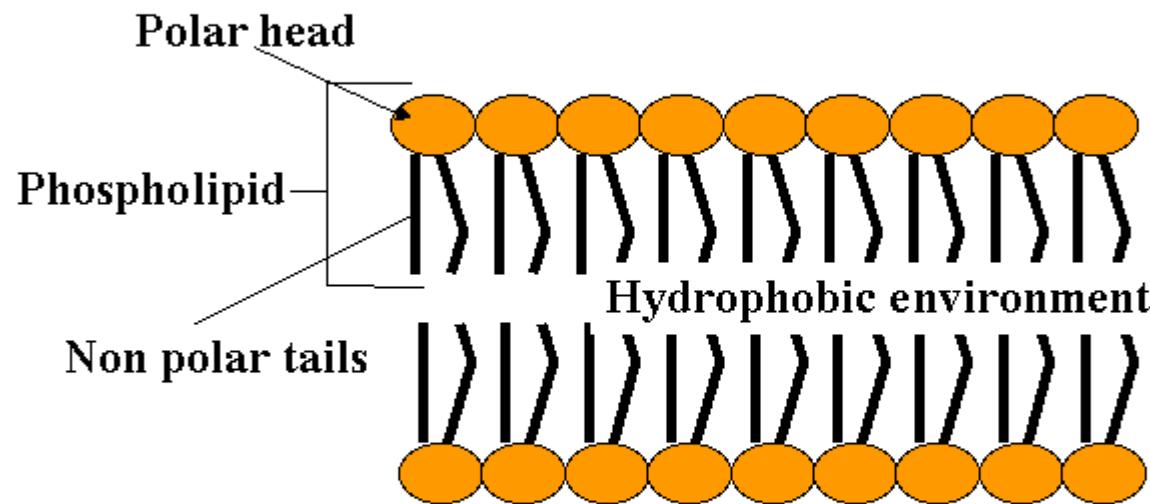


# **From Neuron to Brain: Pedagogic Approach**

KAMALES BHAUMIK



# Plasma membrane



## Movement of charged ions across the cell membrane

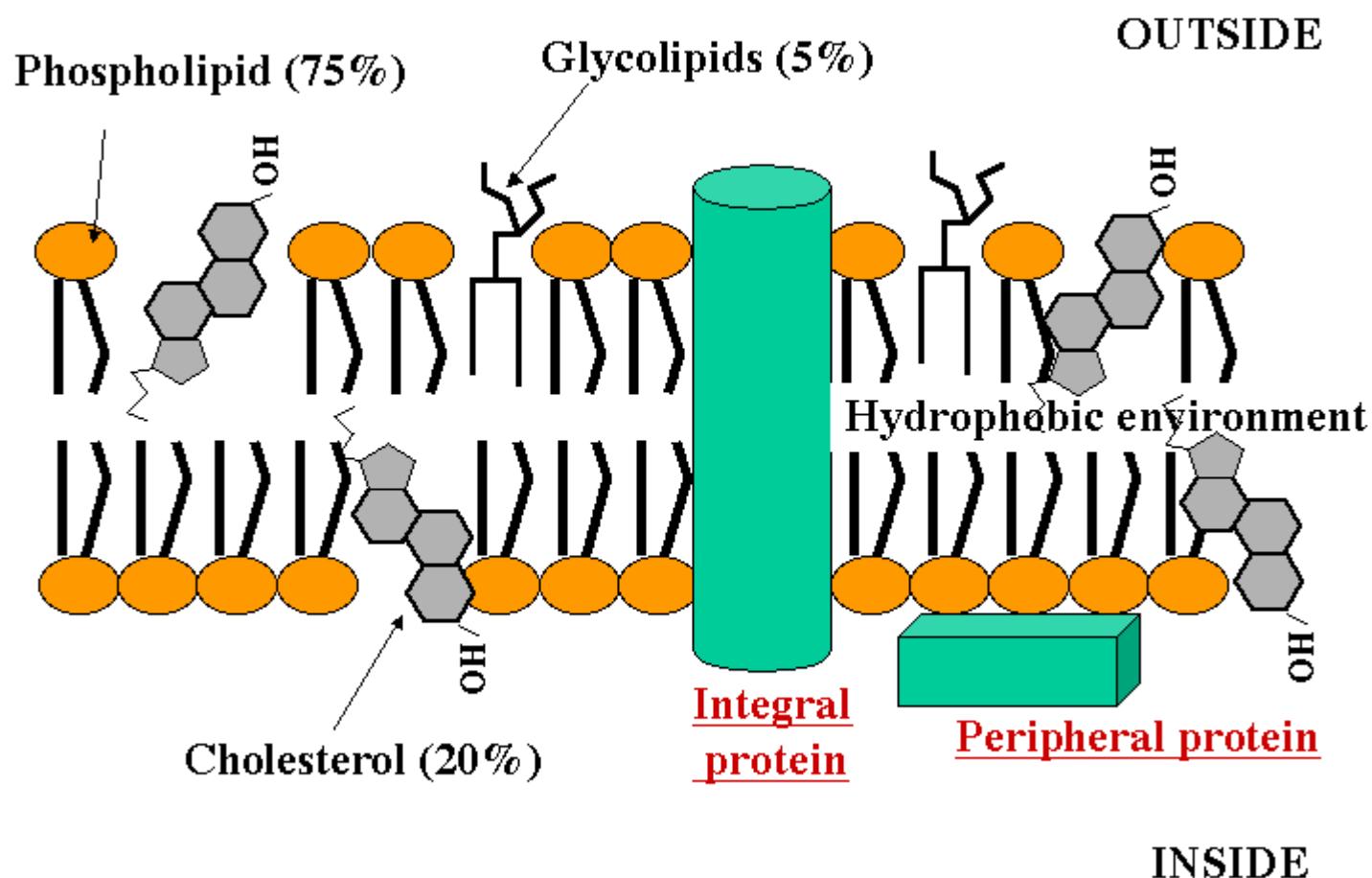
$$V = \frac{q}{\epsilon a}$$

Capacitance  $C = \frac{q}{V} = \epsilon a$

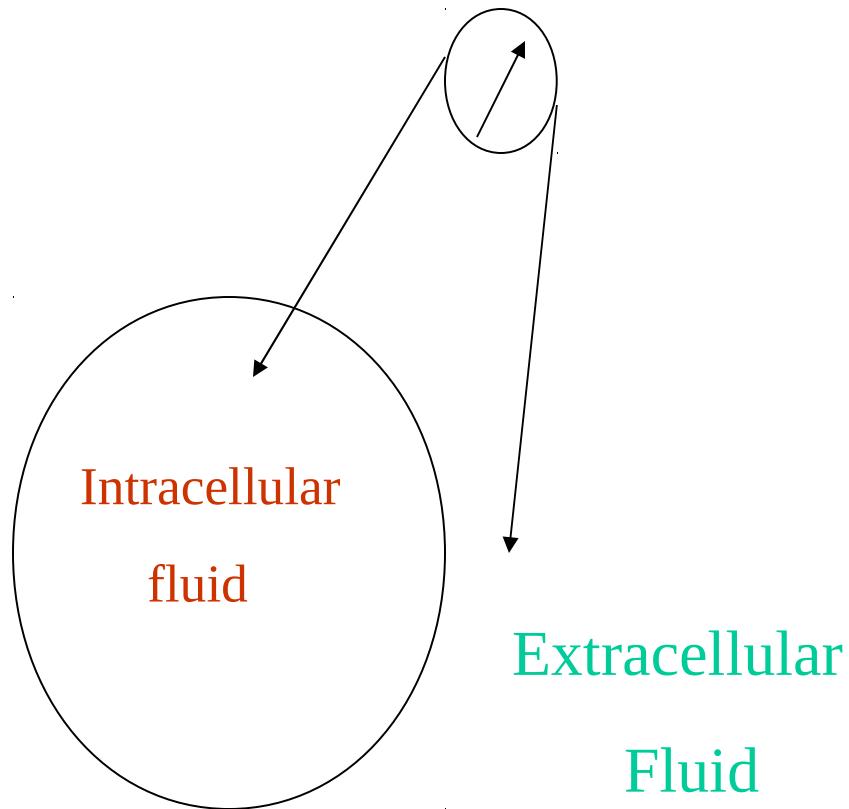
Self-energy  $E = \frac{1}{2} CV^2 = \frac{q^2}{2a\epsilon} \frac{1}{2}$

$$\epsilon_{water} = 80 \quad \epsilon_{lipid} \approx 4$$

# Plasma membrane



## Voltmeter



Inside of the cell is negative with respect to outside.

Most of the cells have a resting membrane potential in the range of -30mV to -80mV

# Resting Membrane Potential

## Nernst Potential

# Nernst Potential

Chemical Potential = Partial Molar Gibbs Free Energy

Chemical Potential  $\mu_c = \mu_c^0 + RT \ln C$

Electrochemical Potential  $\mu = \mu_0 + RT \ln C + zFV$

$\Delta V$	
$V_1$	$V_2$
$C_1$	$C_2$

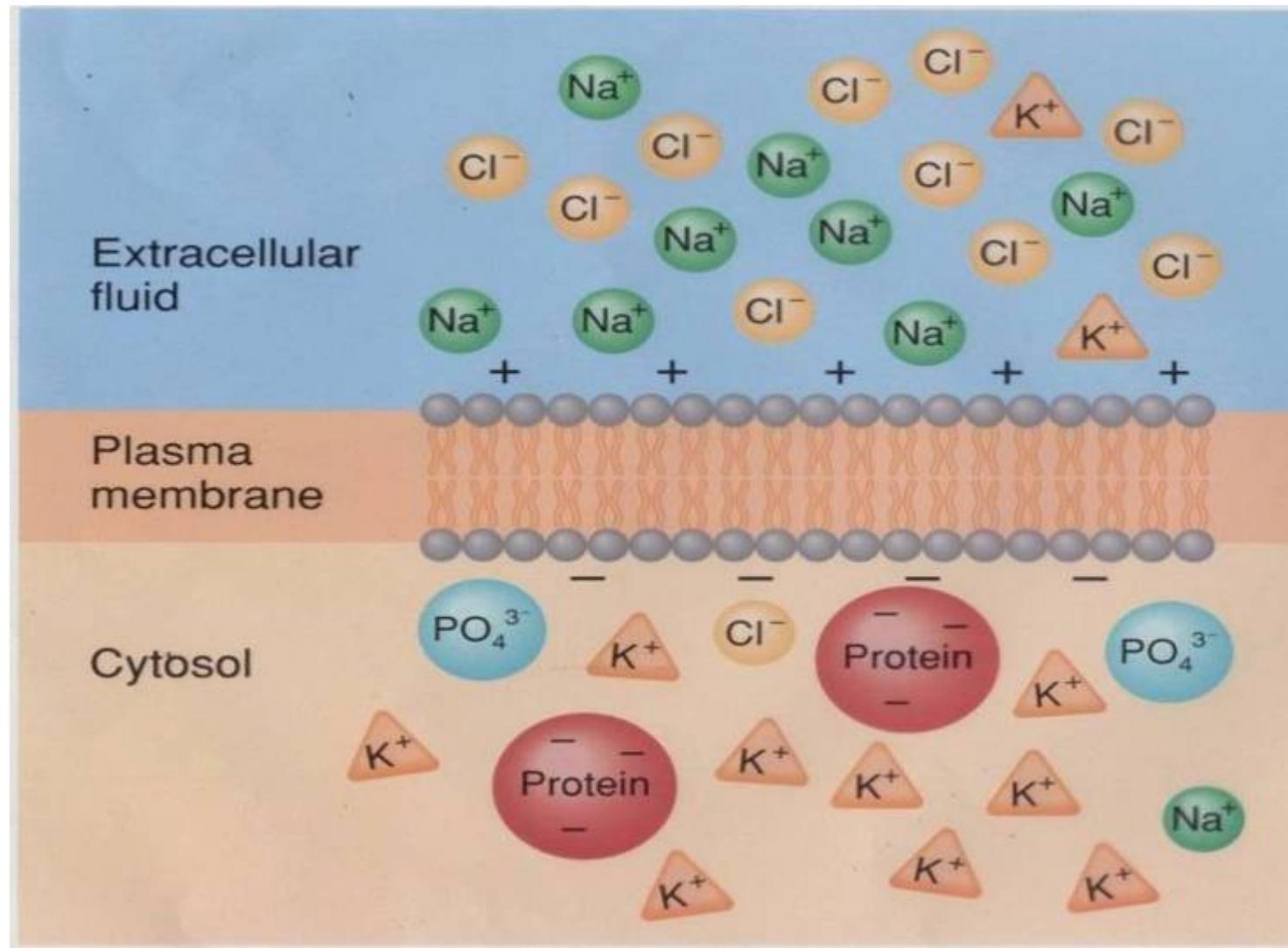
$$\begin{aligned} & RT \ln C_1 + zFV_1 \\ & = RT \ln C_2 + zFV_2 \end{aligned}$$

## Nernst Potential

$$\Delta V = V_2 - V_1 = \frac{RT}{zF} \ln \frac{C_1}{C_2}$$

$$\frac{RT}{zF} = 25 \text{mV}$$

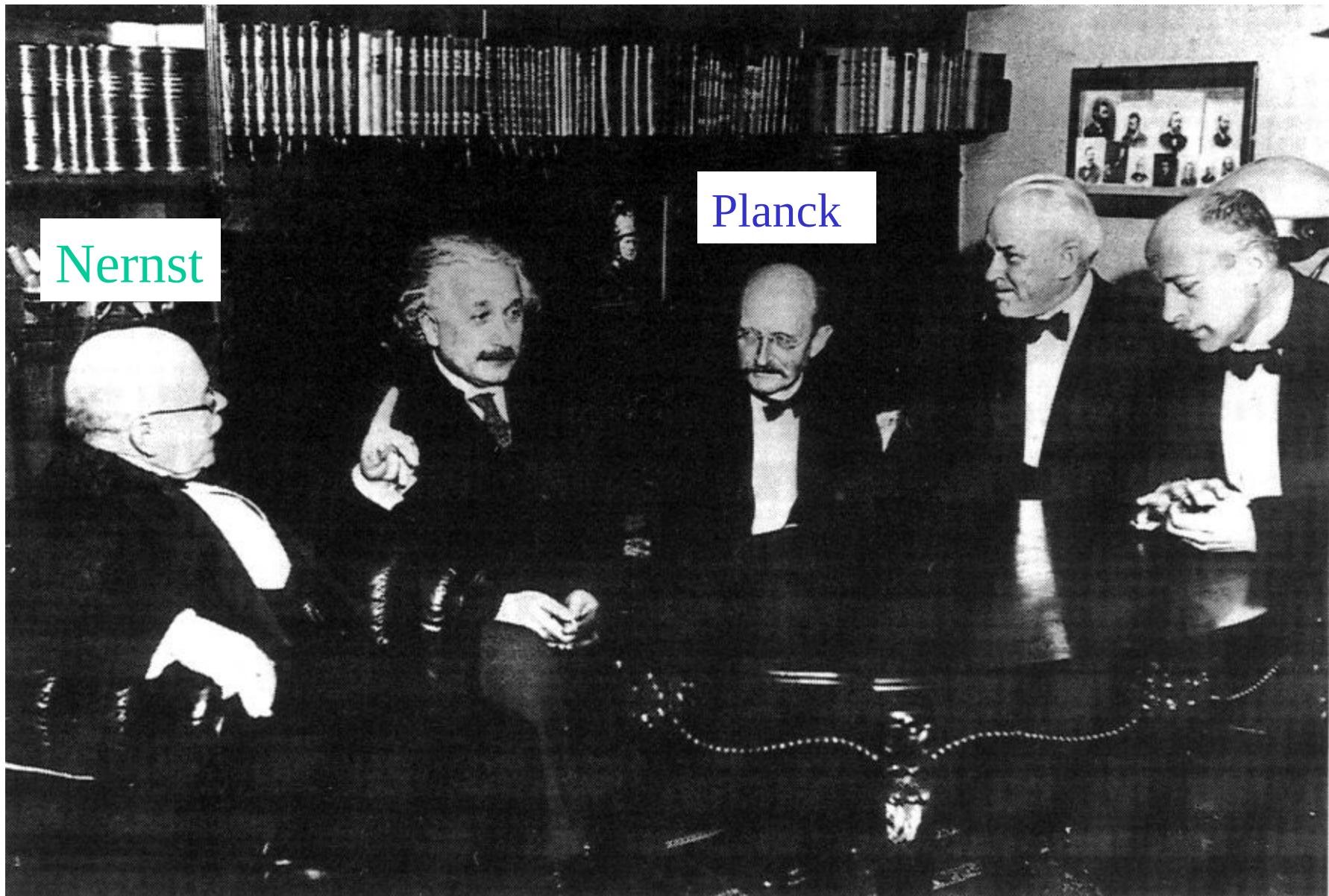
$$\begin{aligned} (\Delta V)_{Na} &= 25 \times 2.303 \log \frac{C_1}{C_2} \\ &\approx 60 \log \frac{C_1}{C_2} \text{mV} \end{aligned}$$



CONCENTRATION (millimoles/liter)			
ION	Extracellular	Intracellular	RELATIVE PERMEABILITY
$\text{Na}^+$	150	15	1
$\text{K}^+$	5	150	30
$\text{Cl}^-$	108	10	0.025
$\text{A}^-$	0	65	0

Nernst

Planck



## Nernst-Planck Equation

$$\mu_i = \mu_i^0 + RT \ln C_i + z_i F V$$

$$J_i = \text{Flux} \text{ mol L}^{-2} T^{-1}$$

$$J_i = -u_i C_i \frac{d\mu_i}{dx}$$

$$J_i = -u_i R T \frac{\partial C_i}{\partial x} - u_i C_i z_i F \frac{\partial V}{\partial x}$$

FICK's Law

OHM's Law

$$D_i = u_i R T \quad Einstein's \text{ Relation}$$

# Integration of Nernst Planck Equation

CONSTANT FIELD MODEL

Goldmann Hodgkin Katz (GHK) Model

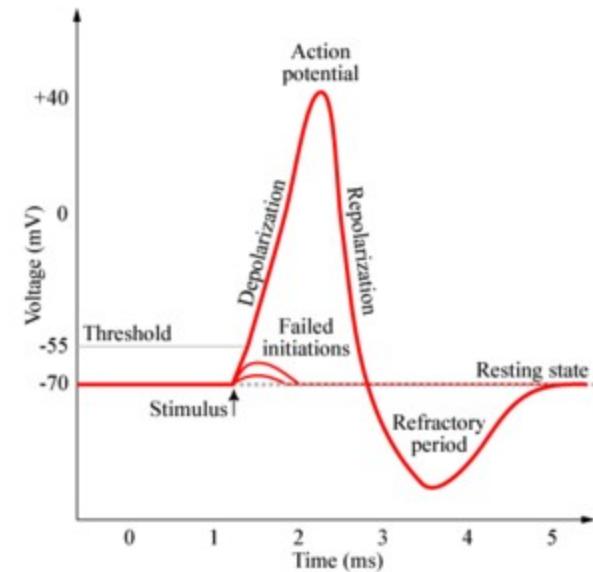
$$V = \frac{RT}{F} \ln \frac{P_{Na}C_{Na}^1 + P_K C_K^1 + P_{Cl} C_{Cl}^2}{P_{Na}C_{Na}^2 + P_K C_K^2 + P_{Cl} C_{Cl}^1}$$

# Excitation of cells at resting membrane potential

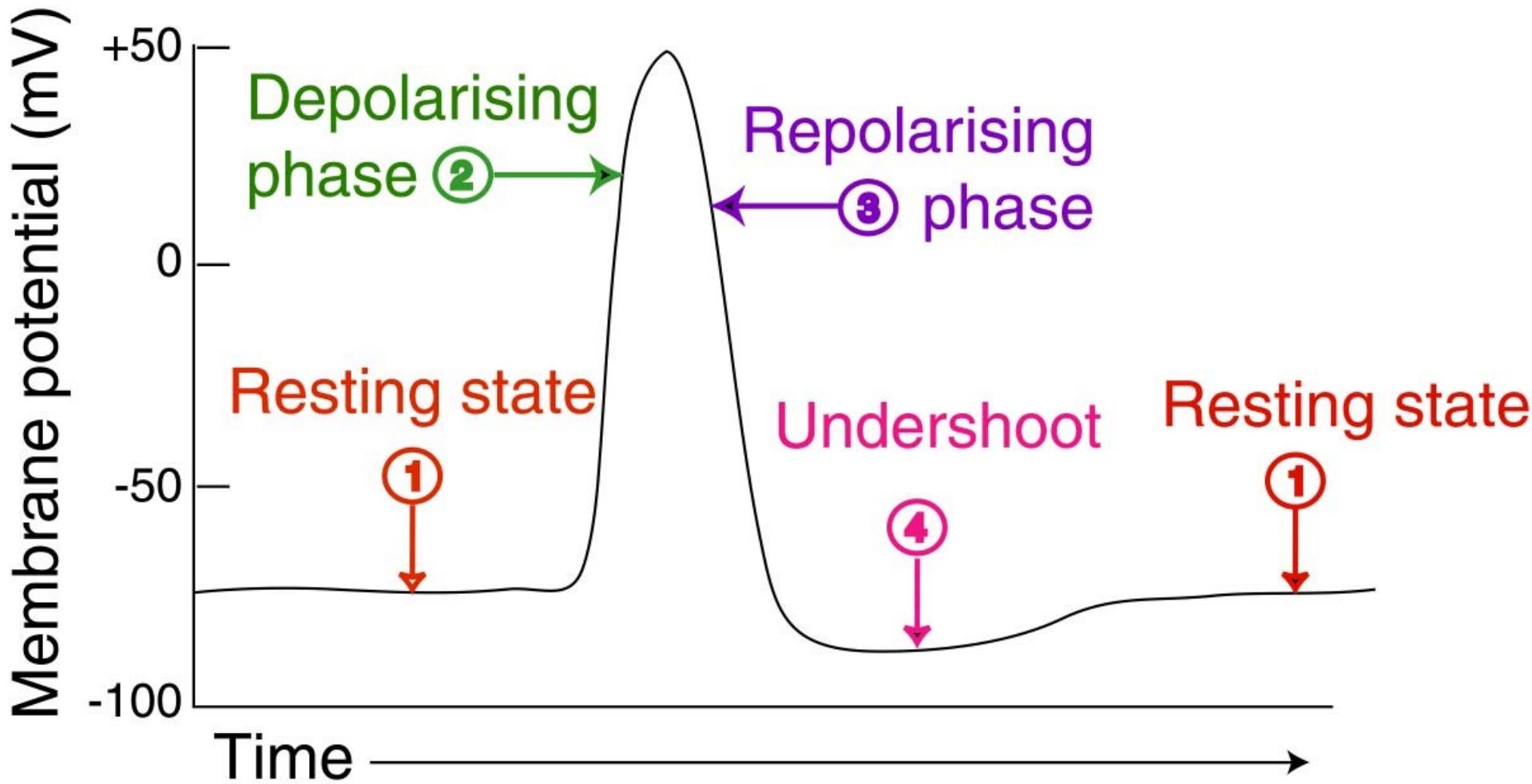
Passive Response

Excitable Cells

Action Potential

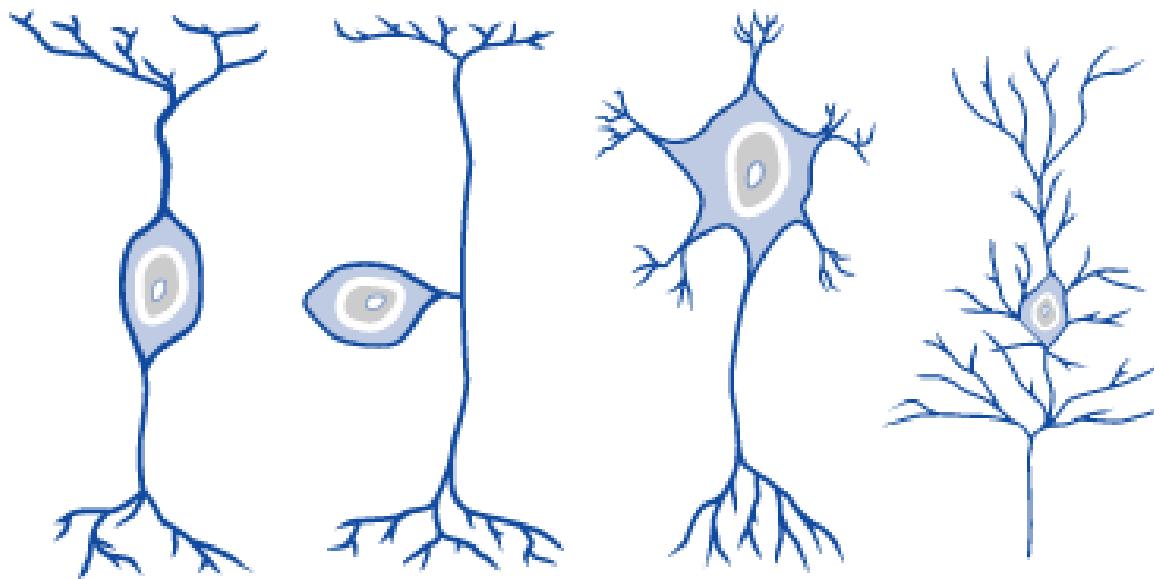


All or None response





## Basic Neuron Types

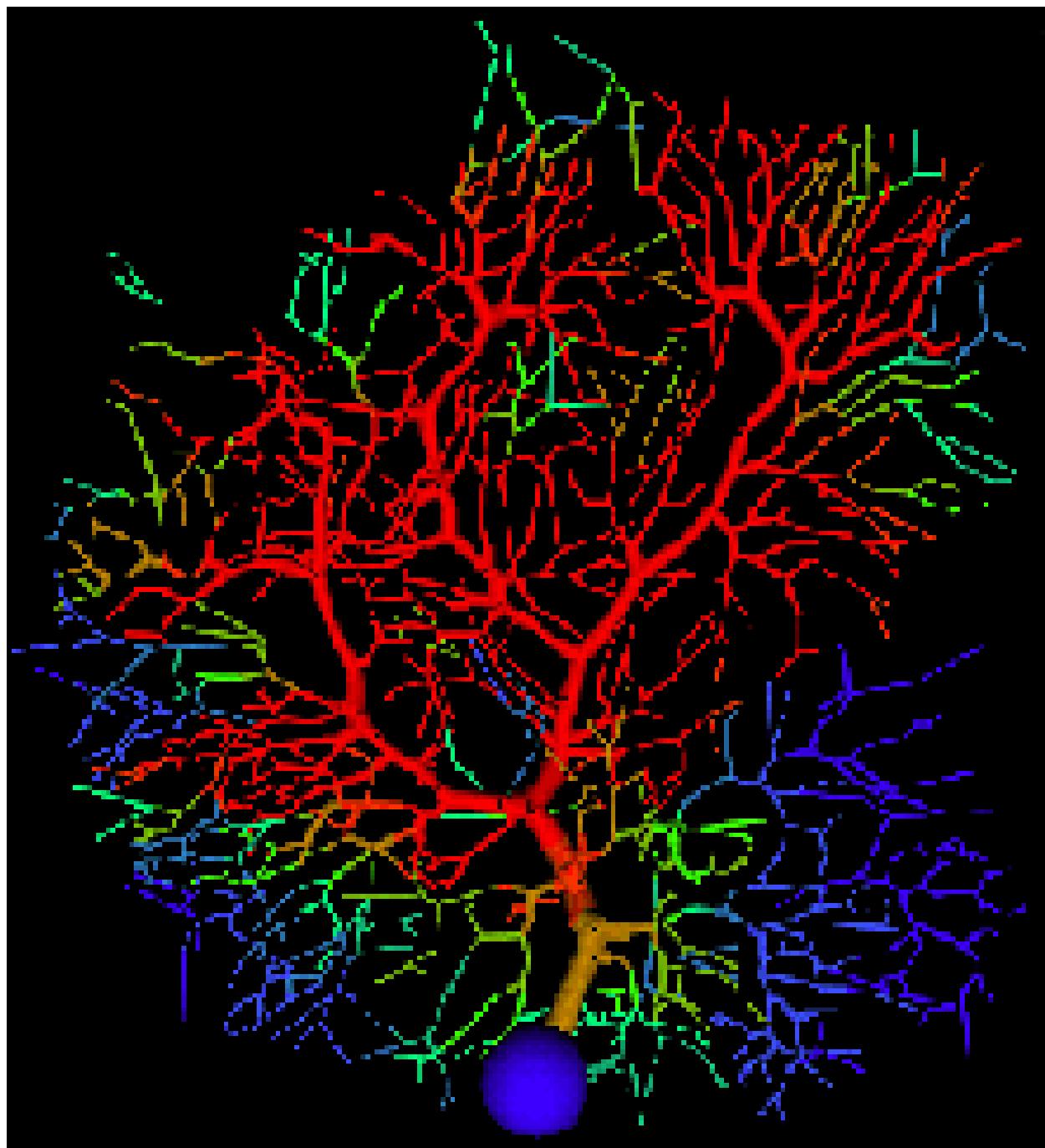


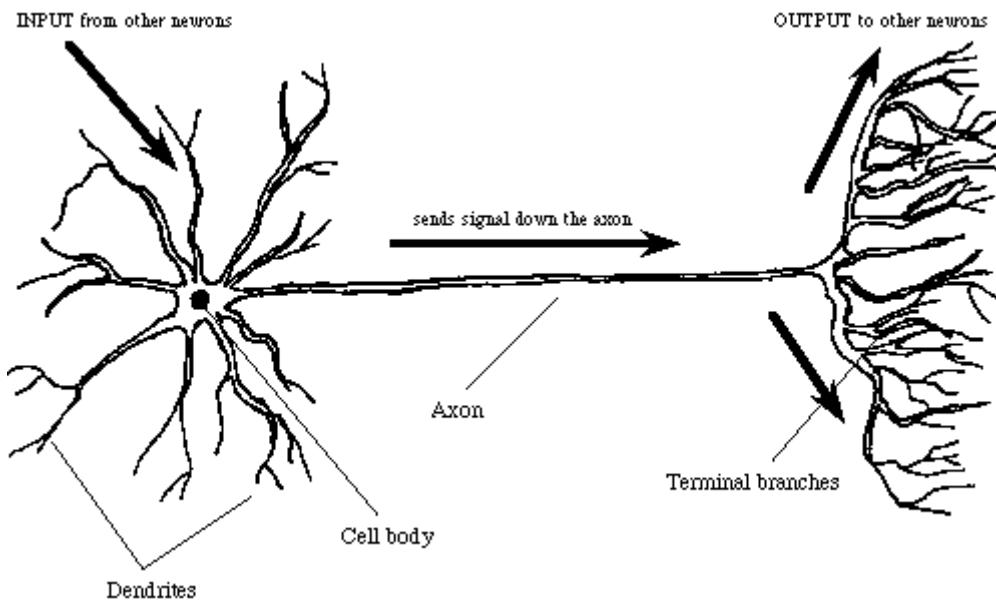
Bipolar  
(Interneuron)

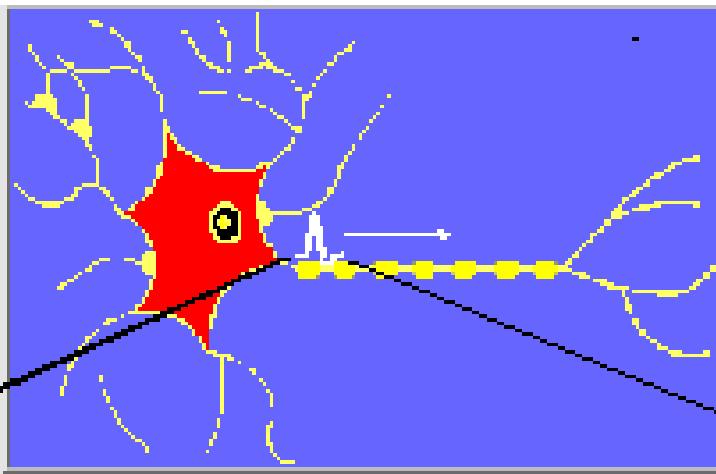
Unipolar  
(Sensory Neuron)

Multipolar  
(Motoneuron)

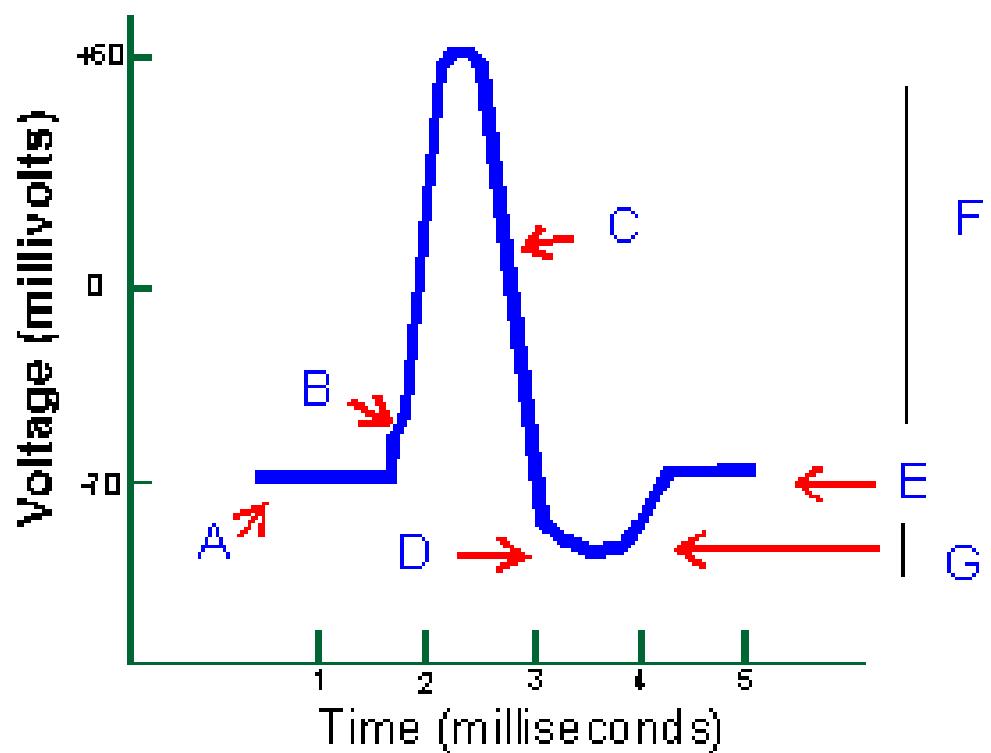
Pyramidal  
Cell







## Action Potential



# Currents Flowing in and out of the membrane During Action Potential

Capacitative current

$$C_m \frac{dV}{dt}$$

Resting membrane potential = K-Nernst potential

Peak of action potential = Na-Nernst Potential

Nonlinearity in the current flow

Necessity of a voltage clamp machine

# Separation of ionic currents

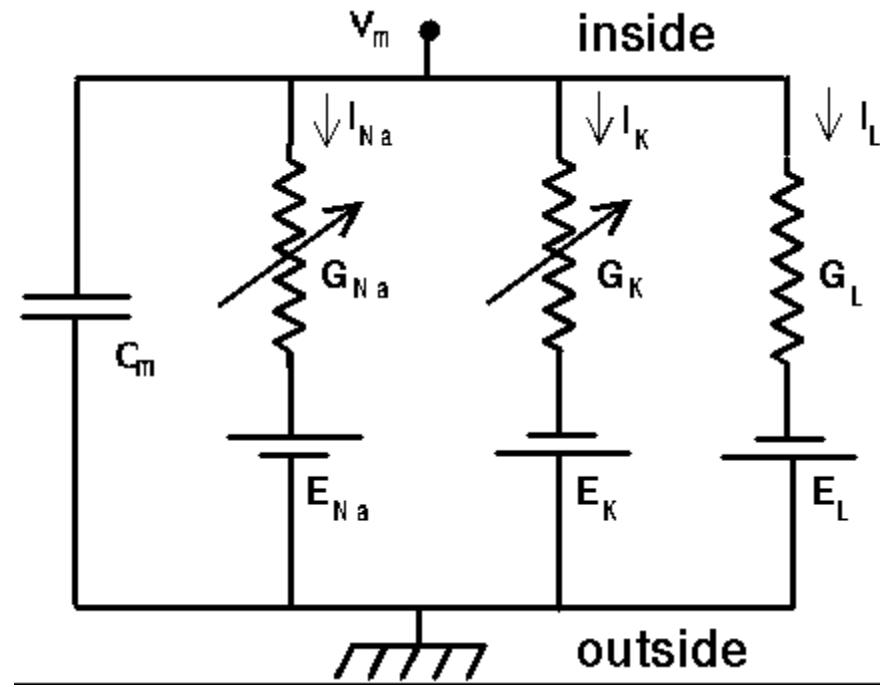
TTX (Tetrodotoxin) Na-Channel Blocker

TEA (Tetra-ethyl-ammonium) K-channel blocker

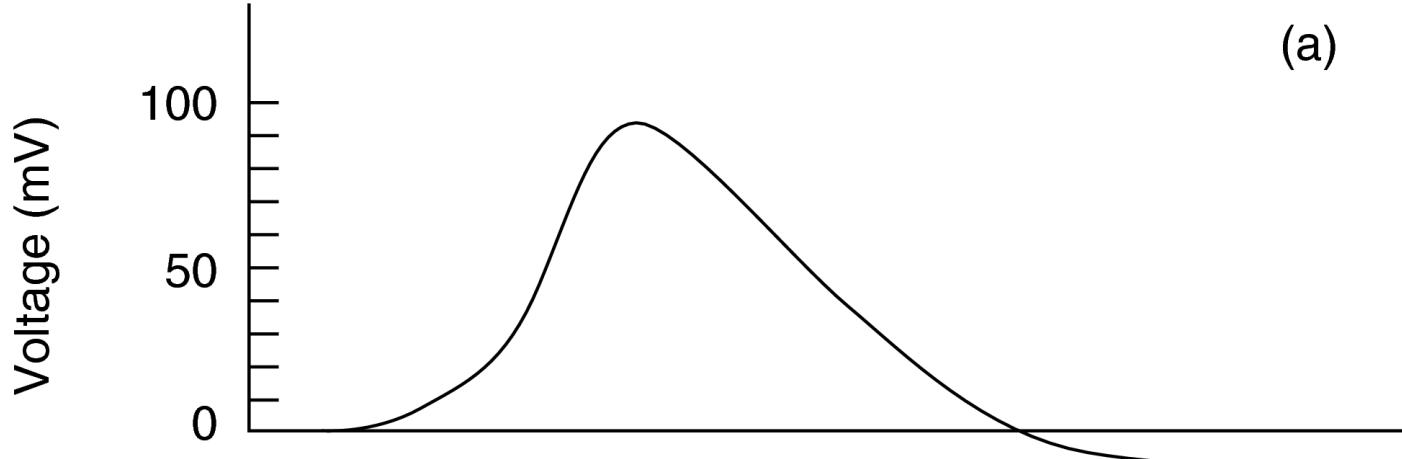
Na and K channel conductance depends on membrane voltage

Chloride and non-specific ions follow Ohm's law

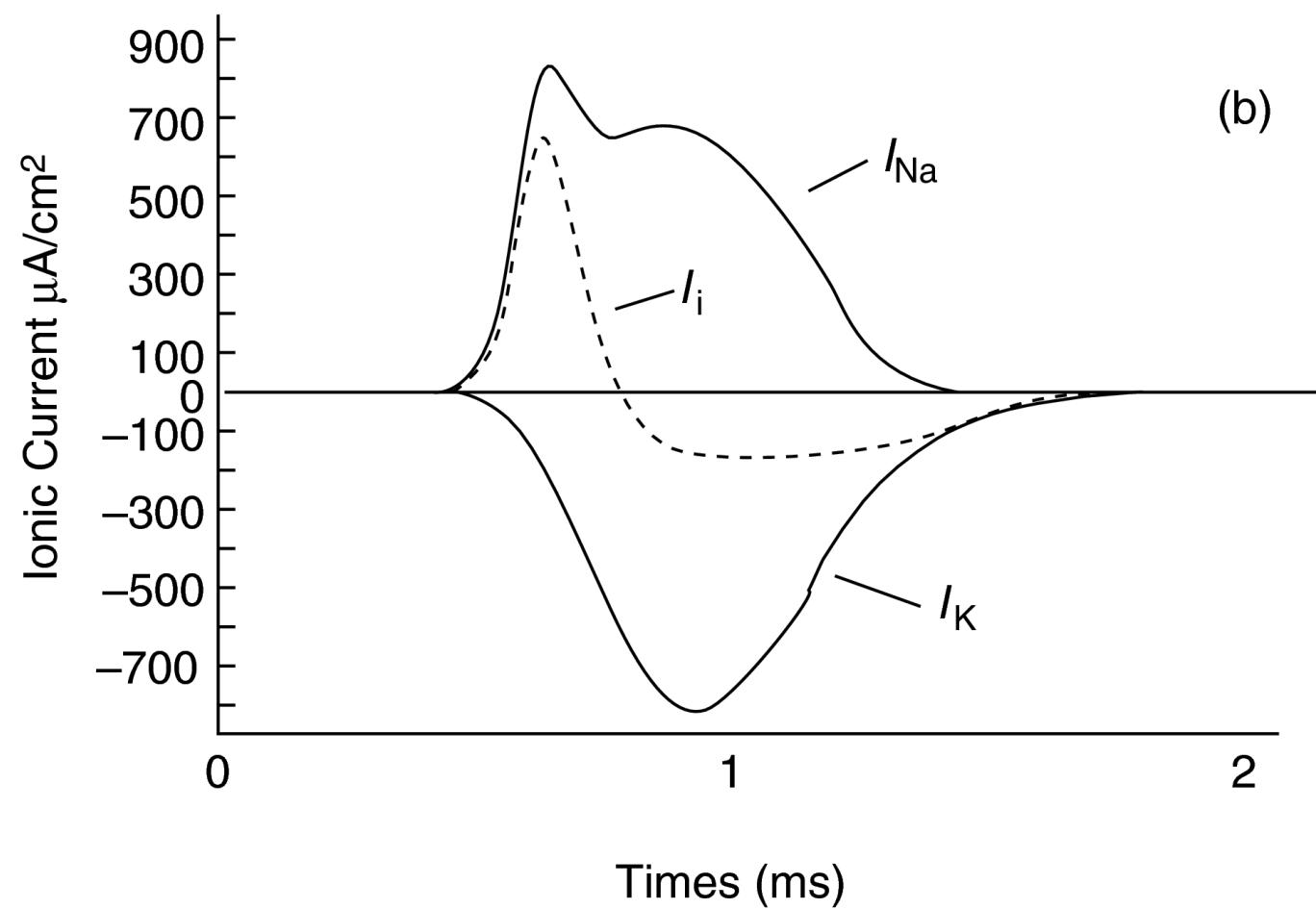
# Equivalent Circuit



(a)



(b)



$$I = C_M \frac{dV}{dT} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_l (V - V_l)$$

$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n$$

$$\frac{dm}{dt} = \alpha_m (1 - m) - B_m m$$

$$\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h$$

$$\alpha_n = \frac{0.01(V+10)}{e^{\frac{V-10}{10}} - 1}$$

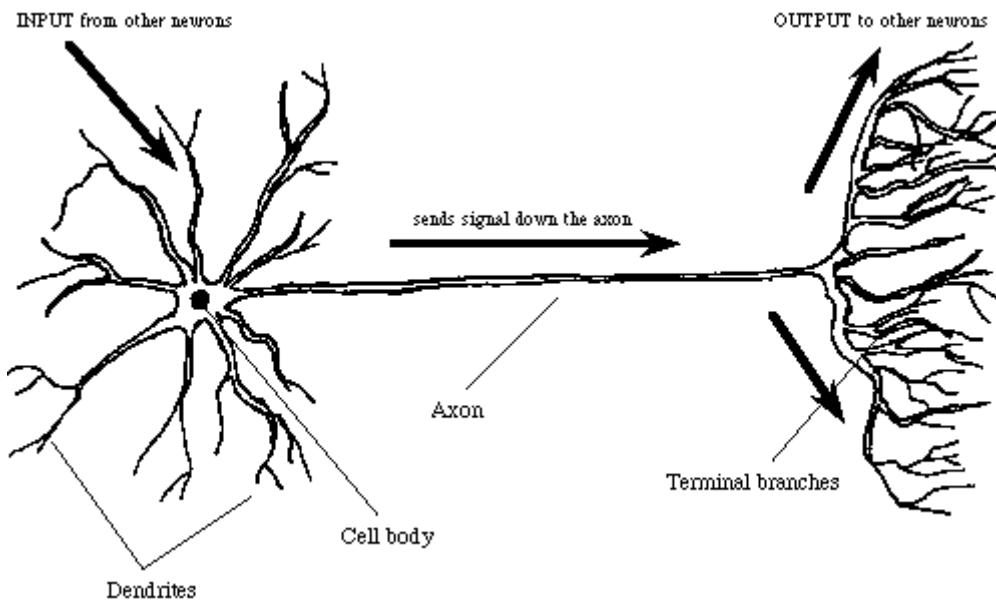
$$\beta_n = 0.125 e^{(V/80)}$$

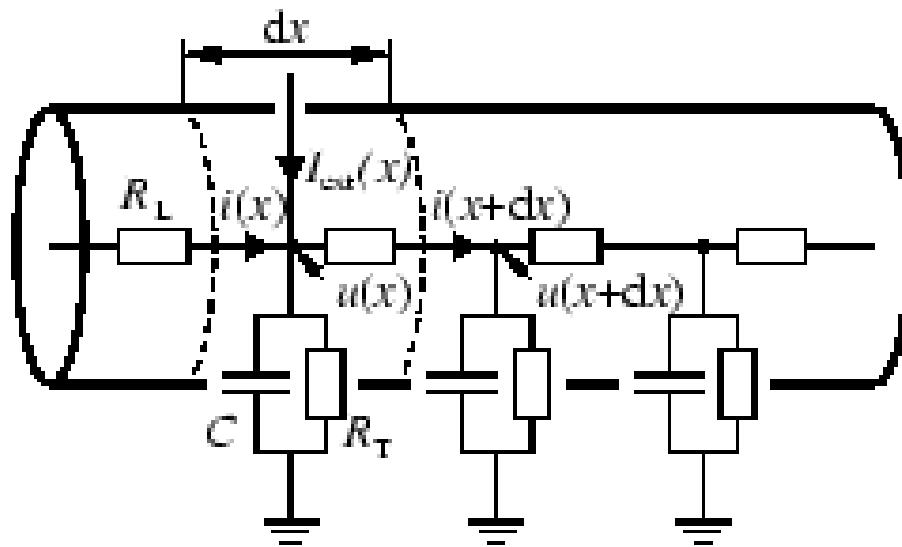
$$\alpha_m = \frac{0.1(V+25)}{e^{\frac{V+25}{10}} - 1}$$

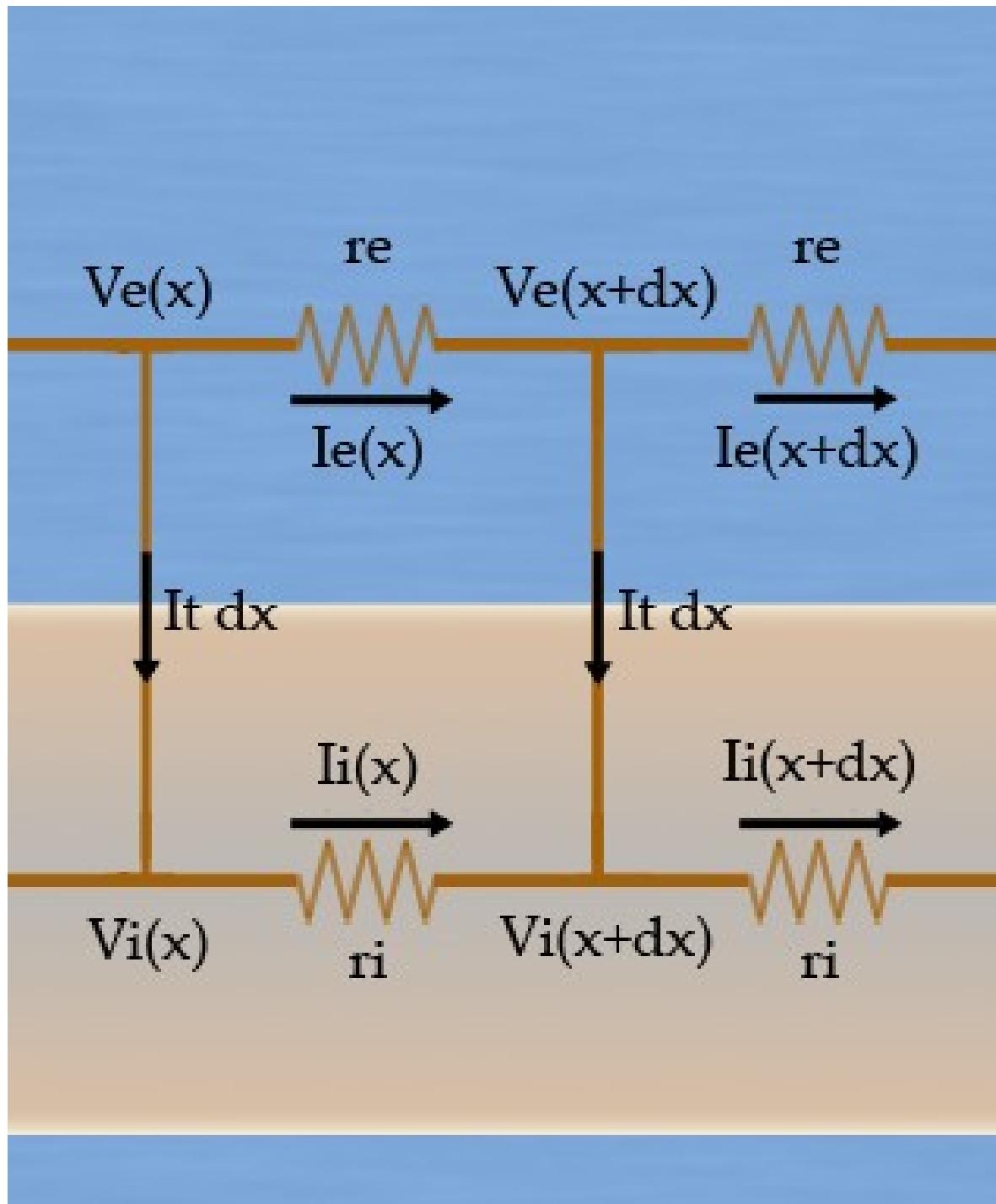
$$\beta_m = 4 e^{(V/18)}$$

$$\alpha_h = 0.07 e^{\frac{V}{20}}$$

$$\beta_h = \frac{1}{e^{\frac{V+30}{10}} + 1}$$





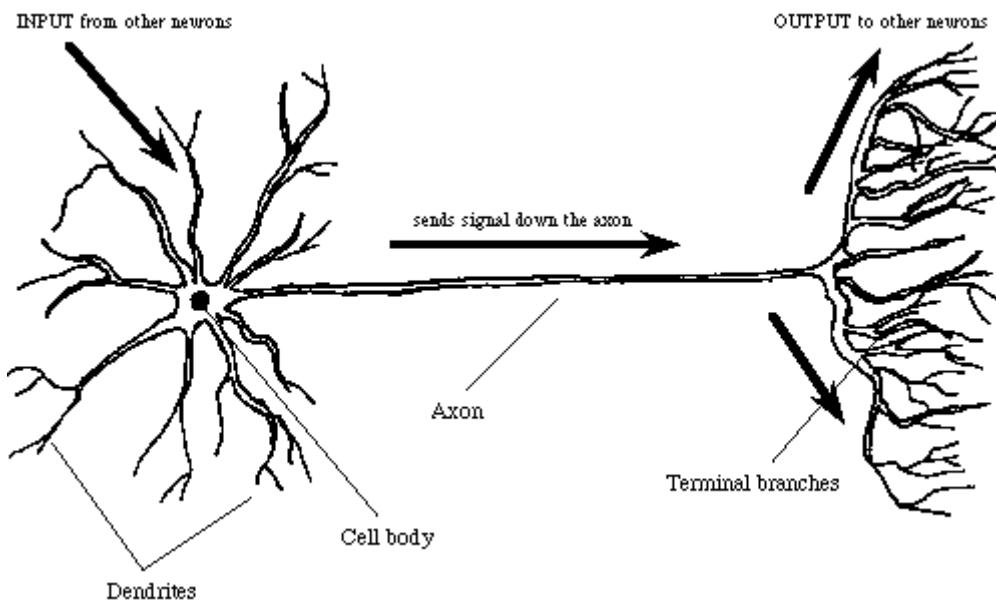


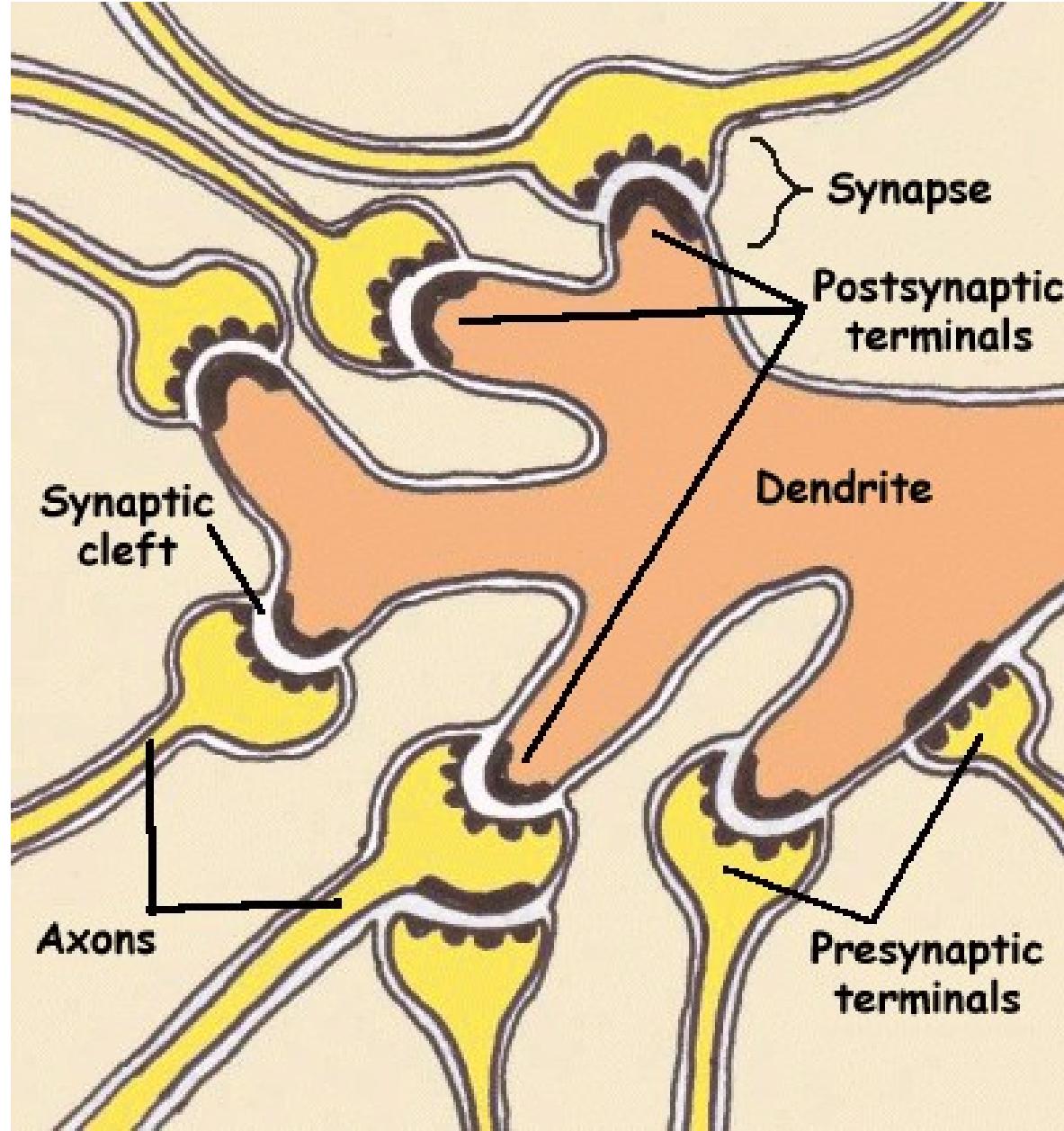
# Propagation of Axon Potential

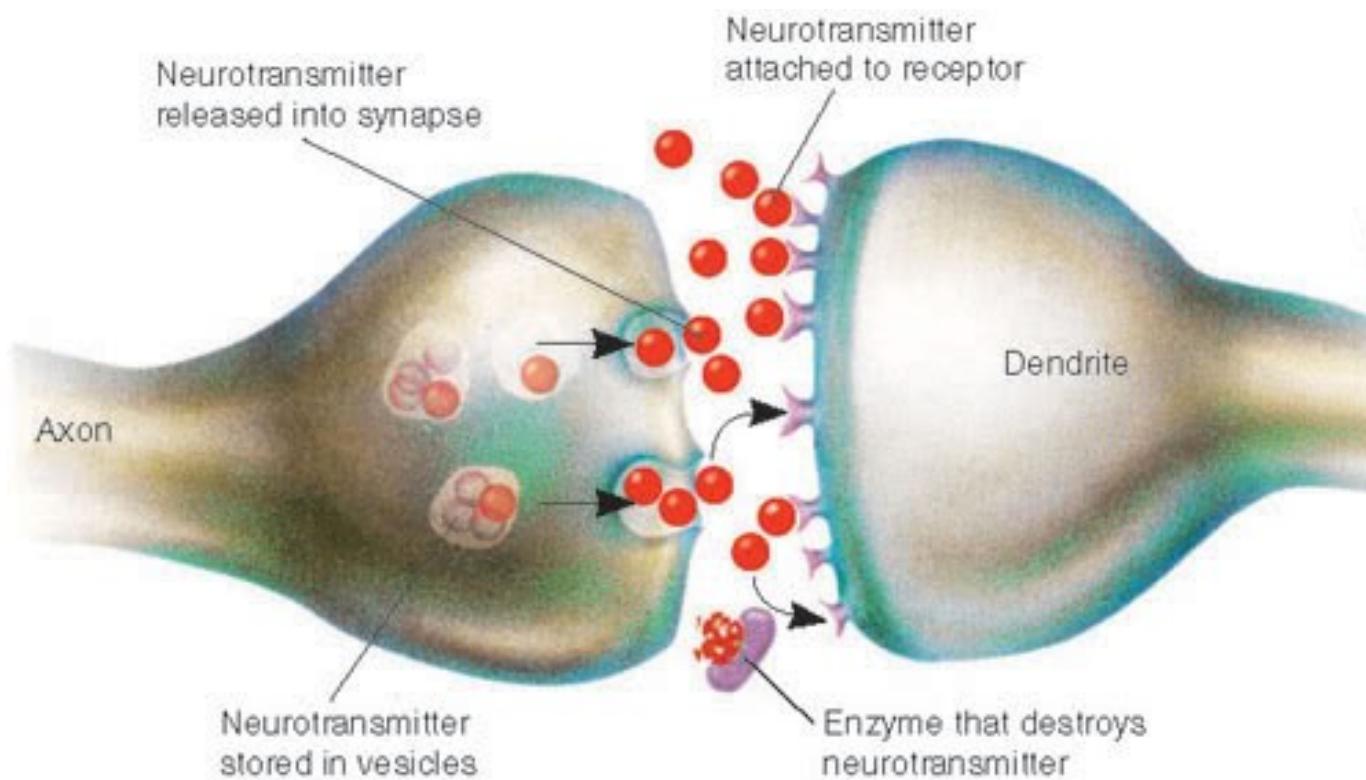
$$\frac{1}{factor} \frac{\partial^2 V}{\partial x^2} = I_m$$

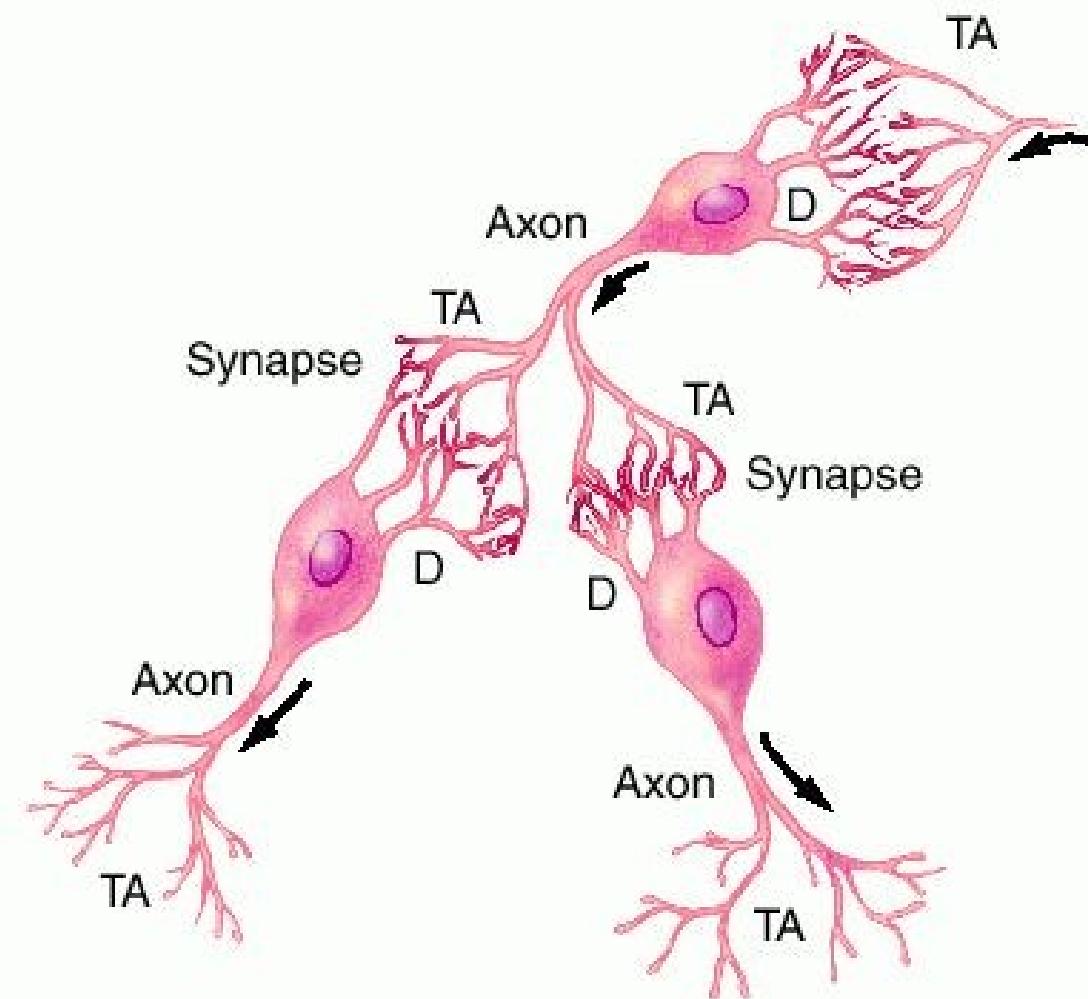
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

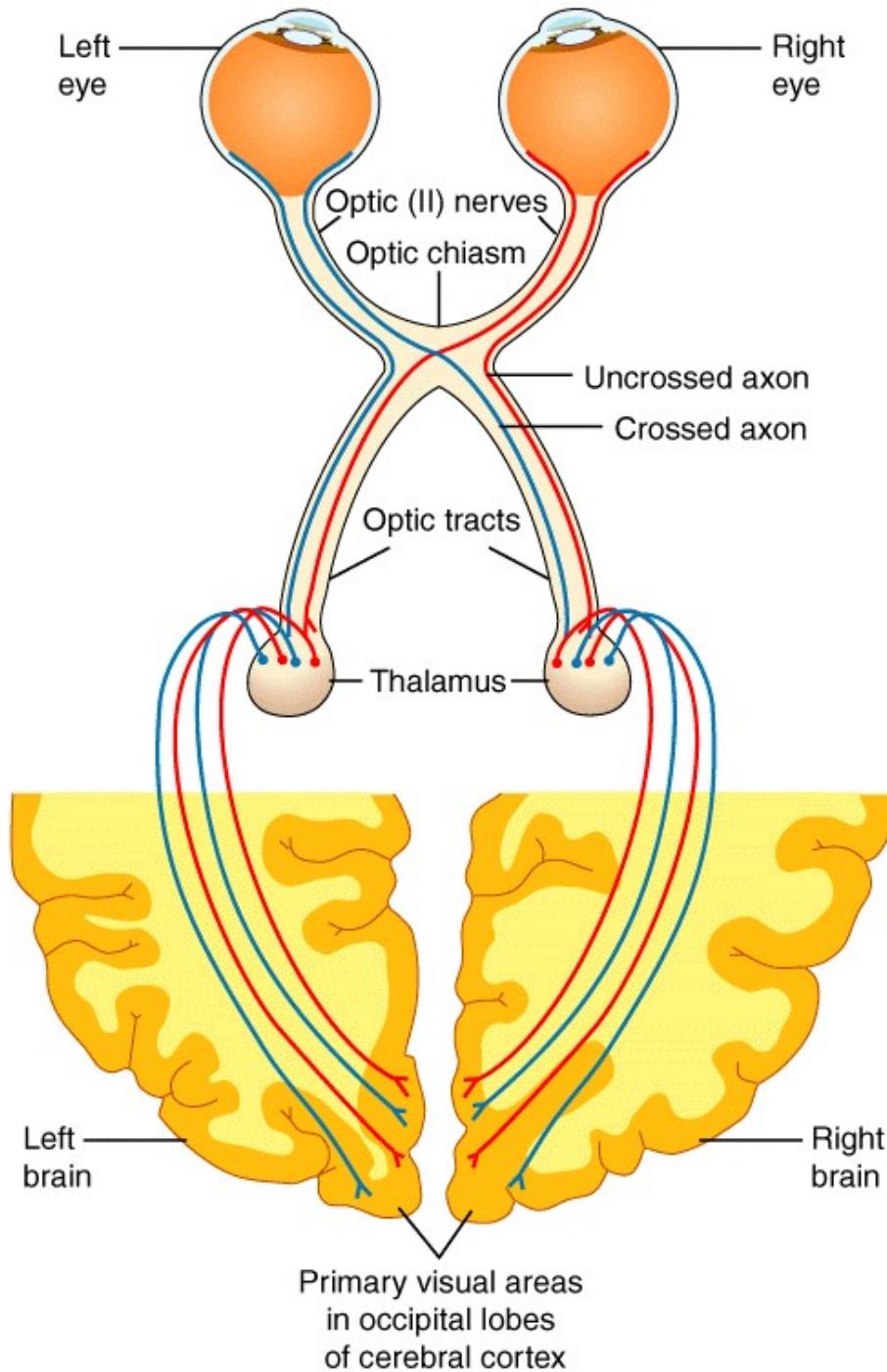
$$\frac{1}{factor} \frac{1}{v^2} \frac{\partial^2 V}{\partial t^2} = I_m = C_m \frac{\partial V}{\partial t} + \sum_i I_i$$

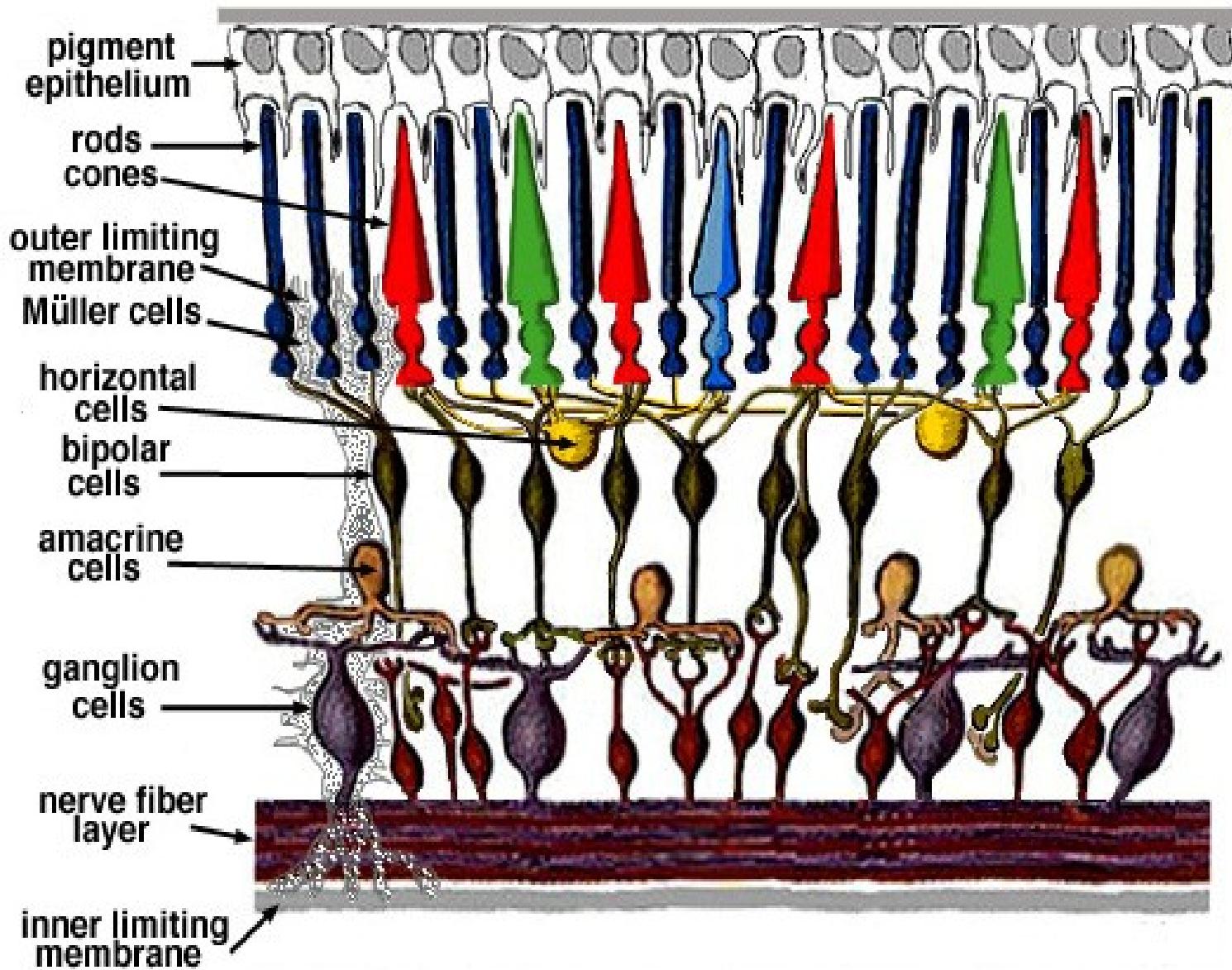


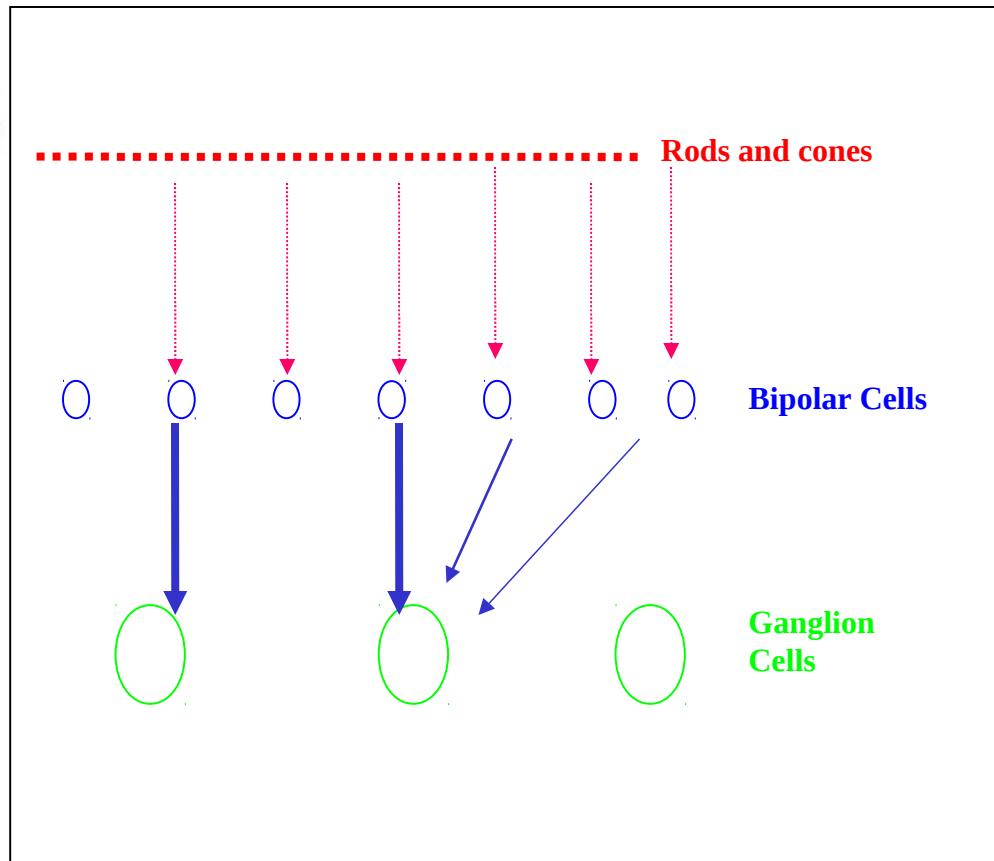
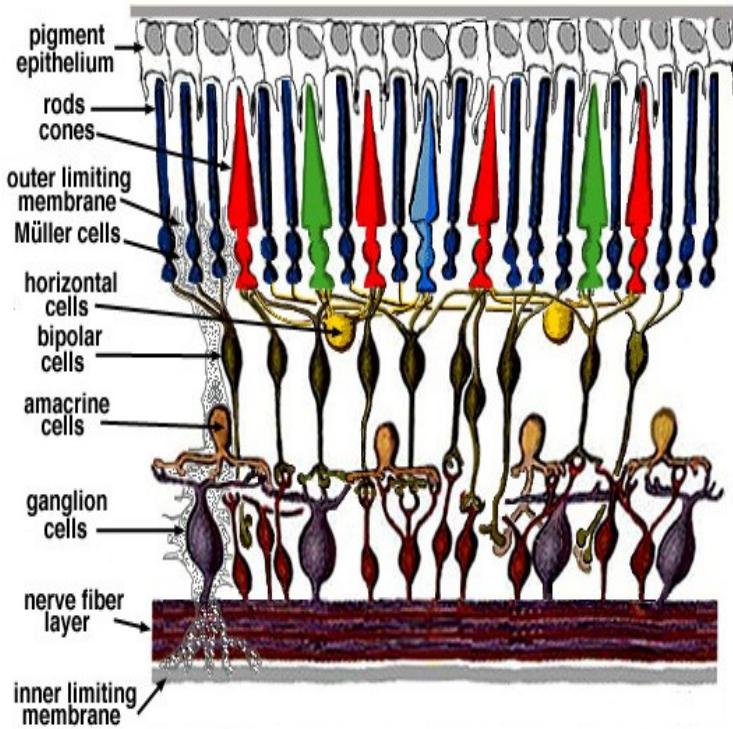




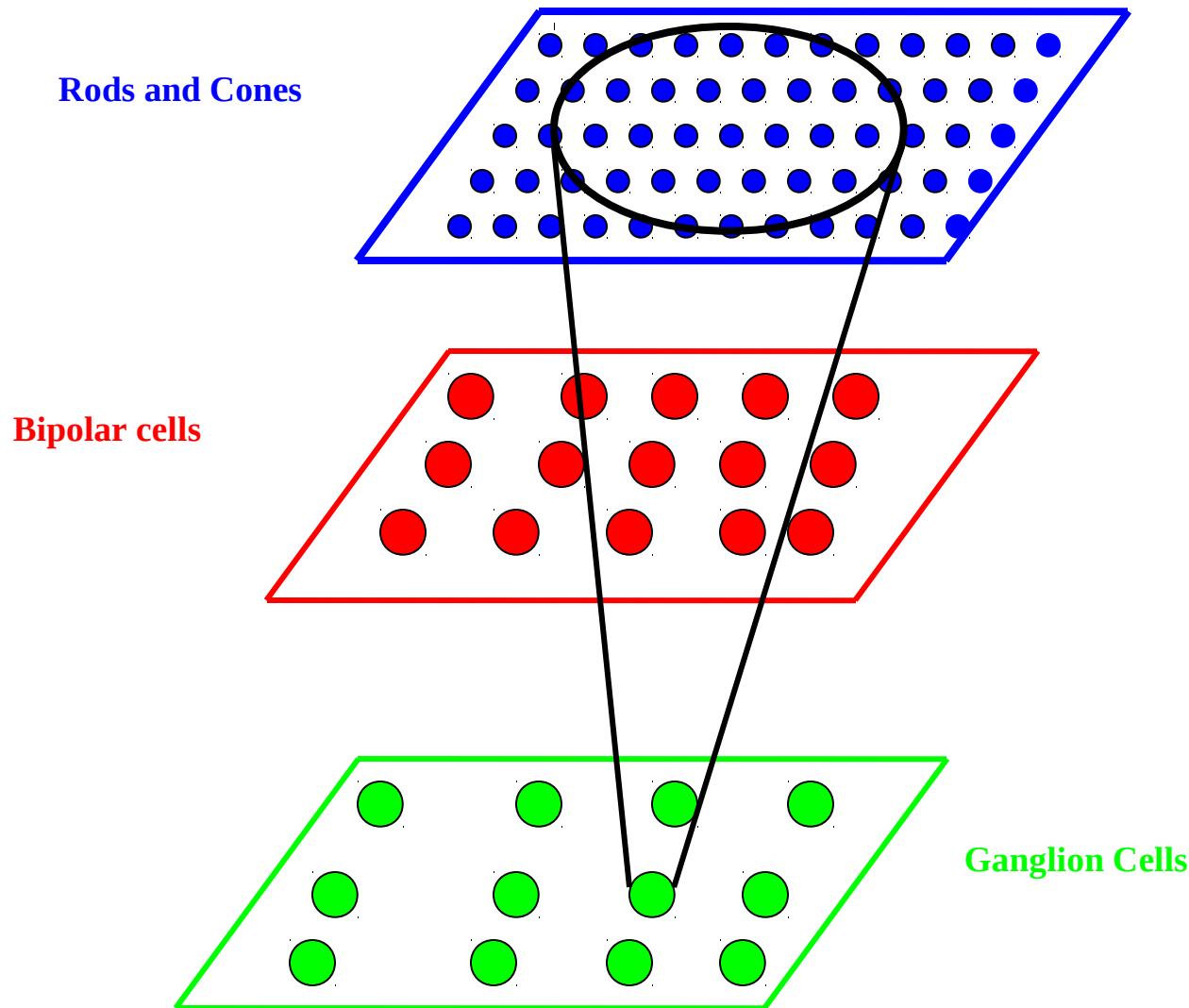


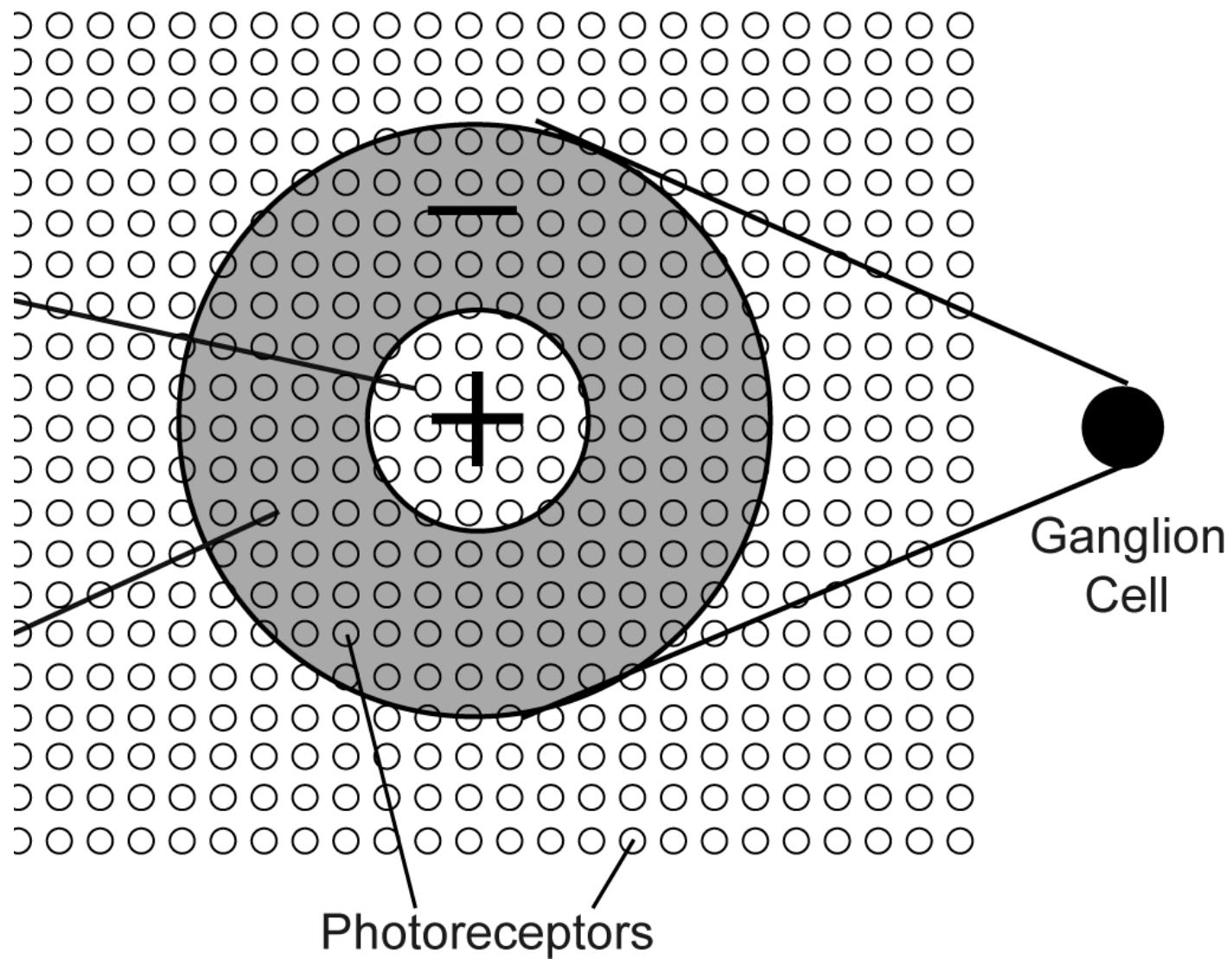


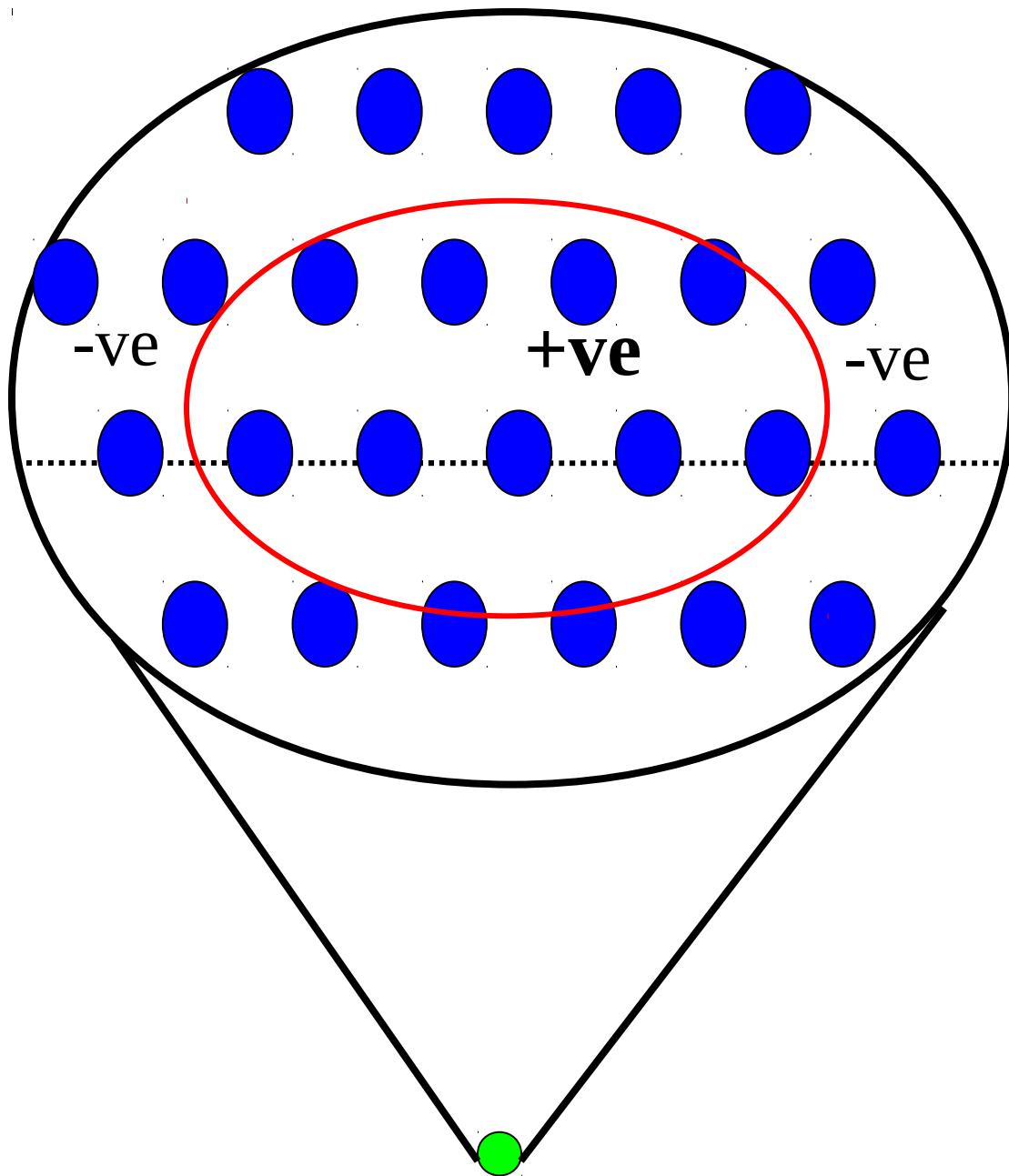


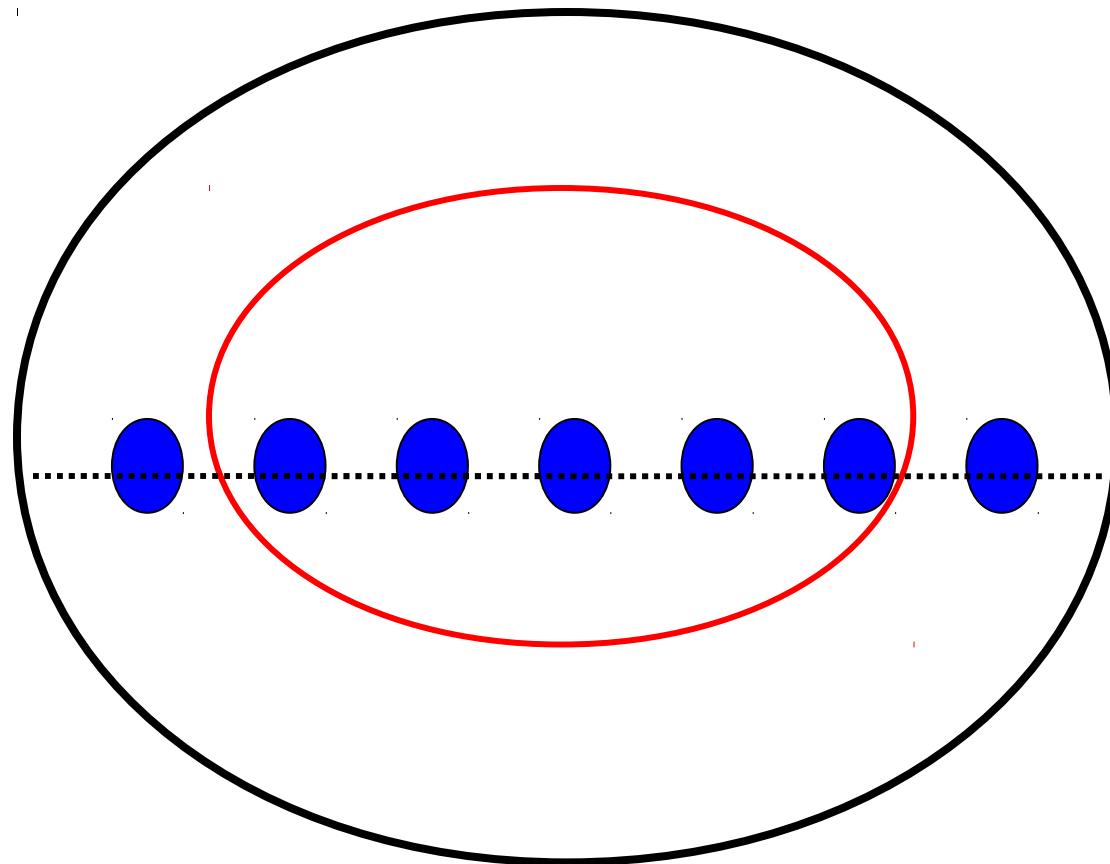


# Receptive field of Ganglion cell



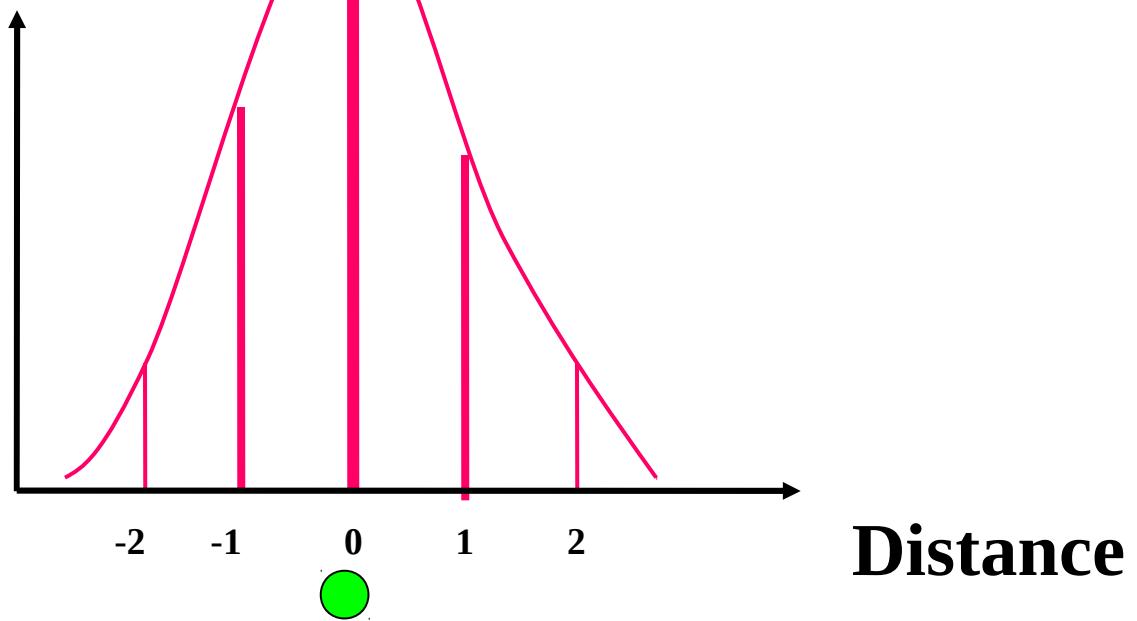








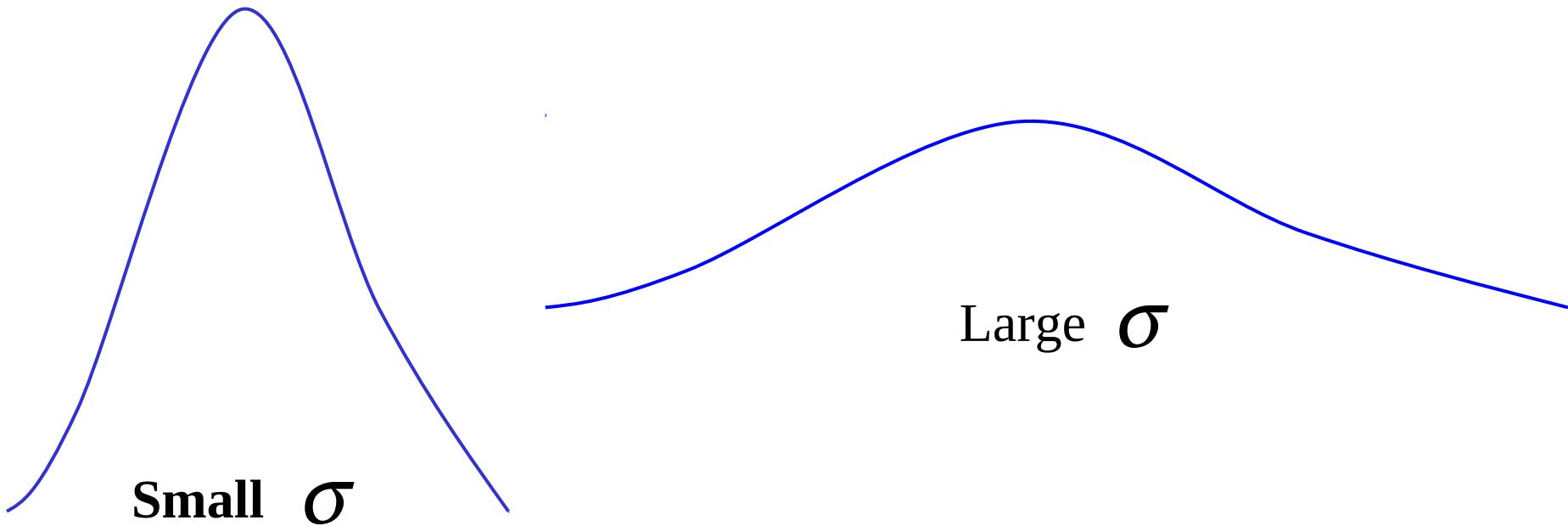
Intensity of  
signal

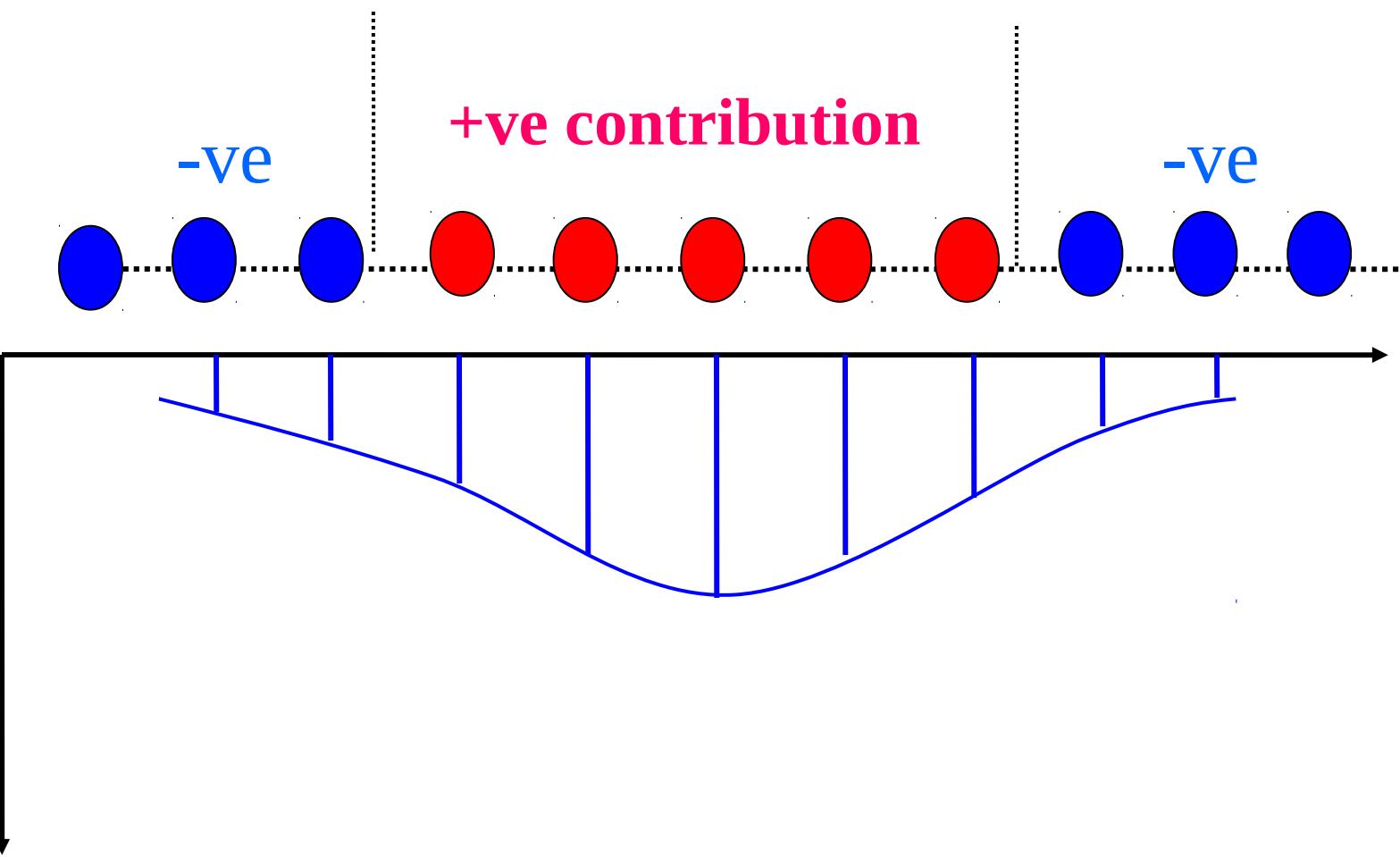


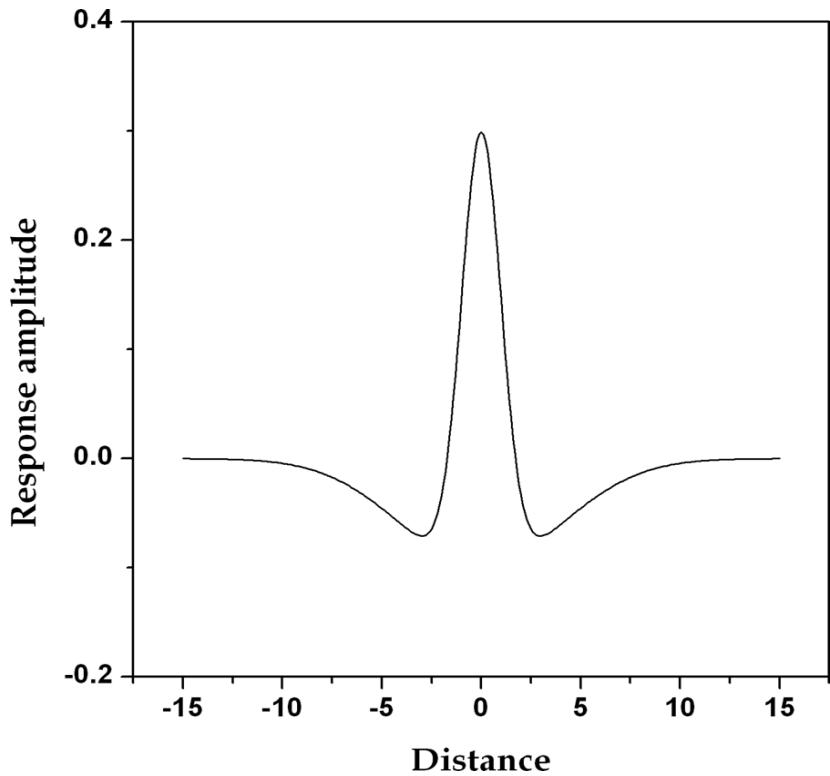
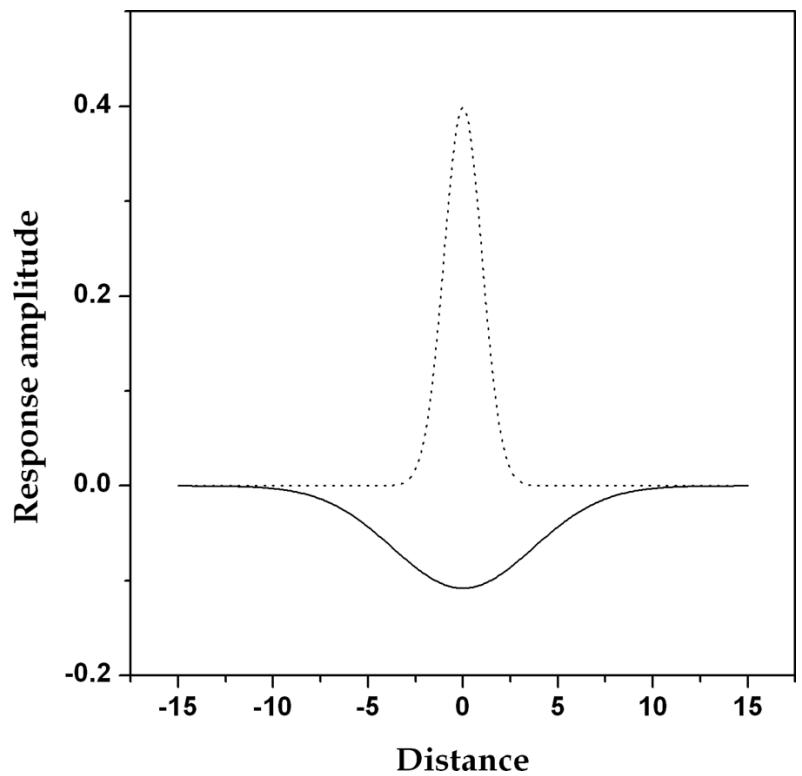
**A Gaussian function, whose mean value is kept at zero, is given by:**

$$g(x, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

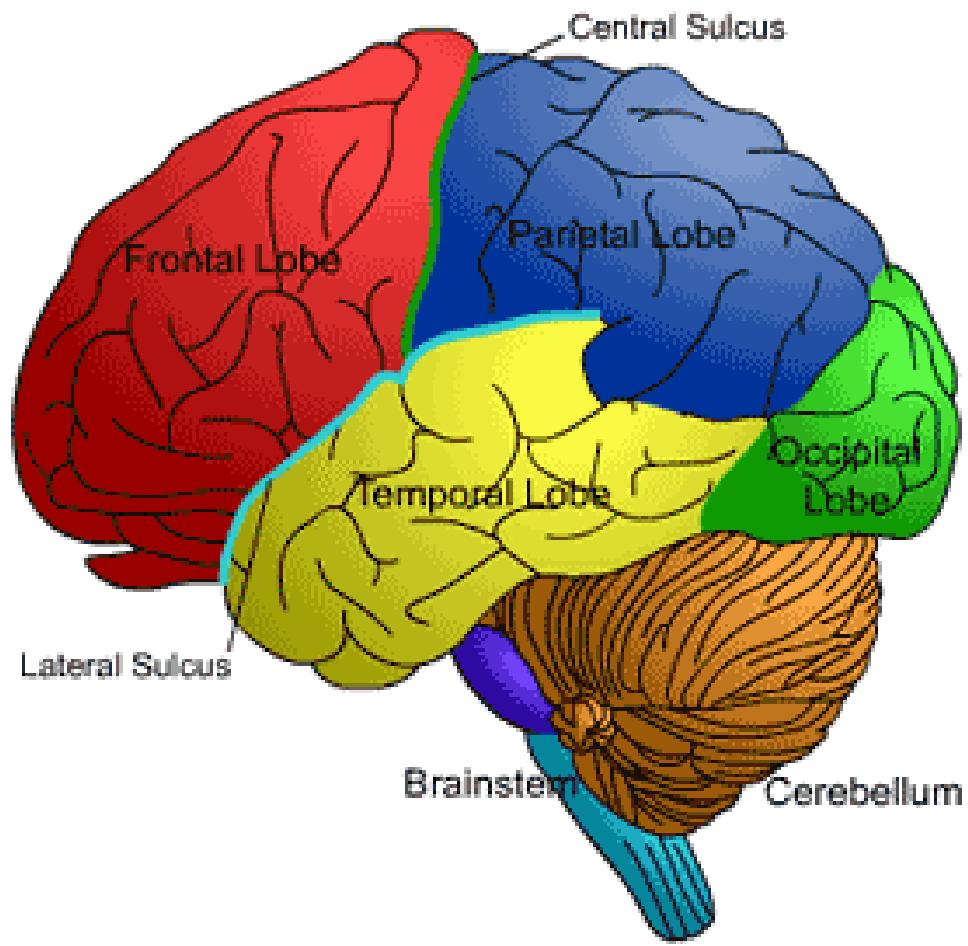
**where  $\sigma$  is called the standard deviation**

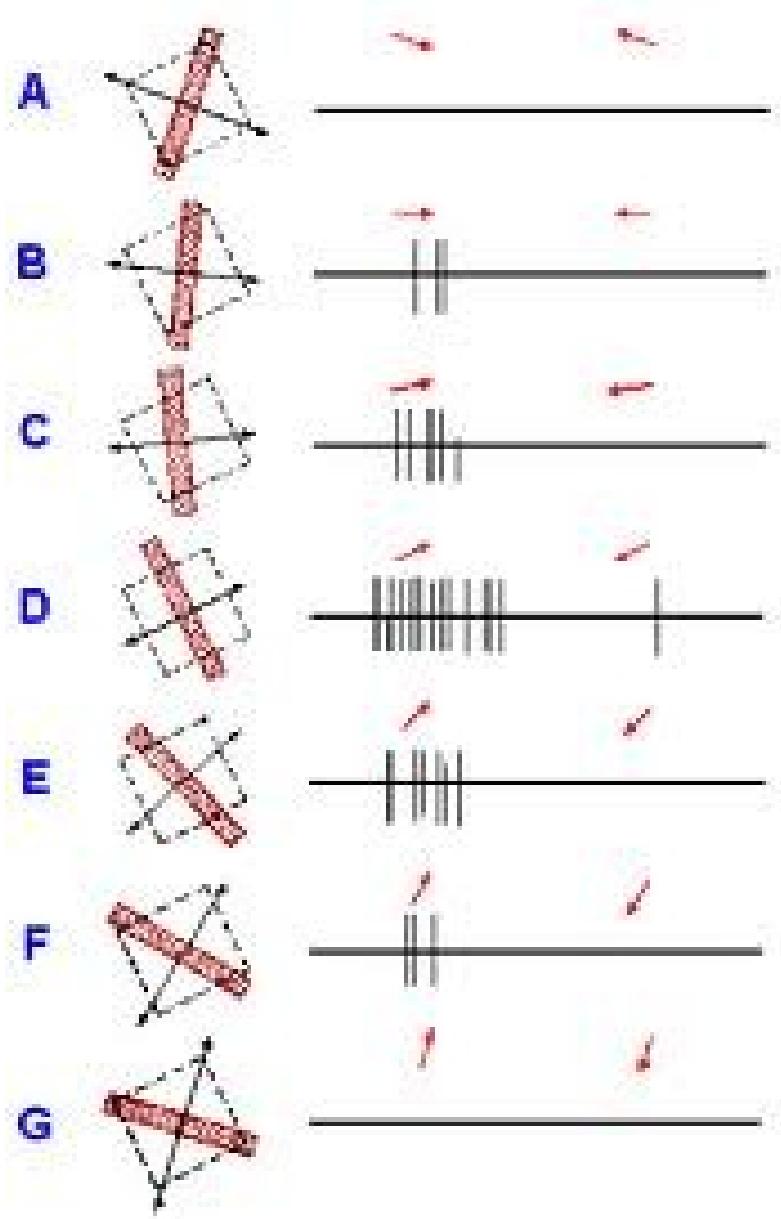


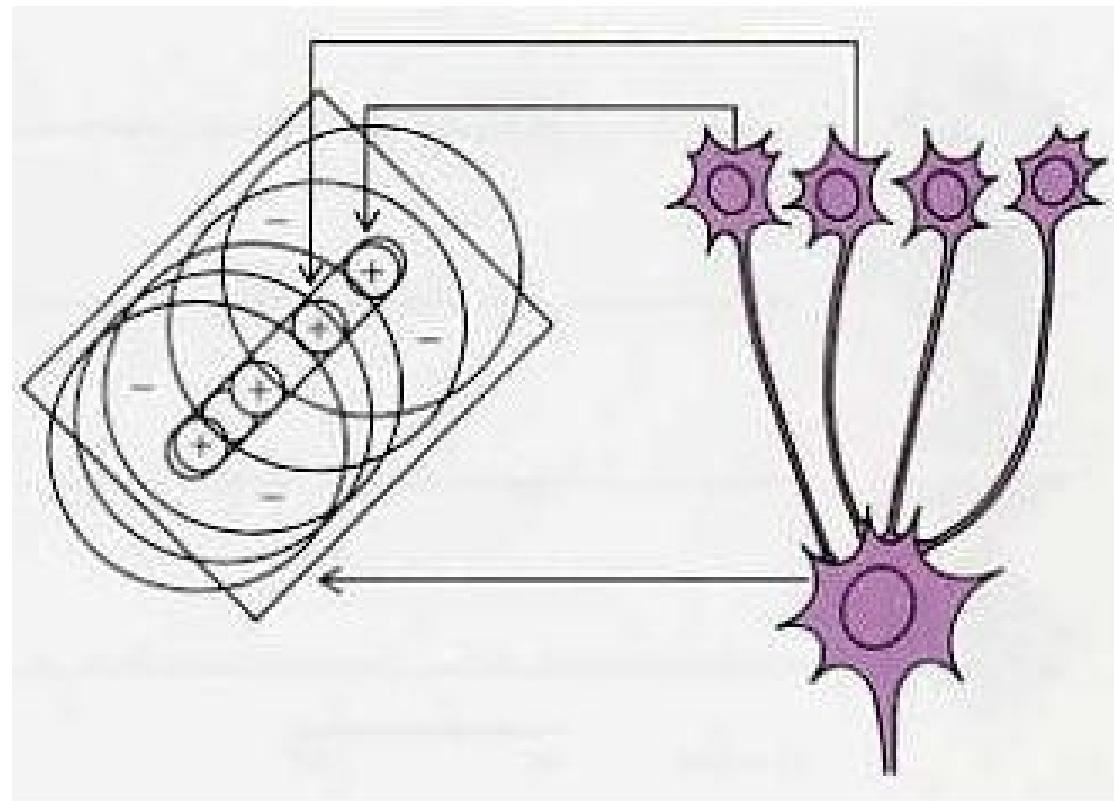


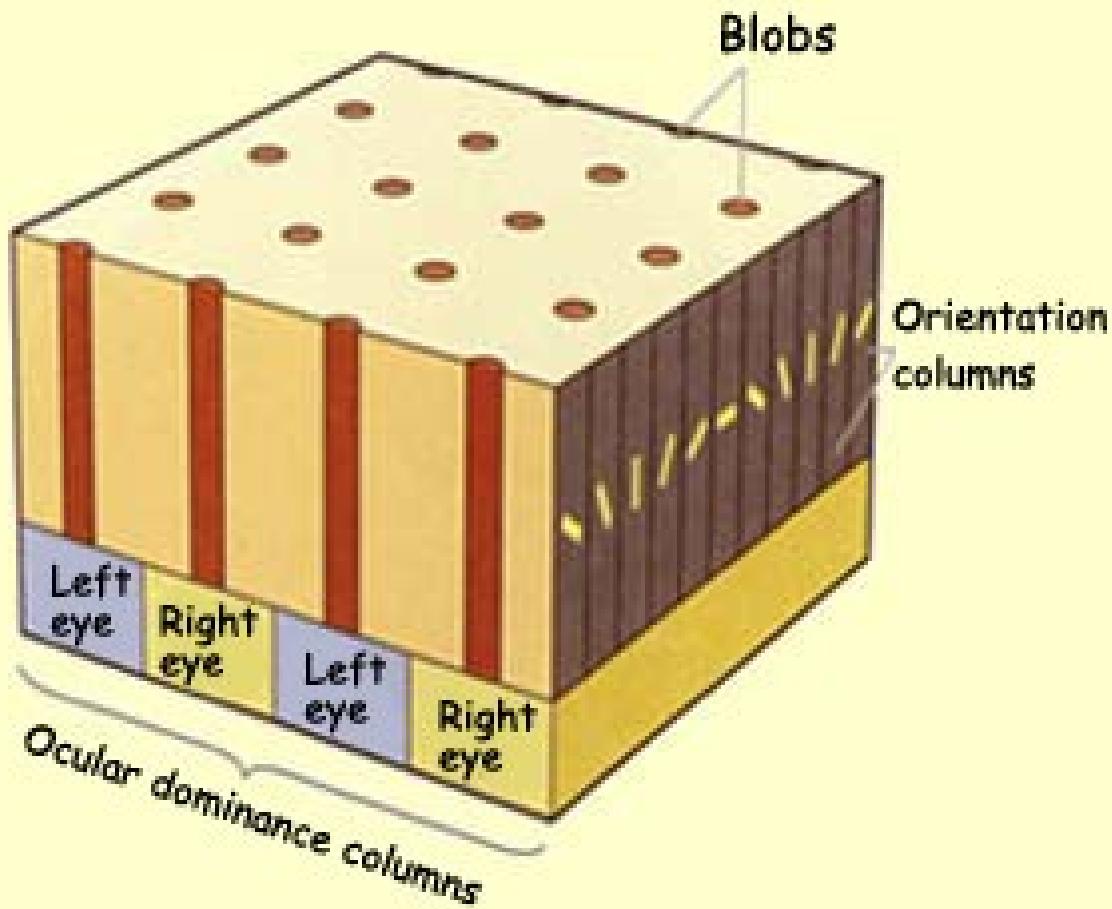


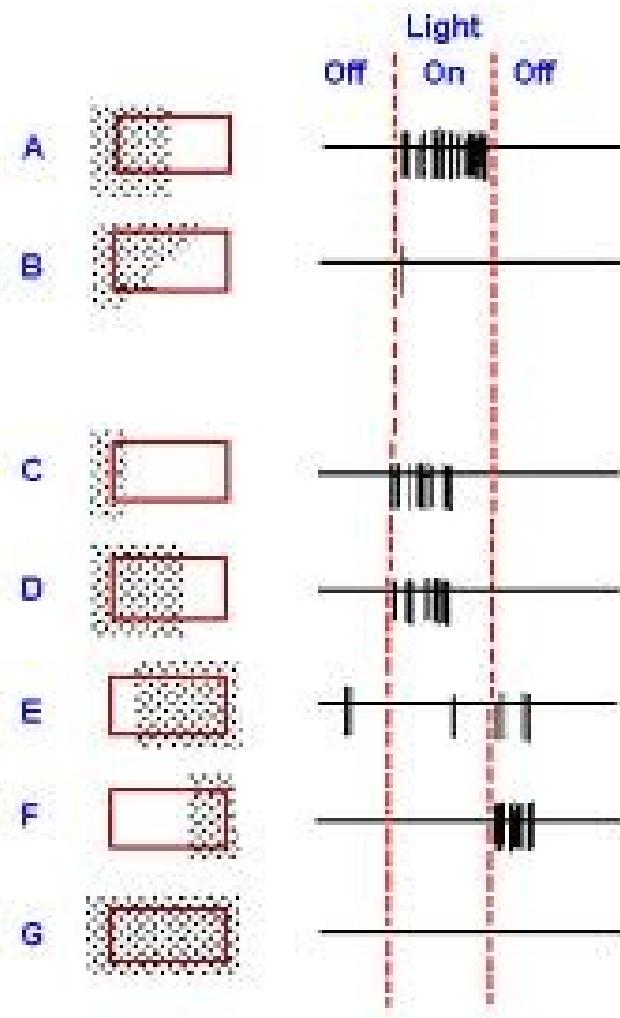
$$DOG(\sigma_1, \sigma_2) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{x^2}{2\sigma_1^2}} - \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{x^2}{2\sigma_2^2}}$$

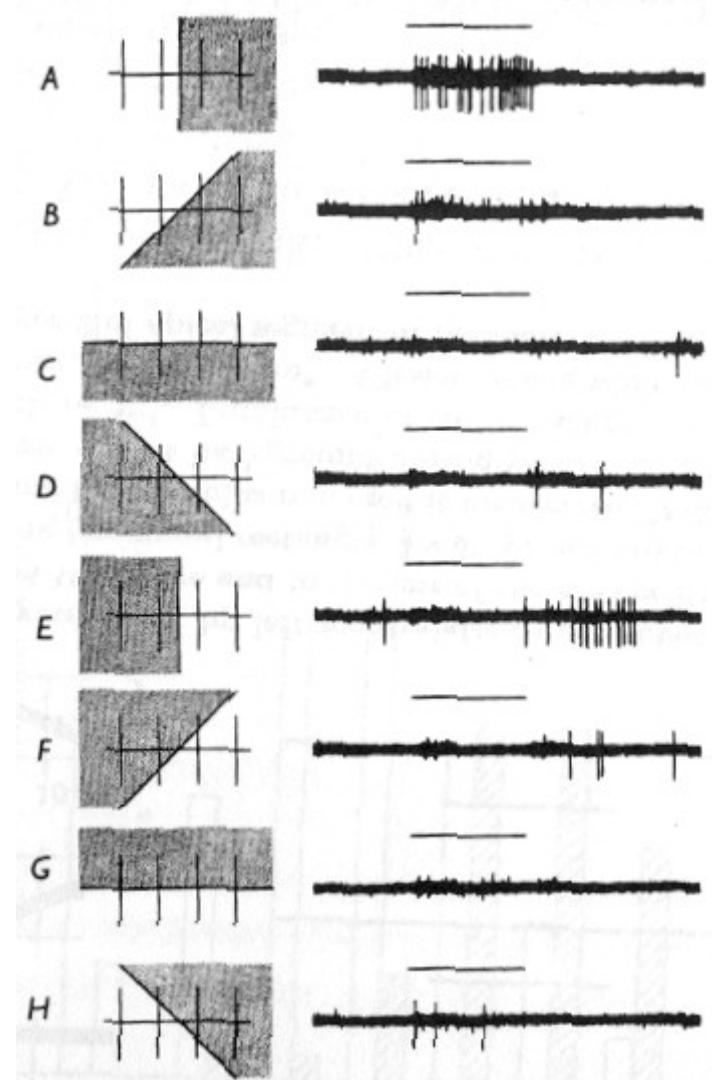


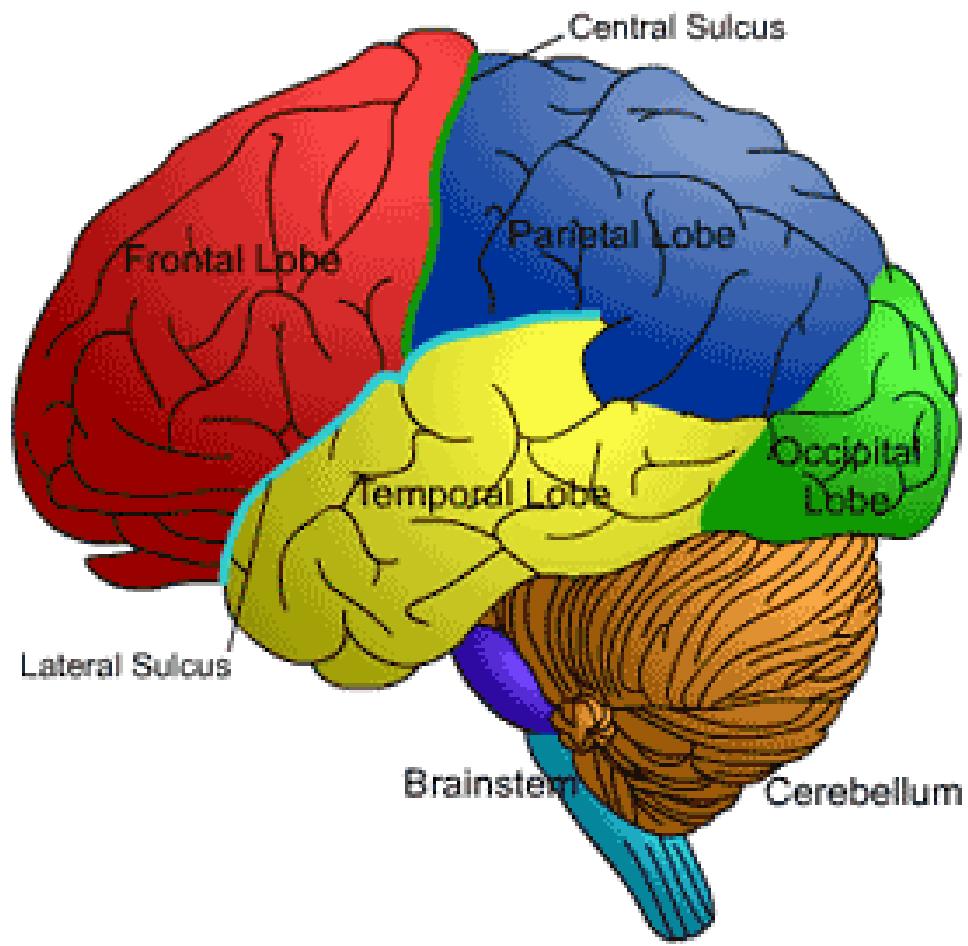


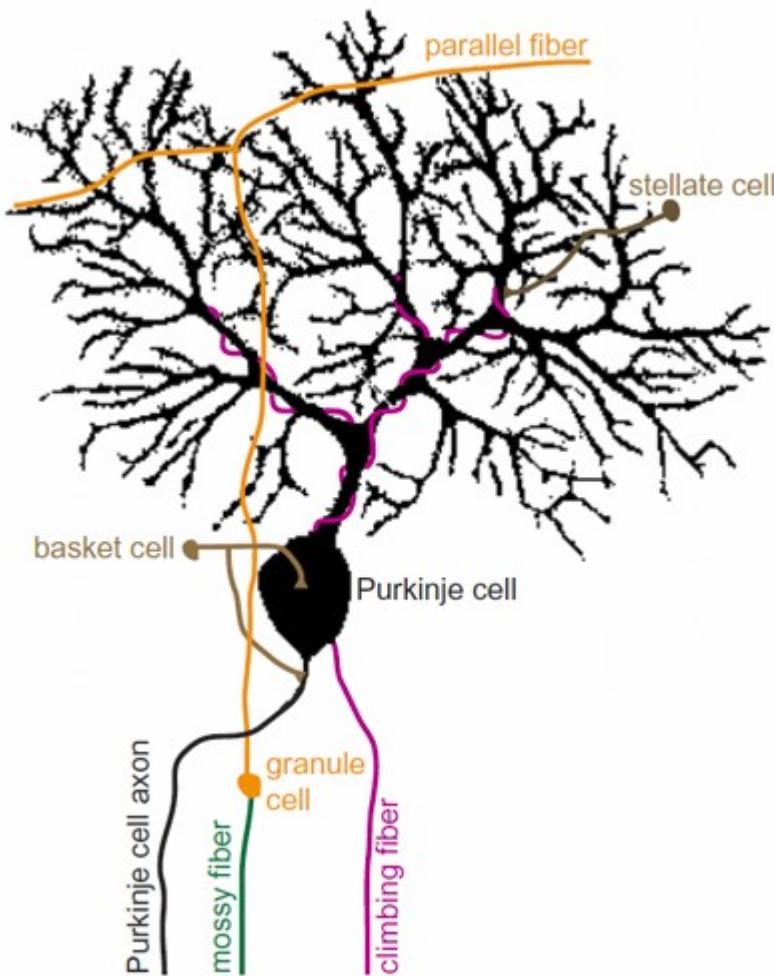


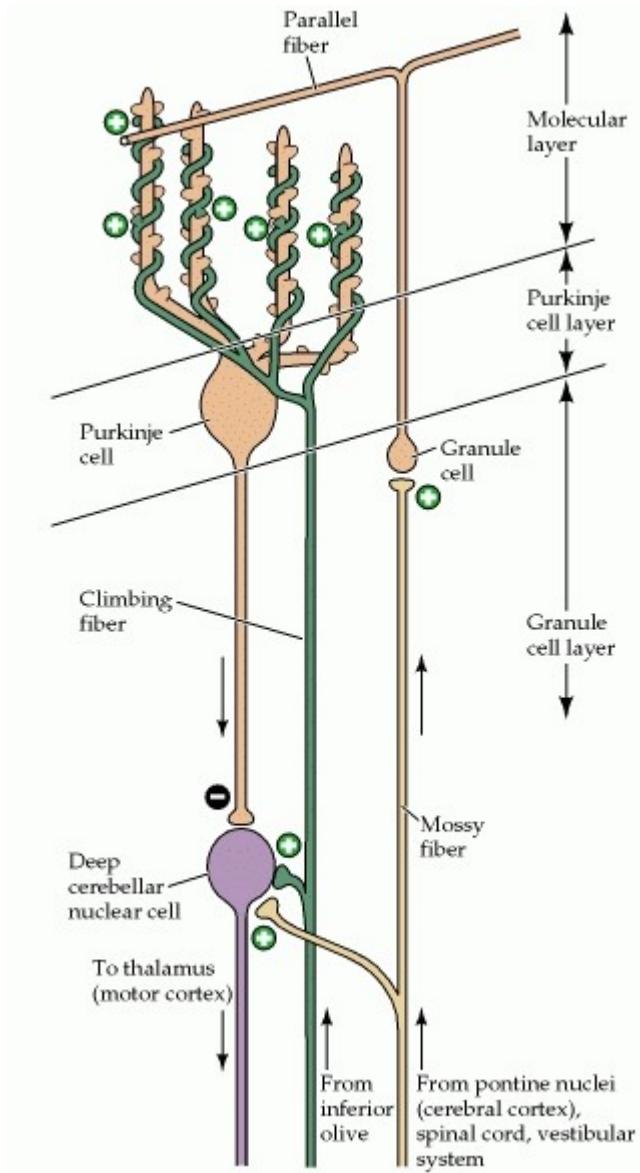


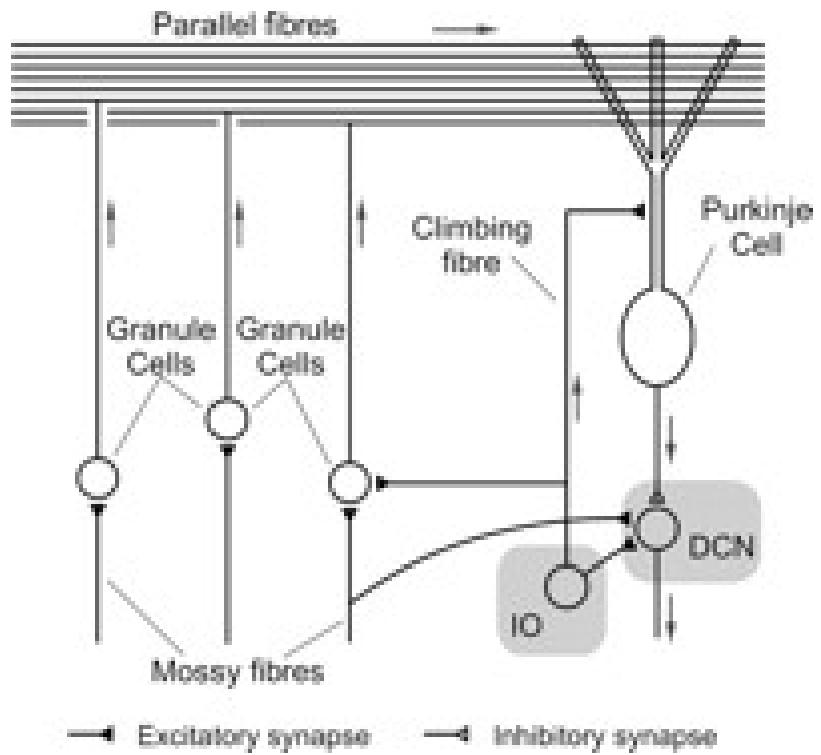












**Thank  
you**