:

$$n_H=6{\cdot}76\times10^{15}.e^{-.92\times10^{-8}z\text{ (in cms)}}.$$

The hydrostatic density gradient, on the assumption of a temperature of 5000°K is $6{\cdot}9\times10^{-8}$cm^{-1}. It is, as if the weight of hydrogen has diminished to $\cdot92/6{\cdot}9=1/7{\cdot}5$ of its value or the temperature has risen to nearly 35000°K.

The electron-density at any point within the chromosphere can be obtained by extrapolation; assuming the exponential law to hold good, calculation shows that $n=n_e e^{-\beta z\text{ (in cms)}}$, where $\beta=\cdot43\times10^{-8}$cm^{-1}. The gradient is almost half that of H, as electrons are not obtained only from the ionisation of H but also from the metallic elements which are almost completely ionised.

The values of electron density in the corona (from 20,000 km upwards) are given by Bumbauch (1937). These are obtained from a thorough discussion of numerous measurements of the brightness of the corona, on the basis of K. Schwarzschild's hypothesis, generally accepted, that the continuous spectrum of the solar corona is due to photospheric light scattered by free electrons. Bumbauch's calculations (Unsöld, p. 440) extend from 20,000 km to nearly ten times the solar radius, and the density varies from 4×10^{8} to nearly 10^{5}/c.c. The figure for the top of the chromösphere (14,500 km) has been extrapolated by Wildt from Bumbauch's figure. Another calculation based on extrapolation of hydrogen densities, gives $n_e=2\times10^{11}$/c.c. at this height. Bumbauch's figures have recently been revised by Allen (1946) and van der Hulst (1947).

The magnetic field in the spot regions

Magnetic field of sun-spots has been systematically measured in the Mount Wilson Observatory since the great discovery by Hale in 1908, as part of the routine programme of the observatory and the results are available in publications of the observatory and in various reports.

For the measurement of the field, the iron line $\lambda=6173{\cdot}346$ Å, which is exceptionally sharp, has been generally used. As this is a line of intensity 5 on the Rowland scale, and in the chromosphere spectrum reaches only a height of 400 km, it can be taken that the fields measured refer to the lowest level of the spot.

But for the purposes of this paper, we require the value of the spot field at higher levels, and away from the axis of the field. There appears to be no systematic observation on this point, though Hale (1908) was aware of the necessity of such observations. He states:

"We have already seen that the strength of the field in spots apparently changes very rapidly along a solar radius, and is small at the upper level of the chromosphere."

We have given in Appendix the full passage from the original paper of Hale which suggests a programme for measurement of the magnetic field at various levels above the reversing layer of spots. To our knowledge these suggestions have not been worked out but they are worth a trial.

Two different and indirect methods, based on Mount Wilson measurement of the field over various distances from the centre of the umbra, are available. These are given below:—

(1) Broxon (1942) while observing the effect of the spot fields on cosmic rays suggested that the magnetic fields of unipolar spots may be fairly approximately represented by the field of a vertically placed magnet having a dipole moment $\mu=\sqrt{2}a\,H_o/8$ situated at a depth of $a/\sqrt{2}$ with the axis coinciding with the axis of the spot, where a=radius of the spot and H_o is the field strength at the spot centre. For complete mapping of such a field, let A'SA be the plane of the spot and MS the axis of the spot, with the

dipole at M. We chose the frame-work XYZ, with X-axis along SA, Z-axis along MSM′ and Y-axis perpendicular to the plane of the paper (Fig. 3). As the field is symmetrical

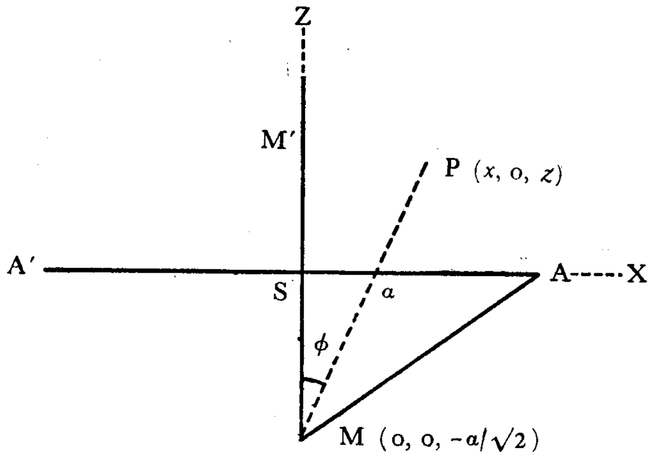

Fig. 3
The co-ordinate system for a circular spot (Broxon type)

about the axis let us find out the field at any point P (x,o,z) in the XZ plane. Let also $R=MP$ and $\phi=\angle PMS$ be the polar co-ordinates of P with M $(0,0,-a/\sqrt{2})$ as origin.

Then the potential Ω at P is given by

$$\left.\begin{aligned}
\Omega &= \frac{\mu\cos\phi}{R^2}=\mu\frac{(z+a/\sqrt{2})}{\{x^2+(z+a/\sqrt{2})^2\}^{\frac{3}{2}}}\\
H_R &= -2\frac{\mu\cos\phi}{R^3};\; H_x=\frac{3\sqrt{2}}{8}H_0\frac{\xi(\zeta+1/\sqrt{2})}{\{\xi^2+(\zeta+1/\sqrt{2})^2\}^{\frac{5}{2}}}\\
H_\phi &= -\frac{\mu\sin\phi}{R^3};\; H_z=\frac{H_0}{4\sqrt{2}}\cdot\frac{2(\zeta+1/\sqrt{2})^2-\xi^2}{\{\xi^2+(\zeta+1\sqrt{2})^2\}^{\frac{5}{2}}}\\
H &= \frac{H_0}{4\sqrt{2}}\cdot\frac{\sqrt{\zeta^2+4(\zeta+1/\sqrt{2})^2}}{\{\zeta^2+(\zeta+1/\sqrt{2})^2\}^2}=\frac{\mu}{R^3}\sqrt{1+3\cos^2\phi}
\end{aligned}\right\}\quad(3\cdot1)$$

with $x=\alpha\xi,\qquad z=a\zeta.$

Since in the above co-ordinate systems,

$$\left.\begin{aligned}
\tan\phi &= x/(z+a/\sqrt{2})=\frac{\xi\sqrt{2}}{(1+\zeta\sqrt{2})}\\
\text{we have}\quad \cot\psi &= H_z/H_x=\cot\phi-\frac{1}{3\sin\phi\cos\phi}
\end{aligned}\right\}\quad(3\cdot2)$$

Expression (3·1) gives the magnitude of the magnetic field at any point on or above the spot and to get field direction, we plot the curves representing lines of force given by

$$R=A\sin^2\phi,\qquad A=\text{arbitrary constant.}$$

To show how the field intensity varies with distance from the spot centre and with height, the quantity H/H_0 has been plotted (Fig. 4) against ξ for values of $\zeta=0$, ·1, ·5, ·6, 1·0. The topmost curve for $\zeta=0$ gives H at different points along the spot surface. At the edge of the spot H/H_0 becomes nearly 14% whereas it ought to be zero. The other curves for $\zeta=\cdot1$, ·3, ·6 and 1·0 give the field intensities at different points on planes parallel to the spot surface at heights of

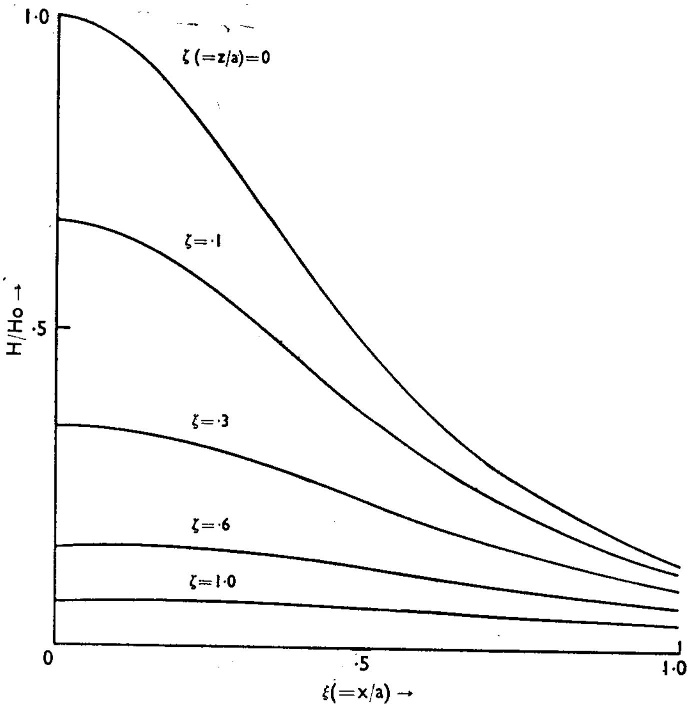

Fig. 4
Variation of the intensity of the magnetic field along and above the spot surface (Broxon type).

$\frac{1}{10}$, $\frac{3}{10}$, $\frac{6}{10}$ and unit times the spot radius. As we go higher and higher above the spot surface, the field becomes more or less constant over the entire plane as is evident from the lowest curve for $\zeta=1$, which is situated at a height equal to the spot radius.

Fig. 5 gives the lines of force at different points. The tangent to these curves at any point gives the direction of the magnetic field at that point. The thick line SA is the spot surface. From (3.2) we note that ϕ-constant lines are the lines of constant ψ. Values of ψ for certain different values of ϕ are represented by arrows in the diagram. $\psi=o$ is the line coinciding with the central line $\phi=o$. Gradually as ϕ increases, ψ also increases, and at the periphery $\psi=\pi/2$, showing that the z-component of the magnetic field is absent at the edges, *i.e.*, the field is perfectly horizontal at the edges.

(2) Chapman (1943) has given another approach to the problem. For finding out the spot fields at different points, he argues as follows:

"If the two ends, whether of the straight or semicircular magnet, are sufficiently far apart, we can calculate the magnetic field near each as if the others were not there. Suppose the surface density σ (of magnetic poles) over the end depends only on the distance r from the centre of the spot; and the magnetic intensity H and its inclination

ψ, will likewise depend only on r. I have calculated ψ for various relationship between σ and r in order to find out what distribution of σ would agree approximately with Nicholson's measurement of ψ for a sun-spot. The details

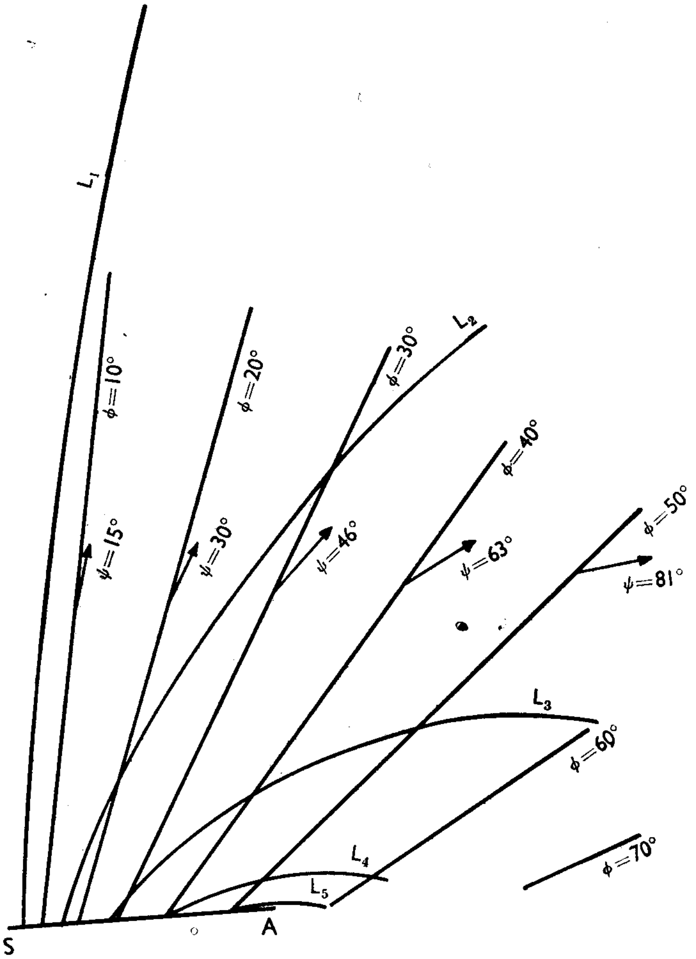

Fig. 5
Lines of force of the spot field according to Broxon.

are omitted here. I give only the results for the assumed variation of surface density,

$$\sigma = \frac{H_0}{2\pi}\left(1-\frac{r^2}{a^2}\right)^2. \qquad \ldots \qquad \ldots \quad (3.3)$$

Taking the same axes as before, the magnetic potential at a point P (x, o, z) is given by

$$\Omega = \frac{H_0 a}{2\pi}\int_0^1\int_0^{2\pi}(1-\rho^2)\frac{\rho d\rho d\phi}{\sqrt{\lambda^2-2\lambda\rho\cos\phi\cos\psi+\rho^2}}, \quad (3.4)$$

where $\lambda^2 = \dfrac{x^2+z^2}{a^2}$, $\tan\psi = \dfrac{x}{z}$, $r/a=\rho$.

This is an elliptic function in λ."

Since for our purpose of investigating the propagation of e.m. waves in the solar atmosphere, we are not much concerned with the actual variation of the field over the surface of the spot, but mainly with the order of magnitude of H on and above the spot and as our line of argument will be the same for different kinds of variations of the magnetic fields, we do not proceed further with Chapman's

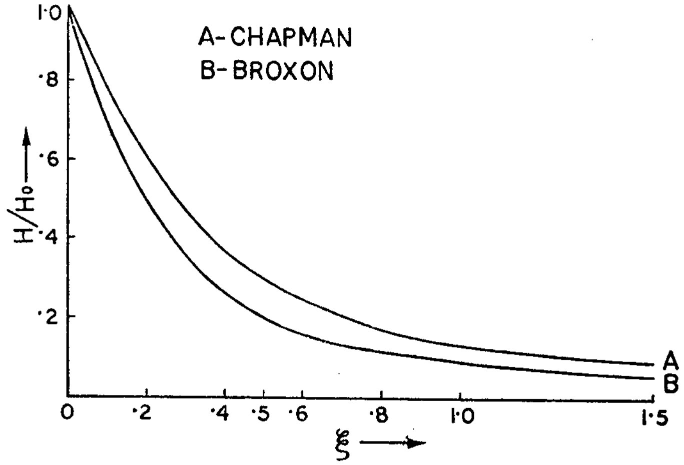

Fig. 6
Variation of the value of the magnetic field along the axis of a spot according to Chapman (A) and Broxon (B). ($\zeta=z/a$), z=distance along the axis, a=radius of spot.

assumptions, but stop here merely comparing the variation of H for the axial case for Chapman and Broxon fields. For the axial case expression (3.4) gives

$$H_z = \frac{H_0}{3}\cdot\{(8\zeta^4+12\zeta^2+3)-8\zeta(1+\zeta^2)^{\frac{3}{2}}\}. \qquad \ldots \quad (3.5)$$

Values of H/H_0 have been plotted against ζ for ξ=o (axial case) for Chapman fields (A) and Broxon fields (B) in Fig. 6. The general trend of the two curves is the same, but Chapman's values are somewhat higher. We find that in both the cases, for large spots of radius a=28,000 km, and H_0=3600 Γ, large fields of the order of 1000 Γ persist at heights of the order of 15,000 km, *i.e.*, the top of the chromosphere. For such spots the fields reduce to approximately 13 to 10% at 30,000 km.

4. Propagation of E.M. Waves

We are now in a position to discuss the actual problem of propagation of radio-waves through the solar atmosphere surrounding the spots, on the supposition that these waves have their origin deep within the spots, and are consequent on the physical processes taking place within the sun which give rise to spots.

As mentioned before, the propagation depends on: N, the electron density, H, the magnetic field, δ the damping coefficient=ν/p where ν is the number of collisions suffered by an electron per sec, p=pulsatance=$2\pi f$, θ the angle of propagation, which is the angle between the direction of propagation and the positive direction of lines of force.

We have discussed N and H, we shall now discuss the other quantities.

(*a*) The damping factor $\delta = \nu/p$.

In the reversing layer (500 kms), we have $T \simeq 6000°K$, n=concentration of atoms mostly hydrogen$=4\cdot10^{15}$/c.c., n_e=electron density$\simeq 10^{14}$ per c.c. The mean free path of electrons is, therefore, $L_e = 4\sqrt{2}\ L$, where L is the mean free path of heavy particles.

$$\text{Hence} \quad L_e = 4\sqrt{2}\frac{1}{\sqrt{2}\,\pi n\sigma^2} = \frac{4}{\pi.4\cdot10^{15}.10^{-16}} \simeq 3\text{ cms.}$$

$\bar{c}$=mean molecular velocity of electrons

$$= \sqrt{\frac{3kT}{m_e}} \simeq 5\cdot10^7 \text{ cms.}$$

Hence $\nu = \bar{c}/L_e \simeq 2.10^7\ \text{sec}^{-1}$.

Taking $\lambda = 2m, f = 1\cdot5.10^8, p = 2\pi f$

and $\delta = \nu/p \simeq 2.10^{-2}$.

The value of δ is thus seen to be very small even on the reversing layer. In the higher layers δ falls very rapidly and therefore it is justifiable to neglect damping altogether.

(*b*) The angle of propagation

Let the globe represent the sun, C its body centre and CE the direction of the Earth. E is the centre of the visible disc. Let S be the position of the centre of the umbra of the spot.

Then the direction of the spot axis, and of the central magnetic field of the spot is along CS. The angle ECS,

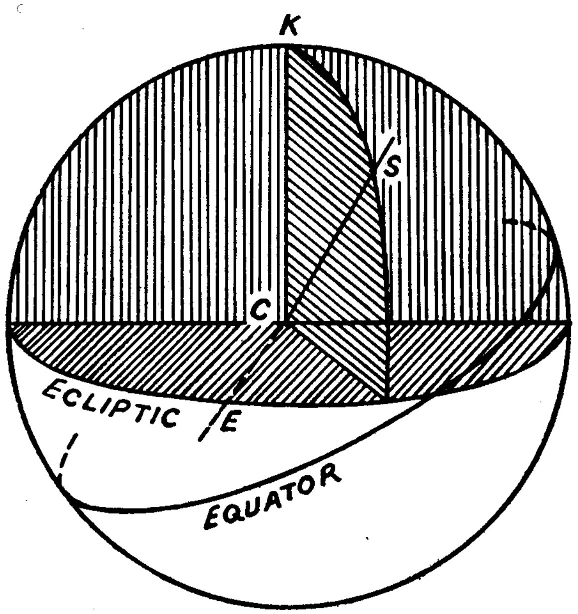

Fig. 7
Position of spot as seen from the earth. S-position of the spot on sun (after Smart, *Spherical Astronomy*).

which is generally denoted by ρ (but we shall denote it by χ as ρ has been used in a different sense), can be obtained by calculation from daily observation of the spot (*vide* Smart, *Spherical Astronomy*, p. 172). When the spot first appears on the east limb, $\chi=90°$, but there is hardly any observation available at the first appearance of the spot. Observations generally start after a day, when $\chi \simeq 75°$, then gradually χ diminishes day by day, but even during central meridian passage χ is generally different from zero, except on very rare occasions. After central meridian passage, χ again increases day by day till it is 90° when the spot disappears on the west limb.

Choice of Axes—The plane ECS may be taken as the (XZ) plane, CE being the axis of Z, the axis of X being perpendicular to CE in the plane ECS. The axis of Y is perpendicular to the plane of ECS (Fig. 8).

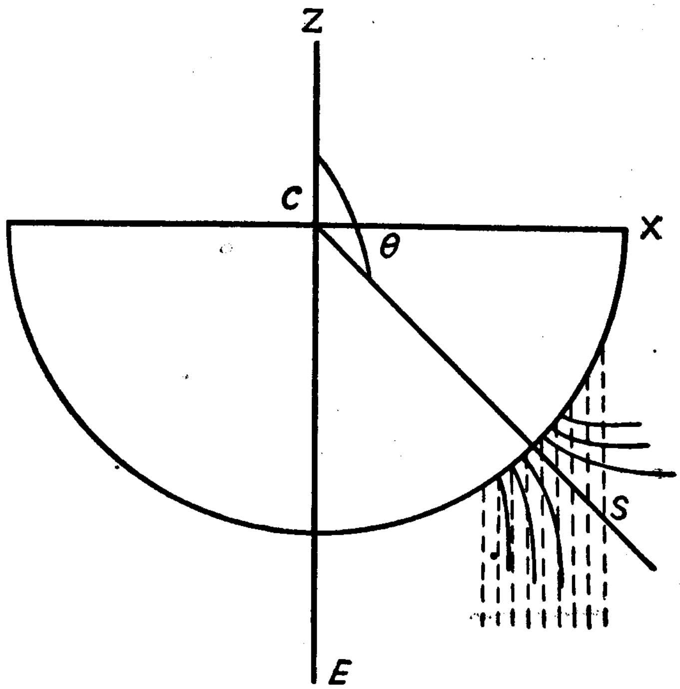

Fig. 8
Plane through Earth, Sun's centre and spot centre, showing how angle of propagation of waves varies in the neighbourhood of spot.

Angle of propagation—(See Fig. 8) According to our assumption, the source of radio-waves is the funnel shaped volume about S, with CS as axis. The volume comprises the whole conical space of the solar atmosphere with the spot as the base, which is traversed by the lines of force. If the lines of force were everywhere parallel to CS, $\theta=\chi$ if the polarity of the spot were R (lines of force away from the surface) and $= |\pi-\chi|$ if the polarity were V (lines of force passing into the surface).

But such is not the case, as the lines of force curve away from the direction of CS, as discussed in § 3. Let us find out the angle θ for any point P $(a\xi, a\eta, a\zeta)$ within the spot.

This is the angle between CE, and the tangent to the lines of force at P. These make an angle ψ defined by (3.2) with CS, at an azimuth $\alpha=\tan^{-1}\eta/\xi$.

θ varies between $\chi+\psi$, $\chi-\psi$ as α varies between o and 2π.

The equations of propagation of the radio-waves through the spot atmosphere can now be written, using the symbols of the previous paper. They are:

$$\left.\begin{aligned}&\frac{d^2}{du^2}(E_x+iF_1E_y)+q_0^2(E_x+iF_1E_y)=o\\&\qquad\text{for the o-wave,}\\ \text{and}\ &\frac{d^2}{du^2}(E_x+iF_2E_y)+q^2_e(E_x+iF_2E_y)=o\\&\qquad\text{for the e-wave,}\end{aligned}\right\}\quad(4.1)$$

where $u=2\pi z/\lambda$, $F_1=\sqrt{1+g^2}-|g|$, $F=\sqrt{1+g^2}+|g|$,

$$g=\omega\sin 2\theta/2\cos\theta\,(r-1)$$

$$\left.\begin{aligned}\text{and}\quad q_0^2&=1-\frac{r}{1+\omega\sin\theta\,(\sqrt{1+g^2}-|g|)}\ \text{for } r<1\\&=1-\frac{r}{1-\omega\sin\theta\,(\sqrt{1+g^2}-|g|)}\ \text{for }\ >1\end{aligned}\right\}\ (4.2)$$

$$\left.\begin{aligned}q_e^2&=1-\frac{r}{1-\omega\sin\theta\,(\sqrt{1+g^2}+|g|)}\ \text{for } r<1\\&=1-\frac{r}{1+\omega\sin\theta\,(\sqrt{1+g^2}+|g|)}\ \text{for } r>1\end{aligned}\right\}\ (4.8)$$

The waves will be able to come out as long as q_o^2 and q_e^2 are $>$ o. Let us first investigate the probable values of q_o^2 and q_e^2 in the spot region. They vary from point to point and we give the values of the relevant quantities for $\lambda=1{\cdot}5m$, $f=2{\cdot}10^8$/sec.

We have $\quad N_0=\dfrac{mp^2}{4\pi e^2}=5.10^8,\quad \omega=1.4\times10^{-2}$ H.

Reference to paper (I)* shows that whatever the values of ω and θ, q_0^2 always tends to zero at $r=1$. This happens at a height of 13,500 km. So all o-waves coming from below this height are turned back.

As regards the e-wave, the transparency extends to the point where $r=1+\omega$. For the present case, we assume that N, the ion-concentration varies according to the law $N=N_0e^{-\beta z}$ (*vide* Table II). The e-wave can leak out from levels where $r\leq1+\omega$. For the present case this happens at $z>3900$ kms. So the whole column, above a height of 3900 km, would be transparent to the e-waves of wavelength 2m (Fig. 9).

It is easy to see that the shorter waves would be able to come from correspondingly deeper layers.

Polarisation and sense of polarisation of the waves—Detailed description of measurements of polarisation is not available; but in all the short notes which have been published it is stated that the polarisation is circular, even when the spots are close to the limb. Some state that the sense of polarisation changes on crossing the meridian (Martyn, 1946), while others do not find any such change (Bowen, 1946).

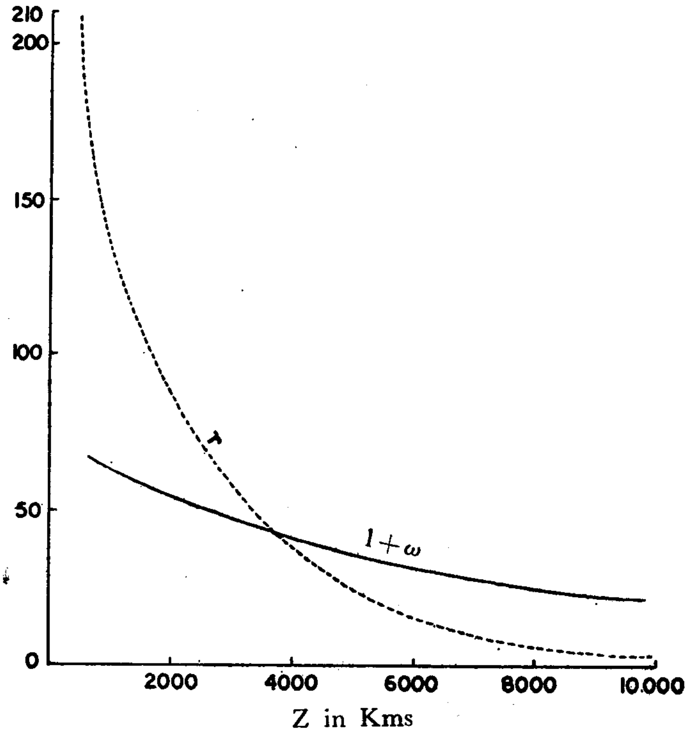

Fig. 9

The dotted curve shows the variation of $r=\dfrac{4\pi Ne^2}{mp^2}$ with height over the spot for $\lambda=2m$. The other curve gives $(1+\omega)$ as a function of height.

It is only possible to make some general statements about polarisation from our theory. We have $H_y/H_x=\sqrt{1+g^2}-|g|$ both for the o-wave, and the e-wave. Since g is changing at every point, so polarisation of the wave, as it is propagated through the solar atmosphere, is constantly changing. A large $|g|$ means "linear polarisation", a small $|g|$ means circular polarisation. Inside the chromosphere $|g|$ is large, hence the waves will be "linearly polarised". But as the waves pass through the corona both ω and r tend to vanish and $\omega\sin^2\theta/2(r-1)\cos\theta$ is a very small quantity, *e.g.*, it is ·066 for $\theta=120°$ at a height of 10^6 km above the photosphere and hence $\rho\to1$, and the waves become circular. Table II gives the values of r and ω for the spot atmosphere at various heights.

The sense of polarisation would depend upon the value of θ.

The waves have to pass through the Earth's ionosphere before we receive them. But in this case

$$r=\frac{4\pi Ne^2}{mp^2}=\frac{N}{5{\cdot}10^9}\simeq10^{-2},\quad \omega=f_h/f=\cdot7\times10^{-2},$$

$$g=\cdot0035\cos^2\theta\simeq0.$$

*Saha, Banerjea and Guha, 1947, *Ind. J. Phys.*, 21, 181.

TABLE II

Level above photosphere in km	N_e	(r)		H (Broxon)	ω
		Normal Atmos-phere	Spot Atmos-phere		
0	$1.6.10^{11}$ cc.	320	176	3600Γ	50·4
500	$1.29.10^{11}$,,	258	141.9	3330 ,,	46.6
1,000	$1.04.10^{11}$,,	208	114.4	3063 ,,	42.9
1,500	$8.37.10^{10}$,,	167	91.85	2885 ,,	40.39
2,000	$6.80.10^{10}$,,	136	74.80	2720 ,,	38.8
3,000	$4.42.10^{10}$,,	88	48.40	2364 ,,	35.10
5,000	$1.86.10^{10}$,,	37	20·35	1830 ,,	25·62
8,000	$5.12.10^{8}$,,	10	5·50	1296 ,,	18·14
10,000	$2.14.10^{8}$,,	5	2·75	1068 ,,	15·00
14,500*	3.10^{8} ,,	·66	·53	720 ,,	10·80
20,000[1]	$2.9.10^{8}$,,	·64	·35	18 ,,	8·65
30,000	$2.5.10^{8}$,,	·50	·28	360 ,,	5·04
40,000	$2.1.10^{8}$,,	·46	·25	187 ,,	2·62
50,000	$1.7.10^{8}$,,	·34	·19	104 ,,	1·46
100,000	$.85.10^{8}$,,	·20	·11	14 ,,	·20
1000,000	$.11.10^{6}$,,	·00	·00	0	0

*This value is not extrapolated from the currve $n=n_0e^{-z}$, but is an experimental value obtained from coronal data.

(1) These values are taken from extrapolation of the values of electron concentration in the corona obtained from scattering of light by free electrons given by Bambauch (See Physik der Stern-atmosphäre, page 440, 1938) and recently corrected by Allen (1946), and Van de Hulst (1947).

Hence $F_1=F_2=1$, *i.e.*, the magnetic field is too weak to cause any change in the polarisation of the micro-waves from the sun.

5. General Deductions

It is not possible to penetrate more deeply into the phenomena with the aid of our theory, unless the observations become more refined and the technique is improved, but some general statements can be made.

The equations of propagation show that the escape of waves and the sense of their circular polarisation are extremely sensitive to the value of the spot field and the concentration of electrons. In our treatment we have taken the source to be single spot (α-type), but observations have shown that majority of spots are bipolar, consisting of a leader and a follower with opposite polarity, whose field-strength undergoes rapid changes with the age. Let us quote from Chapman (1943):

"Sun-spot Dipole Moments.—Hale found that most sun-spot groups at some stage in their life-history, show bipolarity that is to say, they include two spots, or two regions or groups of spots, of opposite magnetic polarity, one red (R) and the other violet (V). At other stages, however, there is often only one spot, of like polarity over all its area; this is called a unipolar spot.

A typical spot group begins as two small spots, or two groups of spots, of opposite polarity, nearly in the same latitude, and 3° or 4° apart in longitude. The two principal spots grow rapidly and separate in longitude to a distance of 10° or more; the rear spot attains its maximum area in 3 or 4 days; the leader attains a larger maximum area in 7, 8 or 9 days, many smaller spots develop within the group, mostly near the two main ones. The dipole moment of the group is greatest when the spots are largest and furthest apart.

Consider a notable case, that of the great bipolar group M. W. 6725, 8° S, consisting of two great spots each of radius 20,000 km, about 14° apart, together with many smaller spots between; for this group $H=3900\Gamma$; considering only the two large spots, the dipole moment is 5×10^{16} km^3, *i.e.*, 60,000 times as great as the Earth's dipole moment, and about 1/170 of that of the sun. The sun-spot dipole moment is of course horizontal, and nearly parallel to the sun's equator, though the leading spot is generally somewhat nearer to the equator than is the follower. At the equator the dipole axis is along the equator, and according to W. Brunner its inclination to the equator increases with latitude to about 16° at 30° latitude."

Many of the spots whose radio emission had been under investigation during the last and the present year, appear to have been of this type.

In such cases, each one of the component spots with different polarity will emit microwaves of different intensity and with circular polarisation of opposite sense. Though the waves from each spot may be co-herent, the waves from the two spots may not be co-herent, like light from two distinct sources. This appears to explain in a general way the observations of Ryle and Vonberg, Martyn, and of Appleton and Hey. But it is desirable that radio observations be coupled with simultaneous observations of the polarity and field strength of the component spots.

When the spot group is of the γ-type (multipolar and without demarcation) the polarisation may be random.

Sudden changes in intensity.—Sudden changes in intensity are expected when H and N change suddenly. This happens during flares which, as Geovanelli (1945) has shown, are connected with the γ-type of spots, where the field strength undergoes rapid changes. This may affect microwave emission in two ways, (1) sudden increase in the value of H, will allow the e-component of the microwave to come from deeper layers and vice versa, (2) if the physical mechanism of emission given out by the senior author (Saha, 1946) be correct, sudden fluctuations of magnetic field will induce more copious emission of microwaves. But this point requires further investigation.

Appendix 1

The line $\lambda=6173.346$ is the weakest member of a d^7p^5D—d^7p^5P multiplet of Fe. Its position is given in table mentioned below:

The multiplet is d^7s^5D—d^7s^5P according to Russell. The excitation potential of the $^5D_0$ level is 4.21 volts ($\nu=34121$ cm^{-1}). The other lines of the multiplet, notably 6137.005, and 6151.630 etc, are also sometimes used for measurement of the sun-spot fields.

The theoretical Zeemann-pattern of 6173.32 is given by $\Delta\nu=\pm\, a(o, 5/2)$.

This is also experimentally verified. The Mount Wilson values of magnetic field are obtained from comparison of laboratory data with observational data, and are therefore independent of particular Zeemann-pattern.

	$^5D_0$	$^5D_1$	$^5D_2$	$^5D_3$	$^5D_4$
$^5P_1$	1 6173·346 16194·19 (400)	5 6213·44 16089·79 (500)	5 6297·83 15874·16 (500)		
$^5P_2$		1 6137·005 16290·09 (200)	6 6219·190 16074·59 (500)	10 6335·34 15780·11 (500)	
$^5P_3$			1 6151·630 16251·35 (350)	6 6265·145 15956·92 (500)	10 6430·86 15543·73 (800)

Hale's suggestion for measurement of magnetic field in the higher layers of the spot

"On considerations it will be seen, however, that the separation of the doublets must depend, in some degree on the distribution of the absorbing vapor in the solar atmosphere, and on the coefficient of absorption of the particular line employed. A striking instance of this kind, affecting lines of the same series, is illustrated in the case of hydrogen, described in a previous paper.[1] Although the H_α line extends to the upper part of the chromosphere and prominences, the mean level represented by its absorption is much lower than that given by H_α. The consequence is that H_α enables us to photograph the solar vortices, the characteristic stream lines of which do not appear at the lower H_δ level. Similarly, if the intensity of a given titanium line falls off rapidly, the level represented by this line may be comparatively low. If, on the other hand, its intensity curve is of such a form as to indicate that the absorption at higher elevations plays an important part, the mean level represented by the line may be considerably higher than in the previous case. To settle this question we must know:

(1) The range of elevation in the spot of the vapors of iron, titanium, and other elements;

(2) The intensities of the lines of these elements at different levels;

(3) The rate at which the strength of the field decreases upward.

In the absence of information regarding the first two points, we may enquire as to the probable relative behavior of titanium, iron, and other elements if the distribution of the vapors at different levels were the same as in the chromosphere. From a discussion of a large number of photographs of the flash spectrum, made by different observers at several eclipses, Jewell has compiled a table showing the heights above the sun's limb attained by various lines in the blue and violet.[2] The heights for titanium range from 100 miles (160 km) for 4466·0 to 3500 miles (5640 km) for 4466·7, while certain strong enhanced lines in the ultra-violet reach elevations of 6000 or 8000 miles (9660 or 12,880 km). For iron the minimum height is 200 miles (320 km) for 4482·4 and the maximum 1000 miles (1610 km) for 4584·0. Chromium ranges from 100 miles for 4280·2 to 1200 miles (1930 km) for 4275·0; manganese from '100 miles or more' for 4451·8 to '800 miles (1290 km) or more' for 4030·9; vanadium from 100 miles for 4390·1 to 200 miles for 4379·4. It thus appears that the range in level represented by the titanium lines is much greater than for the lines of iron, chromium, manganese, and vanadium. If the vapors were similarly distributed in spots, the maximum strength of field indicated by the titanium lines should therefore correspond with the maximum value for iron, but some titanium lines, produced by absorption at higher mean levels, should give lower field strength. Chromium should agree more nearly with iron. Vanadium, if the less refrangible lines reach no greater elevations, should give closely accordant (maximum) values for the field strength. It will perhaps be possible, with the aid of the 30 feet spectrograph, to determine the relative levels in the chromosphere attained by most of the lines in question, but it is a much more difficult matter to do this for sun-spots. I hope, however, that our new spectroheliograph of 30 feet focal length may throw some light on this subject."

[1] Solar Vortices.

[2] "Total Solar Eclipses of May, 28, 1900, and May 17, 1901." Publications of the U.S. Naval Observatory Series, Vol. IV, Appendix 1.

It is evident that these considerations will have no bearing on the present problem unless the field strength decreases very rapidly upward in spots. That this probably occurs is shown by the fact that the D-lines of sodium and the b-lines of magnesium are usually but slightly affected in the spot spectrum,[3] and are displaced through a very small distance when the Nicol is rotated. Thus, at the level represented by these lines, which attain elevations in the chromosphere probably not exceeding 5000 miles, the field strength is reduced to a small fraction of its maximum value.

ACKNOWLEDGMENT

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[3] Except for the strengthening of the wings, which may be produced by some cause other than a magnetic field.

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84. NOTE ON DIRAC'S THEORY OF MAGNETIC POLES

(*Phys. Rev.*, **95**, 1968, 1949)

In a note bearing the above heading, Professor H. A. Wilson[1] has described a simple method for finding out the value of Dirac's free magnetic poles. I may point out that this method was described by me nearly thirteen years ago[2] in a paper "On the origin of mass in neutrons and protons." I may just quote the result:

"It was Dirac who first showed that quantum mechanics demands the existence of free magnetic poles, having the pole strength (or magnetic charge) $ch/4\pi e=e/2\alpha$, where α=Sommerfeld fine-structure constant. Recently, the present author deduced the existence of free magnetic poles from very simple considerations. If we take a point charge e at A and a magnetic pole μ at B, classical electrodynamics tells us that the angular momentum of the system about

$$A \text{———} B$$

the line AB is just $e\mu/c$. Hence following the quantum logic, if we put this$=\frac{1}{2}\cdot h/2\pi$, the fundamental unit of angular momentum, we have $\mu=ch/4\pi e=e/2\alpha$ which is just the result obtained by Dirac."

[1] H. A. Wilson, Phys. Rev. **75**, 308 (1949).
[2] Ind. J. Phys. **10**, 145 (1936).