2. ON THE LIMIT OF INTERFERENCE IN THE FABRY-PEROT INTERFEROMETER

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When a monochromatic source of radiation (for example, that given by a vacuum tube, when excited by an electrical discharge) is examined by a Fabry-Perot interferometer, we obtain bright and narrow rings of maximum intensity separated by wide dark intervals. If the distance between the plates of the etalon be gradually increased, the maxima gradually decrease in brightness, until we reach a limit where we can no longer distinguish between the maxima and the minima. The theory of this phenomenon has been worked out by Lippich, Lord Rayleigh[1], and Schönrock[2], and is shown to be due to the fact that the emission centres (in this case the gaseous atoms) being in motion, a sort of Döppler-Fizeau effect is produced in the line of vision of the observer. They have shown that when the pressure is small, the critical distance D (or the limit of interference) is connected by the following formula with the wave-length (λ) of light, the temperature (T) of the tube, and the mass (M) of the emission centres:

$$\frac{D}{\lambda} = A\sqrt{\frac{M}{T}} \qquad (a)$$

This theorem has been made the basis of a wide series of experiments by Michelson[3], and the French School of opticians including Fabry, Perot and Buisson[4]. Among the various problems to which the formula (*a*) has been applied may be mentioned the following:

(*i*) The temperature of the discharge tube when emitting a monochromatic light.

(*ii*) The temperature of stars and nebulae.

(*iii*) Mass of the emission centres of lines in the spectrum. Probably the mass of the emission centres of many lines of unknown origin in the solar corona and many nebulae (e.g., $\lambda = 5007$Å.U.) which are attributed to hypothetical elements[5], coronium and nebulium may be determined by this method.

The value of the constant A is of much use in all these investigations, and it is generally deduced from theoretical considerations. While going through the literature on the subject, I found that A is generally calculated from approximate and not altogether satisfactory considerations, though an exact solution is not difficult. My object in the present communication is to effect this improvement in the theory. For this we must begin with a preliminary scrutiny of the theory of the Fabry-Perot interferometer.

The Fabry-Perot interferometer consists of two plane parallel plates of glass, both heavily silvered on the inside. If a ray of light is sent through the plates, it undergoes several internal reflections, and at each reflection from either surface, a part issues out. Every incident ray is thus subdivided into a large number of parallel rays. If the angle of incidence is very small, almost normal, as is the case in practice, the number would be infinite. Let us confine our attention to the rays issuing on the side further from the source of light. The parallel rays issuing at some particular angle have path differences amounting to $2d\cos\alpha$, $4d\cos\alpha$, $6d\cos\alpha$, etc., according as they have suffered double reflection once, twice, thrice or any number of times. When these rays are brought together by a converging lens we shall have the interference phenomena. The parallel system is composed of rays transmitted directly, *i.e.*, without reflection—this ray can be represented by $E_0 \cos nt$; rays suffering reflection twice, four times, etc. Since at each double reflection there is a retardation in phase amounting to $2\pi\triangle/\lambda$ and the intensity is reduced by a fraction f, we can represent the rays by the equations

$$f E_0 \cos(nt-\delta),\ f^2 E_0 \cos(nt-2\delta),\ f^3 E_0 \cos(nt-3\delta)$$

where we put

$$\triangle = 2d\cos\alpha \text{ and } \delta = \frac{2\pi\triangle}{\lambda}.$$

The resultant ray is now represented by

$$E = E_0\{\cos nt + f\cos(nt-\delta) + f^2\cos(nt-2\delta) + \ldots\}$$
$$= E_0[\cos nt\ \{1+f\cos\delta+f^2\cos 2\delta+\ldots\} + \sin nt\ \{f\sin\delta+f^2\sin 2\delta+\ldots\}]$$
$$= E_0\left[\cos nt.\frac{1-f\cos\delta}{1-2f\cos\delta+f^2} + \sin nt.\frac{f\sin\delta}{1-2f\cos\delta+f^2}\right].$$

Therefore the intensity

$$I = I_0\frac{1}{1-2f\cos\delta+f^2} = \frac{I_0}{(1-f)^2}\cdot\frac{1}{1+\frac{4f}{(1-f)^2}\sin^2\frac{\delta}{2}}.$$

This is the ordinary theory of the interferometer. The intensities of the maxima and the minima are all in the ratio of $1 : \frac{1}{1+\frac{4f}{(1-f)^2}}$.

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[1] Lord Rayleigh, *Phil. Mag.*, November 1915.
[2] Schönrock, *Ann. D. Physik.*, 1907, Bd. 22, 1907.
[3] Michelson, *Astrophysical Journal*, 1895, Vol. (ii), p. 251.
[4] Buisson et Fabry, *Journal de Physique*,, tome 11, 1912, p. 442-464.
[5] Nicholson, *Phil. Mag.*, 1911, Vol. 22, p. 864.

If we take $f=\cdot 75$, this ratio becomes 49 : 1, the angular separation being $\alpha=\lambda/\triangle$. If the theory held rigorously, we could observe interference with large values of $\triangle$. But this is not the case. For example, in the case of the sodium D_1 line, no interference can be obtained when $\triangle$ exceeds 3 cm. This is due to the fact that the radiant particles are themselves in motion, and the theory cannot be perfect unless we take account of this fact.

According to Maxwell's distribution law, the number of particles having their velocity between v and $v+dv$ is $Ae^{-\beta v^2}\,dv$. The frequency of radiation emitted by these particles is $n\,[1+(v/c)]$ where n is the wave frequency of light emitted by particles at rest. In the expression for retardation in phase, we must therefore replace λ by $\lambda/[1+(v/c)]$ and write $\frac{2\pi\triangle}{\lambda}\,[1+(v/c)]$ in place of $2\pi\triangle/\lambda$.

The intensity of light emitted by molecules having their velocity between $v+dv$ and v is

$$dI=B\,\frac{e^{-\beta v^2}\,dv}{1-2f\cos\delta\,[1+(v/c)]+f^2}.$$

The total intensity

$$I=B\int_{-\infty}^{\infty}\frac{e^{-\beta v^2}\,dv}{1-2f\cos\delta\,[1+(v/c)]+f^2}.$$

We have by trigonometry,

$$\frac{1-f^2}{1-2f\cos\delta+f^2}=1+2f\cos\delta+2f^2\cos 2\delta+\ldots..$$

Now, we have

$$\int_{-\infty}^{\infty}e^{-\beta v^2}\,dv.\quad \sin\frac{n\delta v}{c}=0,$$

$$\int_{-\infty}^{\infty}e^{-\beta v^2}\,dv.\quad \cos\frac{n\delta v}{c}=\sqrt{\frac{\pi}{\beta}}\;e^{-\left(\frac{1}{4\beta}\right)}\;(n\delta/c)^2.$$

We have therefore

$$I=\frac{B_0}{1-f^2}\sqrt{\frac{\pi}{\beta}}\left[1+2\sum_1^{\infty}f^n\,e^{-\left(\frac{1}{4\beta}\right)(n\delta/c)^2}\cos n\delta\right].$$

Now let I_1=the maximum value of I, corresponding to $n\delta=0$,

I_2=the minimum value of I, corresponding to $n\delta=\pi$.

Then the visibility factor V is, according to Michelson

$$=\frac{I_1-I_2}{I_1+I_2}=\frac{fe^{-\left(\frac{1}{\beta}\right)}\left(\frac{\pi\triangle}{\lambda c}\right)^2+f^3e^{-\left(\frac{3^2}{\beta}\right)}\left(\frac{\pi\triangle}{\lambda c}\right)^2+\ldots.}{\frac{1}{2}+f^2e^{-\left(\frac{2^2}{\beta}\right)}\left(\frac{\pi\triangle}{\lambda c}\right)^2+f^4e^{-\left(\frac{4^2}{\beta}\right)}\left(\frac{\pi\triangle}{\lambda c}\right)^2+\ldots}.$$

Now $(1/\beta)\left(\frac{\pi\triangle}{\lambda c}\right)^2$ is of the order 10^8. We can, therefore, safely omit terms containing f^2, f^3, etc.

V is therefore$=2fe^{-\left(\frac{1}{\beta}\right)}\left(\frac{\pi\triangle}{\lambda c}\right)^2$.

From the kinetic theory of gases, we have $\beta=(m/2KT)=(wM/2KT)$,

where m=weight of the radiant atom in grams,

w=weight of the hydrogen atom,

M=atomic weight of the radiant gas,

K=universal gas constant,

T=temperature.

Then we have, since

$$-\frac{1}{\beta}\left(\frac{\pi\triangle}{\lambda c}\right)^2=\log_e\left(\frac{V}{2f}\right),\ \frac{\triangle}{\lambda}=\frac{c}{\pi}\sqrt{\frac{wM}{2KT}\log_e\left(\frac{2f}{V}\right)}.\qquad\ldots\ldots(b)$$

Lord Rayleigh took account of the first two interfering beams only, but by this he had evidently the Michelson interferometer in his mind. But I think that when we are applying the result to the Fabry-Perot interferometer, we should take into account all the infinite number of interfering beams, and the effect of reflection. This is exactly what has been done in the present communication.

The exact evaluation of the constant $\frac{c}{\pi}\sqrt{\frac{w}{2K}\log_e\left(\frac{2f}{V}\right)}$, cannot be done unless the reflecting power of the plates, and the value of V be known. f will depend upon the silvering of the plates, while V will vary with the observer. Thus Lord Rayleigh takes the visibility factor equivalent to $\cdot 025$, while Schönrock takes it equivalent to $\cdot 05$. Assuming that $V=\cdot 025$ and $f=\cdot 75$, we have

$$\frac{\triangle}{\lambda}=1\cdot 50\times 10^6\sqrt{\frac{M}{T}},$$

while according to Lord Rayleigh

$$\frac{\triangle}{\lambda}=1\cdot 42\times 10^6\sqrt{\frac{M}{T}}.$$

As it is, the discrepancy between the two values if calculated by two different methods is not much. But for particular apparatus, and for particular observers, the discrepancy may be considerable. It is to be hoped that investigators will take notice of these facts.

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