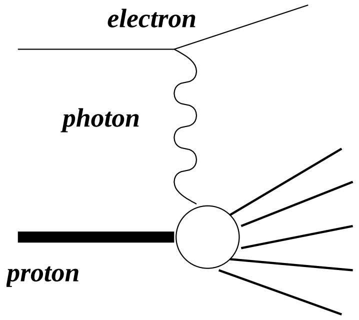
Understanding Hadron Structure

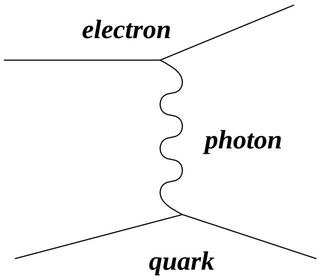
A. Harindranath (SINP)

Plan

- Inelastic lepton-hadron scattering - a tool for revealing constituents of matter
- Operational meaning of constituent picture: Infinite momentum frame and partons
- Quantum chromo dynamics
- Light front dynamics
- Summary

Constituent Picture of Matter





Tool: Deep Inelastic Lepton-Hadron Scattering (DIS):

P Target four momentum

q Virtual photon four momentum

$q = (q^0, \mathbf{q}), \quad q^2 < 0$. Let $Q^2 = -q^2$, $\nu = P.q$

Define $x = \frac{Q^2}{2\nu}$, Bjorken scaling variable.

Differential cross section $\propto W(Q^2, \nu)$, hadron structure function.

Extracted from experimental data.

Tells us about the structure of hadron in detail knowing the nature of interaction responsible for scattering.

Consider a nuclear target.

At very low resolution,

$\frac{1}{\sqrt{Q^2}} \gg Radius_{Nucleus}$

$q^0 \approx Nuclear\,energy\,level\,spacing$

Data shows elastic scattering from nucleus

excitation of nuclear energy levels

But as $\frac{1}{\sqrt{Q^2}} \ll Radius_{Nucleus}$

$q^0 \gg nuclear\,level\,spacing$

yet $\frac{1}{\sqrt{Q^2}} \gg Radius_{Nucleon}$

Data shows a quasi-elastic peak which corresponds to elastic scattering from nucleons, point-like constituents of the nucleus.

For point-like nucleons, only elastic scattering can occur.

In the impulse approximation, we expect the structure function to scale.

Scaling function tells us about the momentum distribution of nucleons in the nucleus.

As $q^0 \approx Nucleon\, energy\, level\, spacing$
data shows excitation of nucleon resonances
But as $\frac{1}{\sqrt{Q^2}} \ll Radius_{Nulceon}$
$q^0 \gg Nucleon\, energy\, level\, spacing$
constituents of the nucleon are revealed.
For point-like constituents, only elastic scattering can occur.
In the impulse approximation, we expect the data to scale.
Existence of scaling $\rightarrow$ point-like constituents inside the nucleon - partons (Feynman)
$W(Q^2, \nu) \rightarrow F(\frac{Q^2}{2\nu}) = F(x)$
Scaling function tells us about the electric charges, spins, and momentum fraction distribution of the constituents of the nucleon.
Partons are fractionally charged! (quarks)
From structure function studies, quarks carry only half the momentum of the nucleon. Rest carried by gluons (stuff that bind quarks).

Operational Meaning of Constituent Picture

Distribution of constituents present at any instant in the system (wavefunction)
Requires probing the system over a time duration small on the scale of internal motion of constituents
Natural for nonrelativistic systems But nucleon is a relativistic system
It seems that only a fuzzy picture of the instantaneous state of the nucleon could be obtained. Feynman to the rescue!
Wavefunction of a relativistic system has non-trivial frame dependence
Normally thought of as bad news But turn it into good news
Therefore, one may hope that certain properties of the system are more easily seen from the wave function in one frame of reference than in another.

FEYNMAN: Study hadron in a frame of reference in which it moves with nearly the speed of light

Due to time dilation, internal motion of the constituents slow down!

$P \to \infty$ [Infinite momentum frame]

Note only does the internal motion of the constituents slow down, transverse dynamics looks non-relativistic! Consider energy momentum relation for a free relativistic particle:

$E = \sqrt{k^2 + m^2}$. $k^2 = k_L^2 + k_T^2$, $k_T^2 = k_x^2 + k_y^2$.

For large k_L, $E \simeq k_L + \frac{1}{2}\frac{k_T^2+m^2}{k_L}$.

Wave function in terms of longitudinal momenta k_L, transverse momenta k_T of the constituents.

$k_L = \eta P$, k_T small

Simple kinematical considerations $\Longrightarrow$

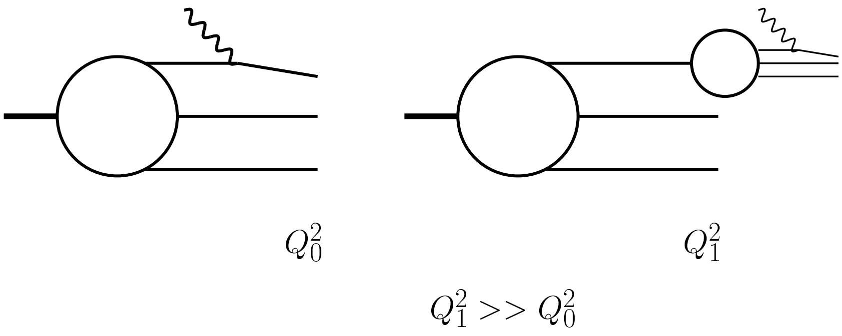
longitudinal momentum fraction

$\eta = x$, Bjorken scaling variable.

Constituents of the hadron = partons

Scaling function $F(x) = xq(x)$

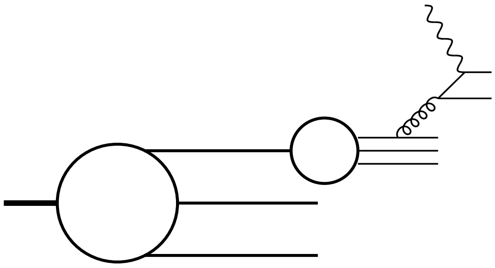
$q(x)$ Number density of partons with longitudinal momentum fraction between x and $x + dx$



Quasi Elastic Scattering:
Data implies finite number of constituents in the nucleus $\Longrightarrow$ One can ignore the dynamical aspects of the *stuff* responsible for nuclear binding $\Longrightarrow$ Non-relativistic character of the system

Deep Inelastic Scattering:
Data implies infinite number of constituents in the nucleon

Scaling is not perfect, $F(x, Q^2)$ (logarithmic Q^2 dependence) $\Longrightarrow$ Dynamical aspects of the *stuff* responsible for nucleon binding cannot be ignored $\Longrightarrow$ Relativistic character of the system.



Quantum Chromo Dynamics

Problems! Problems!

Problem 1: In the parton picture of DIS, at very high energy, nucleon behaves as a collection of almost free constituents. (Scaling violations very weak)

Can one justify parton picture in Quantum Field Theory?

In Quantum Electro Dynamics, Coulomb potential between two electrons $\sim \frac{\alpha}{r}$.

In momentum space $\sim \frac{\alpha}{q^2}$.

Quantum corrections $\sim \frac{\alpha(q^2)}{q^2}$.

$\alpha(q^2)$ increases with increasing q^2. $\alpha(r)$ grows with decreasing r.

For parton picture to emerge, we need the opposite behaviour: coupling $\alpha_s(q^2) \to 0$ as $q^2 \to \infty$.

Theory of strong interactions should possess Asymptotic Freedom.

In 1973 it was discovered that Quantum Chromo Dynamics (QCD) has asymptotic freedom.

Problem 2: Free quarks and gluons have never been observed in nature. To explain this absence QCD should have the property of confinement.

Quarks and gluons are permanently confined inside hadrons.

What happens when we try to take the quarks apart?

Consider the simplest system: a meson containing a quark and an antiquark.

Just as when we try to seperate the north pole and south pole of a bar magnet, result is two bar magnets each with its own north and south pole, trying to separate the quark and antiquark in a meson, results in two mesons!

Does QCD have confinement?

Evidence for confinement comes from putting the theory on a space-time lattice and performing numerical calculations on supercomputers!!

Solving QCD: Still an ongoing problem

How to calculate $q(x)$?

How to calculate parton wavefunctions???

Hamiltonian approach is the most direct route to wavefunctions $\Longrightarrow$

We need to develop Hamiltonian approach to Quantum Chromo Dynamics, the underlying theory of strong interactions.

We want to keep the constituent picture very much alive, even though we want to describe the nucleon, a relativistic system

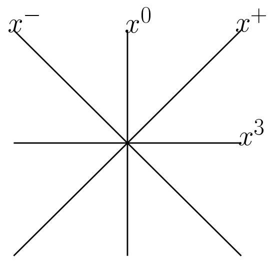
What we need to do is

formulate the problem so as to make

nonrelativistic intuitions relevant to relativistic field theory.

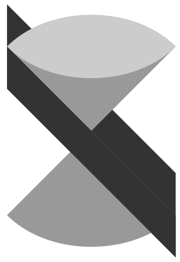
How is this possible?

Abandon equal time dynamics and embrace



Light Front Dynamics Dirac, RMP (1949)

Light Front Dynamics



$x^{\pm} = x^0 \pm x^3,\; x^{\perp} = (x^1, x^2)$

x^+ "time", x^- longitudinal coordinate

$x^2 = x^+ x^- - (x^{\perp})^2$

$[\; x^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \;]$

$k.x = \frac{1}{2}k^+ x^- + \frac{1}{2}k^- x^+ - k^{\perp} \cdot x^{\perp}$

For an on mass shell particle $k^2 = m^2 \rightarrow$

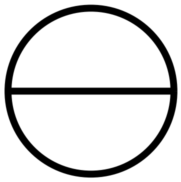
energy $k^- = \frac{(k^{\perp})^2 + m^2}{k^+}$.

longitudinal momentum $k^+ = k^0 + k^3 \geq 0$

Dependence on $k^{\perp}$ just like in the nonrelativistic dispersion relation!!

We haven't made any infinite momentum approximation!

Apparent Triviality of the Vacuum



In conventional
Quantum Field Theory
Only restriction
$\Sigma_i k_i^+ = 0$

In Light front
Field Theory
Only restriction
$k_i^+ = 0$

If $k^+ = 0$ is removed $(k_i^+ \rangle \epsilon)$ Fock space vacuum $\mid 0\rangle$ is an eigenstate of the light front QCD Hamiltonian $\Longrightarrow$ Constituent picture

To build the hadron, we need not first construct the ground state (Vacuum) of the theory.

Further, Longitudinal boost becomes scaling, transverse boosts become Galilean boosts

Thus boost become kinematical (non-relativistic)

In a relativistic theory, internal motion and the motion associated with center of mass can be separated $\Longrightarrow$
Can construct boost invariant wavefunctions

Furthermore, the relativistic fermion is described by a two component field.

We start seeing the "remarkably non-relativistic character of the extreme relativistic limit" (Bjorken)

Detailed calculations revealed that structure functions in deep inelastic scattering are equal light front time correlation functions in QCD
just as structure functions in quasi elastic scattering are equal time correlation functions in non-relativistic physics.

Summary

We briefly described how successive layers of matter are revealed by inelastic scattering. Polarized and unpolarized deep inelastic lepton-hadron scattering experiments continue to reveal details of the quark-gluon structure of the hadron. The theory behind this structure is Quantum Chromo Dynamics (QCD) with the unusual properties of asymptotic freedom and confinement. Understanding the low energy structure of QCD is still a highly challenging problem. Before the advent of QCD, in order to have an intuitive understanding of deep inelastic scattering experiments, Feynman utilized the notion of infinite momentum frame and put forth the parton picture of the hadron. We discussed salient features of light front dynamics which provides the theoretical basis for Feynman's partons in QCD.