10. ON SELECTIVE RADIATION PRESSURE AND THE RADIATIVE EQUILIBRIUM OF THE SOLAR ATMOSPHERE

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§1

In a paper recently communicated to the Astrophysical Journal, an attempt has been made to prove that the quantum theory affords a basis for the existence of radiation-pressure on atoms and molecules.[1] It is well-known that according to the older continuous theory, the pressure of light is evanescent, on obstacles of the atomic or molecular size. But this conclusion is contrary not only to the requirements of many astrophysical data, but also to the experimental results of Lebedew.[2] In some recent communications to the M. N. R. A. S., and the Astrophysical Journal, Professor Eddington[3] has developed a very elegant theory on the "Radiative Equilibrium in the interior of stars," and has successfully explained many of the observational results about the evolution of stars discovered by Russell,[4] Hertzsprung and others. The theory of Eddington is based on the assumption of the existence of radiation-pressure on atoms. We may just quote his own words.[5]

"As there seems to be a rather widespread impression that gases are not subject to radiation-pressure, it may be advisable to state the theory briefly. The pressure is simply a consequence of absorption or scattering. A beam of radiation carries a certain forward-momentum proportional to its intensity, after passing through a sheet of absorbing medium, a weaker beam emerges carrying proportionately less momentum: the difference of incident and emergent momentum is retained by the medium and constitutes the pressure. The medium, in fact, absorbs the momentum of the beams in the same proportion as it absorbs the energy. The calculations of radiation-pressure on small solid particles are simply calculations of absorption and scattering by these particles; it is not possible to apply such methods to atoms and molecules, which absorb by some internal mechanism. But the relation between absorption and pressure is a perfectly general one, depending only on the conservation of momentum."

In the paper mentioned above, I have tried to prove that the existence of radiation-pressure on gaseous atoms follows as an easy deduction from modern theories of emission and absorption. *It has also been suggested, that the action of light pressure is selective.* Let us consider in greater detail what is meant by this term. Suppose a continuous spectrum from a bright back-ground passes through a layer of gas. Then the gaseous atoms will be acted upon by only those pulses of light in the continuous spectrum, which the gas is itself capable of emitting and absorbing. If, for example, the gas be composed of Sodium atoms; then only radiant energy contained in the spectral regions about the D_1, D_2-lines and sometimes the other lines of the principal series will act upon the Na-atoms. The remaining part of the continuous light will be without action on the Na-atoms (more later on). Regarded from this point of view, the theory may properly be styled *as the theory of Selective Radiation-pressure.* The object of the present paper is to show that this theory taken along with the modern theories of atomic structure, is capable of explaining many problems in solar and stellar physics, particularly the problems of the radiative equilibrium of the solar atmosphere.

The range of phenomena covered by the works of Eddington is entirely different from ours. Eddington has considered the aggregate effect of light pressure in the interior of stars, *i.e.*, the region where the gaseous atoms are under such a high pressure that they no longer emit, or absorb waves of a particular type, but waves of all lengths. The mass of gas behaves very much like a continuous body, and the radiation pressure is just the same as that given by the continuous theory, for only the aggregate effect is considered. But the class of phenomena which will be discussed in this article refers to the atmospheres of luminous bodies, where the pressure is so low that the gaseous atoms are capable of emitting their own characteristic radiation. The general problem of radiative equilibrium has already been discussed by Schwarzschild.[6] Before taking up these discussions, I shall give a brief sketch of the problems before us.

§ 2. *The Problems of the Solar Atmosphere.*

It is well known that the customary division of the sun into the photosphere, the reversing layer, and the

[1] M. N. Saha, *Astrophysical Journal*, 1919.
[2] Lebedew—*Annalen der Physik*, Vol. 32, p. 411 (1910).
[3] Eddington—M. N. R. A. S., Vol. 77, p. 25; *Astrophysical Journal*, Vol. 46.
[4] Russell—*Nature*, Vol. 93.
[5] Eddington—M. N. R. A. S., Vol. 79, p. 28, *et seq.*

[6] Schwarzschild—*Gött Nachrichten*, 1906, p. 41.

chromosphere is based upon the results of spectroscopic observations alone. The correlation of these data to actual physical conditions of temperature, pressure, and distribution of mass is a rather tough problem, and one may find in this connection views which are poles asunder. When we speak of the photosphere, we tacitly assume it to be a sharply defined body like a piece of white-hot iron. The reversing layer and the chromospheres are assumed to be similar to the lower and the upper layers of our own atmosphere. In the discussion which follows, we stick to the view that the photosphere has a sharp, though gaseous boundary, and radiates like a black body at a temperature of 7600°K.[7]

The problems may be briefly grouped under the following headings:—

(1) The enormous distance to which the atmosphere extends over the photospheric disc.

(2) The anomalous distribution of elements in the solar atmosphere.

(3) The radiating power of the different parts of the solar disc.

(4) Unsteady phenomena, *viz.*, Spots and prominences.

The main points of the first problem are very well-known. The value of the gravitational acceleration on the disc of the sun is 27.7 times the value of gravity on the earth, while the temperature is nearly 6000°K. The radial gradient of the density (*i.e.*, rate of decrement of mass per unit volume with height) should therefore be very large, no matter in whatsoever way the temperature may vary in the atmosphere. Let us consider in succession, the three theories of equilibrium.[8]

(1) The Isothermal Equilibrium:—Temperature is supposed to be uniform throughout the atmosphere and equal to 6000°K.

Let N_0=number of atoms of a certain element per unit volume just over the photospheric disc, and N=corresponding number at a height z. Then

$$\ln\left(\frac{N}{N_0}\right)=\frac{Mgz}{k\theta}=\frac{mgz}{R\theta} \tag{1}$$

Where R=gas constant$=8.30\times10^7$, k=Boltzmann's gas constant, M=weight of an atom, m=weight of a gram-atom, θ=Absolute temperature.

Taking $\theta=6000$°K, $g=27.7\times981$ *cm.* the logarithmic decrement is $\frac{27.7}{6000}\Big/\frac{1}{300}=\frac{27}{20}$ times the value of the corresponding quantity on the earth. In the case of the Hydrogen atom the density reduces to $\frac{1}{10}$ of its value for a height of 393 km.; for Calcium the corresponding height is only 10 kms. At a height of 1000 kms., the density of Calcium $\rho=\rho_o\left(10^{-100}\right)$. ρ_o=density on the photosphere, *i.e.*, there will be found scarcely one molecule in the whole volume over the disc. Generally $\rho=\rho_0\times10^{-\frac{mz}{393}}$, m=atomic weight, z=height in kilometres.

(2) Let us suppose that the temperature does not remain constant, but vary according to the law of adiabatic compression and rarefaction. This will be the case when the atmosphere is the seat of very violent, and turbulent motion, as is the case, to a smaller extent, in the lower atmosphere of the earth (the troposphere). Probably for the lower reversing layer, and the photosphere, the adiabatic law holds good.

The equation of state is

$$\frac{P}{P_o}=\left(\frac{\rho}{\rho_o}\right)^{\gamma}=\left(\frac{\theta}{\theta_o}\right)^{\frac{\gamma}{\gamma-1}} \tag{2}$$

γ=ratio between the specific heats; from (1), we have

$$\theta-\theta_o=\frac{\gamma-1}{\gamma}\,\frac{Mg}{k}(z-z_o)$$

Taking $\gamma=1.66$, the temperatures should fall by 1°C, for every 70 metres of Hydrogen. The atmosphere has a definite limit (t=P=ρ=0). This limit is 420 km. for Hydrogen and 10.5 for Calcium.

(3) The adiabatic equilibrium can take place only in the regions of violent motion. In the upper regions, convection currents are almost absent (except for such occasional outbursts known as the prominences), and the exchange of heat can take place only by radiation.

The theory of radiative equilibrium is due to Schwarzschild. It is based upon Kirchoff's laws of emission and absorption, and primarily deals with the problem of variation of temperature with height and the darkening of the solar disc towards the edge. It is noteworthy that Schwarzschild does not attempt to account for either the great extension of the solar atmosphere or the anomalous distribution of elements. This is due to the fact that following the old continuous theory he regards the atoms and molecules as infinitely small fragments of black body, and finds the radiation pressure to be evanescent on them.[9]

The arguments of Schwarzschild may be greatly simplified by following a method due to Fabry[10]. Fabry has shown that at any point of free space, traversed by radiation, the motion of temperature has no meaning in itself. Bodies having different physical properties will rise to different temperatures, varying within very wide limits[11].

Suppose we have a spherical black body at a height 'z' over the photospheric disc, and let us suppose that the solar atmosphere has been somehow lifted up. Equilibrium will be established when the heat radiated by the body will be equivalent to the heat received. Let θ_o=temperature

[7] Feliz Biscoe, *Astrophysical Journal*, Vol. 46, p. 355 (1917)

[8] Most of this discussion is taken from Schwarzschild's paper referred to above, and a paper by D. Brunt, M. N. R.A. S., Vol. 72.

[9] Schwarzschild:—Münchener Berichte—1909.

[10] Fabry, *Astrophysical Journal*, Vol. 47.

[11] The maximum temperature must, however, be less than the temperature of the radiating body.

of the disc, θ=temperature of the body. Then according to Stefan's law we have

$$\theta=\theta_o\left(\frac{\Omega}{4\pi}\right)^{\frac{1}{4}},$$

where Ω=solid angle subtended by the body at the sun,

$$=2\pi\left(1-\frac{2z}{r}\right) \text{ approximately,}$$

$$=2\pi,$$

when z is very small. The value of z being generally very small in comparison to r, the radius (the maximum value of $\frac{z}{r}=\frac{14000}{7\times10^5}=\frac{1}{50}$ in the case of the H—K lines), the above assumption is quite justified.

Let us now see how far the actual conditions in the sun differ from these assumptions. F. Biscoe[12] has recently discussed the vast amount of data collected by the Smithsonian Astrophysical Laboratory on the distribution of intensity for different wave-lengths from different parts of the solar disc. He finds that the photosphere radiates like a black body at a temperature of 7500°K. The discussion of course, refers to those parts of the solar spectrum which contain no strong absorption lines. To account for the Fraunhofer lines, we have to assume that the photosphere is bounded by concentric spherical layers of radiant gas,• the temperature gradually decreasing with height. These gases pick out and absorb from the continuous spectrum those pulses which they themselves are capable of emitting, so that these regions of absorption appear relatively dark. The intensity of the dark regions corresponds to the intensity of the outermost layers of the emitting and absorbing gas.

If we suppose that a small spherical black body is placed at a point within the solar atmosphere, the radiation received by it is composed of (1) the radiation from the photosphere attenuated by scattering and general absorption; (2) radiation from the radiant gas of the solar atmosphere interior to the body; (3) radiation from the radiant gas exterior to the body.

Schwarzschild[13] finds that the combined radiation from the first two causes may be put equivalent to $\sigma\theta'^4_o$ where θ'_o is called the effective temperature. The radiation from the layers exterior to the body $=\int_{-\infty}^{z} \mathrm{E}\rho\epsilon dz$, where E= volume-emission per unit mass, ϵ=absorption per unit mass.

The integral is equal to

$$=\sigma\theta'^4_o\,\frac{\epsilon}{g}\int_{-\infty}^{z}\rho g dz=\sigma\theta'^4_o\,\frac{p}{g}.$$

Hence $2\sigma\theta^4=\sigma\theta'^4_o\left[1+\frac{\epsilon}{g}p\right]$, σ=Stefan-constant.

$$\theta=\frac{\theta'_o}{2^{1/4}}\left(1+\frac{\epsilon}{g}p\right)^{1/4}=\tau\;\left(1+\frac{\epsilon}{g}p\right)^{1/4}$$

[12]F. Biscoe—*Astro. Journal*, Vol. 46 (1917).
[13] Schwarzschild—Loc. cit.

These assumptions and calculations are very rough. With the aid of this relation between temperature, and pressure, we can easily calculate the density. We have

$$z=\frac{k}{\mathrm{M}g}\int\frac{4\theta^4 d\theta}{\theta^4-\tau^4}=\frac{k}{\mathrm{M}g}\left[4\frac{\theta}{\tau}+\log\frac{\theta-\tau}{\theta+\tau}-2\tan^{-1}\frac{\theta}{\tau}+\text{const}\right],$$

$$\text{and } \rho=\frac{p}{\mathrm{R}\theta}=\frac{g}{\mathrm{R}\epsilon}\left(\frac{\theta^4-\tau^4}{\theta\tau^4}\right)$$

Brunt[14] has calculated the variation of temperature and density with height on the basis of the above formulae; the figures are reproduced below:—

z in *km.*	z in angular measure	θ	$\theta-\tau$	ρ.
∞	∞	5050°K$=\tau$	0	0
2.1×10^5	5′	,,	0	10^{-1636}
4.2×10^4	1′	,,	0	10^{-3351}
3.5×10^4	·5″	,,	0	10^{-34}
3.97×10^2	·5″	5051	1	3×10^{-6}
2.75×10^2	·4″	5060	10	3×10^{-5}
0		6000	950	3×10^{-3}

This table shows that Schwarzschild's theory leads to an incomprehensible atmosphere with uniform temperature. At a height of 3500 km., where according to the evidence of flash-spectra, lines of H_2, Ca, Fe... are quite plentiful, we obtain a density of 10^{-34}, or only one molecule of Hydrogen in 10^{15} cc of the gas. Schwarzschild's theory therefore fails to account for the great extension of the solar atmosphere.

For a more rigorous application of these formulae we require more precise information about the pressure, and rate of variation of pressure in the photospheric level, and in the reversing layer, as well as the distribution of elements in these levels (*i.e.*, number of atoms of a particular kind per unit volume). The level attained by an element is generally obtained from the length of the arc of the characteristic line of the element in the flash spectrum[15]. Now this level is generally different for different lines (*vide* Sec. 4) of the same element. Apart from this difficulty, we have to remember that no conclusion is possible about the minimum radiation density[16] of an element from the chromospheric level alone, unless our knowledge is supplemented by auxiliary laboratory experiments. These points will be further considered in the next section. But from what has been said, it is quite clear that none of the three theories sketched above can account for the observed extension of the solar atmosphere.

It seems to be the general opinion of the astrophysicists that there is some sort of repulsive force on the sun which

[14] Monthly Notices, R. A. S., Vol. 73.
[15] Mitchell—*Astrophysical Journal* Vol. 38, December.
[16]Minimum radiation-density—The minimum number of radiating particles per unit volume which is required for affecting the photographic plate within a certain interval.

neutralizes the greater part of gravity. It is also supposed that the prominences, particularly, the eruptive ones, are due to some cause which enables "this force of levity" to overcome largely, and in the case of eruptive ones, to preponderate over the force of gravitational attraction on the Sun. We quote the opinions of two distinguished astrophysicists in this connection.

"The rising prominences in some cases observed at Kodaikanal move with an accelerating velocity to be driven entirely away into space by a force opposed to gravity."

(Evershed, *Astrophysical Journal*, Vol. 28, p. 79).

Professor Julius, whose views about the interpretation of astrophysical phenomena are so radically different from those of Evershed, writes in a similar strain:—(*Astrophysical Journal*, Vol. 38, p. 132).

"From the astrophysical point of view, one of the questions material to the explanation of solar phenomena is: What can be presumed about the *general radial gradient of the density in the layers* we are concerned with ?

The subject has been treated very fully and ingeniously on the basis of thermodynamics by Emden in his book 'The Gas-Kugeln'. Emden arrives at the conclusion already mentioned above that the fall of the density must be extremely rapid, but the inference is open to doubt, for in his calculations, Emden presupposes gravitation to be the only radial force acting on solar matter. According to the present state of our physical knowledge, however, we decidedly must admit that *on the sun, gravitation is counteracted by the pressure of radiation*, and by the emission of electrons, and perhaps of other charged particles.

Basing on purely theoretical grounds an estimate of the intensity of that counteraction would, for the present, be as rash as denying its existence; but some evidence in favour of its essentiality is given by the fact that many solar phenomena are much better understood if we assume a radial gradient many times smaller than the one that would correspond to gravitational conditions only. In this connection we call attention to the puzzling properties of quiescent, hovering prominences. Father Fenyi, in his interesting discussion of the long series of prominence observations made at the Haynald Observatory, Kalosca, is very positive in his assertion that "several well-established facts concerning quiet prominences can be accounted for only if in the solar atmosphere gravity is reduced by *certain repulsive forces to a small fraction* (*something of the order of 1/80*) of its commonly accepted value."

In the papers[17] already referred to, Eddington has calculated the compensation of gravity due to radiation-pressure in the interior of a star of given mass, average density, and luminosity. If G denotes, the acceleration due to gravitational attraction alone, and γG the repulsive acceleration due to radiation-pressure, the effective value of gravity reduces to $(1-\gamma)$ G. For a star of the size and mass of the sun, Eddington calculates that $\gamma=.106$, for a molecule of weight 2, and .943 for a molecule of weight 54, so that the effective values of gravity in the interior are .894G for H_2, and .057G for a molecule of weight 54. But as we shall see later on, these calculations apply only to the interior of the star. In the atmosphere, quite a different procedure is to be adopted.

The origin of the "force of levity" has been looked for in two other directions excepting radiation pressure—*viz.*, (1) the existence of electrical forces, (2) diminution of gravitational attraction with temperature. There is not much theoretical or experimental investigation to support the second case, while the first case is rather obscure and problematic. Radiation-pressure has been so long at a 'discount' because relying upon the deductions of the continuous theory, we had to admit that it was evanescent on particles of the atomic size. But if the views presented in my paper already referred to be found acceptable, this objection can no longer be held as valid.

[17] Eddington, M. N. R. A. S., Vol. 77, pp. 16 and 596.