Topological sector of two dimensional ϕ^4 theory in Discrete Light Cone Quantization

A. H (Theory Division, SINP, Calcutta, INDIA)
in collaboration with
Dipankar Chakrabarti, L'ubomir Martinovič, James P. Vary

Dipankar Chakrabarti, A.H, L'ubomir Martinovič, James P. Vary, Phys. Lett. **B582**, 196 (2004), hep-th/ hep-th/0309263.
Dipankar Chakrabarti, A.H, James P. Vary, hep-th/0504094, to appear in PRD.

SLAC Theory Seminar
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Plan

- √ Issues
- √ Discrete Light Cone (Front) Quantization (DLCQ)
- √ DLCQ Hamiltonian for ϕ_2^4
- √ Spectrum - Degeneracy of States
- √ Extraction of Kink Mass
- √ Other Observables
- √ A Transition in the Spectrum at Strong Coupling
- √ Other Signals for Transition
- √ Implication
- √ Summary

Issues:

Motivated by the work of Rozowsky and Thorn, PRL **85**, 1614 (2000).

Can one achieve Spontaneous Symmetry Breaking (SSB) without zero momentum mode and with a finite Fock basis? (SSB cannot occur in a system with finite degrees of Freedom)

Can one calculate the mass and other properties of the kink in a finite Fock basis? (In variational calculations, kink can be approximated by coherent states)

Can one investigate the structure of low lying states in the spectrum at strong coupling in the Fock language? (Can one look for signatures of the onset of kink condensation which is believed to be the mechanism for symmetry restoring phase transition?)

Light Front (Cone) Quantization (1+1 dimensions)

Dirac, RMP (1949)

$x^{\pm} = x^0 \pm x^3$ x^+ " time ", x^- longitudinal coordinate

$x^2 = x^+ x^-$ $[x^2 = (x^0)^2 - (x^1)^2]$

$k.x = \frac{1}{2} k^+ x^- + \frac{1}{2} k^- x^+$

On mass shell particle $k^2 = m^2 \longrightarrow$ energy $k^- = \frac{m^2}{k^+}$

no square root

longitudinal momentum $k^+ \geq 0$

Longitudinal boost becomes scale transformation. Thus boost becomes kinematical (non-relativistic).

In a relativistic theory, internal motion and the motion associated with center of mass can be trivially separated.

We start seeing the " remarkably non-relativistic character of the extreme relativistic limit " (Bjorken)

Discrete Light Cone Quantization (DLCQ)

DLCQ as a practical approach to solve Quantum Field Theory

H.-C. Pauli and S. J. Brodsky, Phys. Rev. D **32**, 1993, 2001 (1985)

Also

C. B. Thorn, Phys. Rev. D **17**, 1073 (1978)

T. Maskawa and K. Yamawaki, Prog. Theo. Phys. **56**, 270 (1976)

A. Casher, Phys. Rev. D **14**, 452 (1976)

Exploit the semi-positive definiteness of longitudinal momentum.

Compactify x^-: $-L \leq x^- \leq +L$.

With Anti Periodic Boundary condition (APBC),

$k^+ \to k_n^+ = \frac{n\pi}{L}, \quad n = 1, 3, 5, \ldots$

Two Dimensional ϕ_2^4 theory

Lagrangian density

$\mathscr{L} = \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4.$

The Hamiltonian density

$\mathscr{P}^- = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4.$

defines the Hamiltonian

$P^- = \int dx^- \mathscr{P}^- \;\equiv \frac{L}{2\pi}H$

where L defines our compact domain

$-L \leq x^- \leq +L.$

The longitudinal momentum operator is

$P^+ = \frac{1}{2}\int_{-L}^{+L} dx^- \partial^+\phi\,\partial^+\phi \equiv \frac{2\pi}{L}K$

where K is the dimensionless longitudinal momentum operator.

In DLCQ with Anti Periodic Boundary Condition, the field expansion has the form

$$\phi(x^-) = \frac{1}{\sqrt{4\pi}} \sum_n \frac{1}{\sqrt{n}} \left[a_n e^{-i\frac{n\pi}{L}x^-} + a_n^\dagger e^{i\frac{n\pi}{L}x^-} \right].$$

Here $n = \frac{1}{2}, \frac{3}{2}, \ldots$.

(No zero momentum mode.)

DLCQ Hamiltonian

The normal ordered Hamiltonian

$$H = -\mu^2 \sum_n \frac{1}{n} a_n^\dagger a_n +$$
$$\frac{\lambda}{4\pi} \sum_{k\leq l, m\leq n} \frac{1}{N_{kl}^2} \frac{1}{N_{mn}^2} \frac{1}{\sqrt{klmn}} a_k^\dagger a_l^\dagger a_n a_m \delta_{k+l,m+n} \; +$$
$$\frac{\lambda}{4\pi} \sum_{k,l\leq m\leq n} \frac{1}{N_{lmn}^2} \frac{1}{\sqrt{klmn}} \left[a_k^\dagger a_l a_m a_n + a_n^\dagger a_m^\dagger a_l^\dagger a_k \right] \delta_{k,l+m+n}$$

with

$$N_{lmn} = 1, \; l \neq m \neq n,$$
$$= \sqrt{2!}, \; l = m \neq n, \; l \neq m = n,$$
$$= \sqrt{3!}, \; l = m = n,$$
$$N_{kl} = 1, \; k \neq l,$$
$$= \sqrt{2!}, \; k = l.$$

(Simplest structure for the Hamiltonian.)

Diagonalization

The Lanczos method is used in a highly scalable algorithm allowing us to proceed to sufficiently high values of K. All results presented here were obtained on clusters of computers (< 30 processors) using the Many Fermion Dynamics (MFD) code adapted to bosons

odd sector		even sector	
K	dimension	K	dimension
15.5	295	16	336
31.5	12,839	32	14,219
39.5	61,316	40	67,243
44.5	151,518	45	165,498
49.5	358,000	50	389,253
54.5	813,177	55	880,962
		60	1,928,175

Dimensionality of the Hamiltonian.

Spontaneous Symmetry Breaking:

Hamiltonian exhibits the $\phi \rightarrow -\phi$ symmetry:

Even and odd particle sectors are decoupled. In the absence of interaction (tachyonic theory), lowest excitations in the odd and even particle sectors consist of the maximum number of particles carrying the lowest allowed momentum $\rightarrow$

system is unstable in the continuum limit.

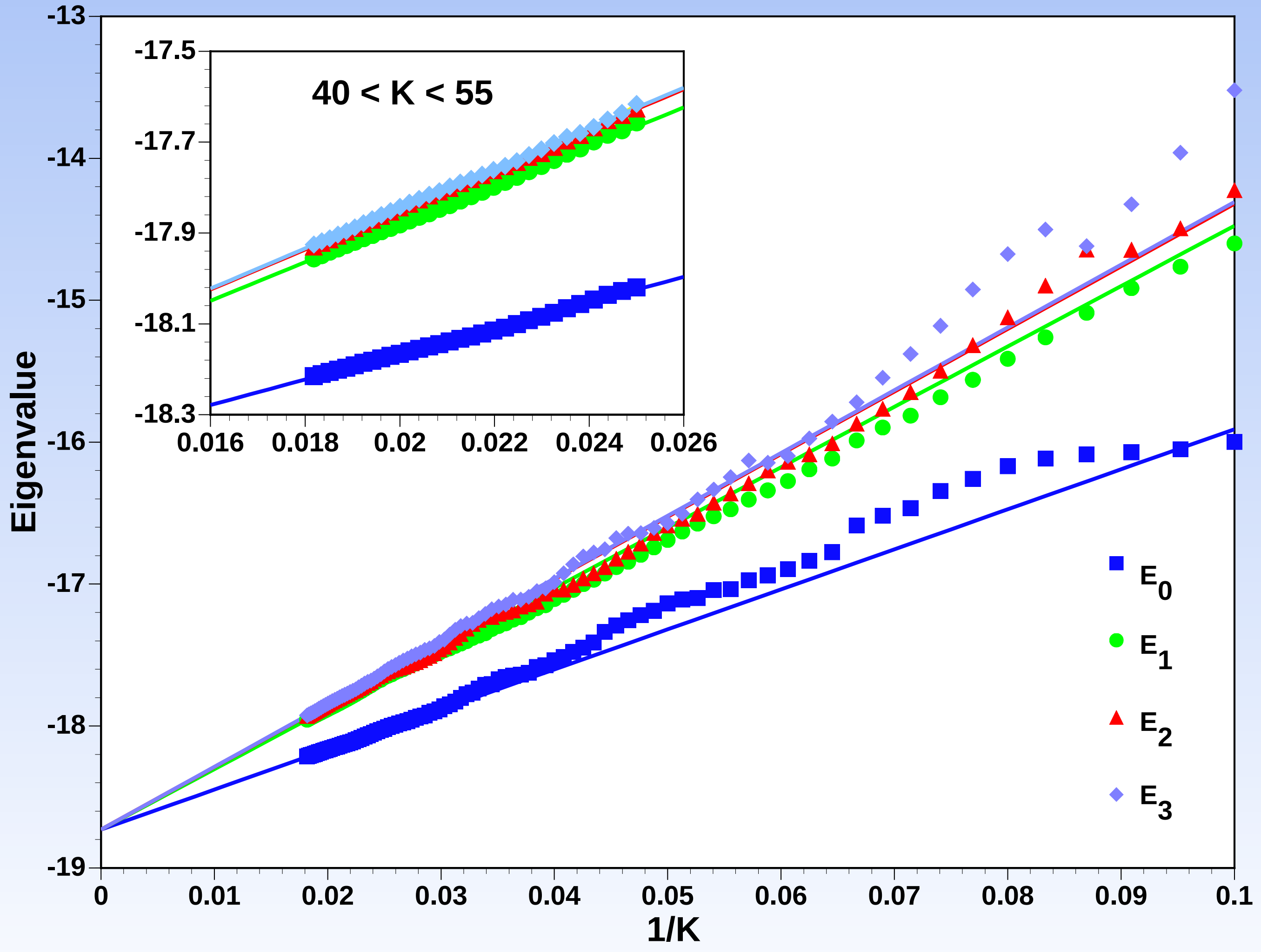
ϕ^4 interaction provides stability to the system. Thus, the possibility arises that excitations in the even and odd particle sectors become degenerate. (Rozowsky and Thorn)

Signal for SSB: look for degeneracy between even and odd sectors.

Denote the lowest excitations in the even and odd sectors by $\mid e\rangle$ and $\mid o\rangle$. Form the linear combinations,

$\mid \pm\alpha\rangle = \mid e\rangle \pm \mid o\rangle$.

Now it becomes possible for $\langle \pm\alpha \mid \phi \mid \pm\alpha\rangle$ to be non-zero.



40 < K < 55
Eigenvalue
1/K
E_0
E_1
E_2
E_3

Extraction of Kink Mass:

All the low-lying states have negative eigenvalues!

How to extract the particle masses?

Coherent State Variational Calculation (CSVC) (Rozowsky and Thorn) shows how to: Fit the data for the eigenvalue of H to the form

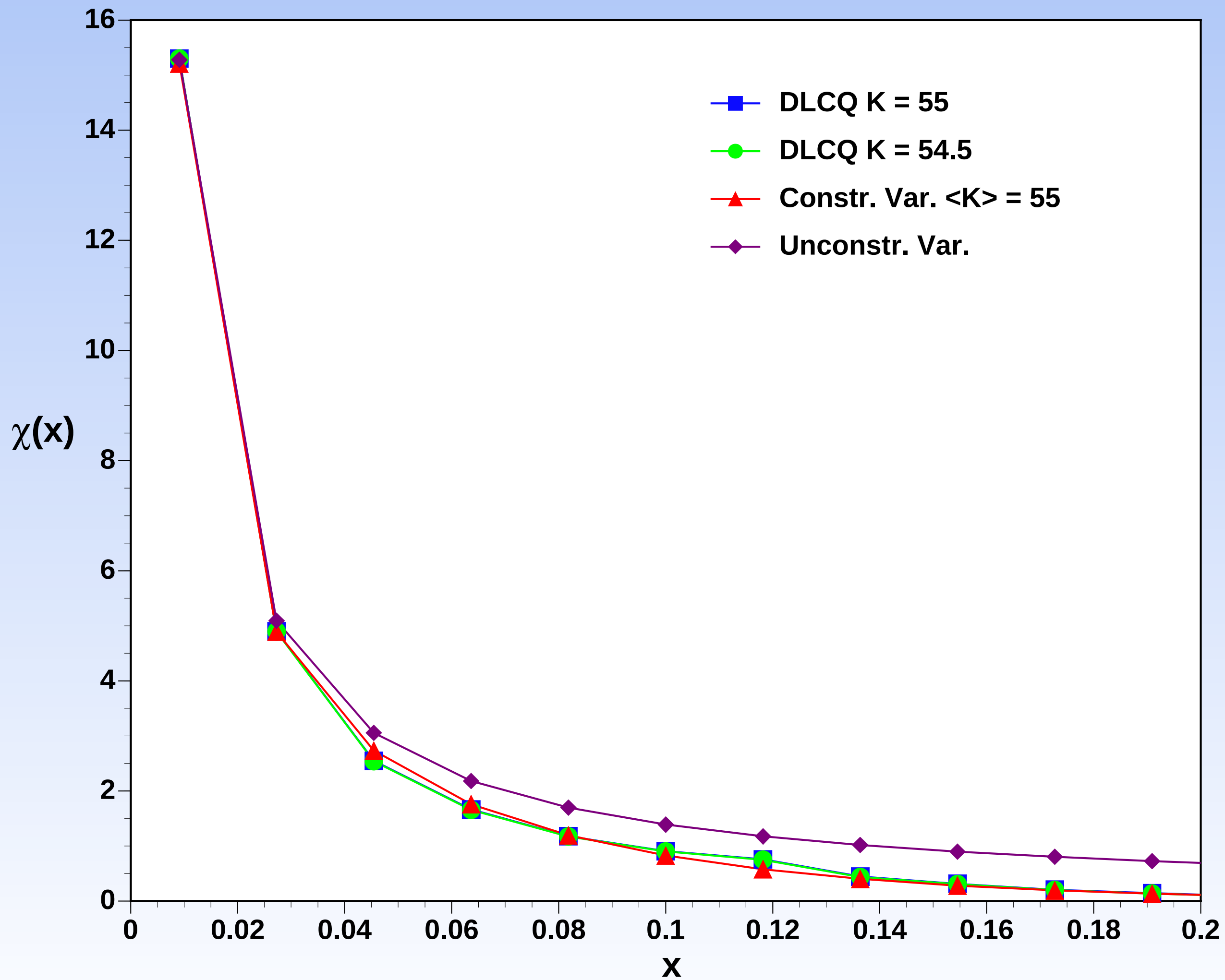
$C_1 + C_2/K$.

According to CSVC, C_1 is the vacuum energy density and $C_2^{1/2}$ is the kink mass.

λ	vacuum energy		soliton mass		
	class.	DLCQ	class.	semi-class.	DLCQ
1.0	-18.85	-18.73 $\pm$0.05	5.66	5.19	5.3 $\pm$0.2

Number density for the kink state

$$\chi(x) = \langle | \, a^{\dagger}(x) a(x) \, | \rangle \qquad x = \frac{n}{K}$$



Excited States:

In the semi-classical analysis, the lowest states, in order of excitation, are kink, excited kink, kink plus boson, and the continuum states.
Kink state yields a characteristic parton distribution which peaks at the lowest momentum mode available.

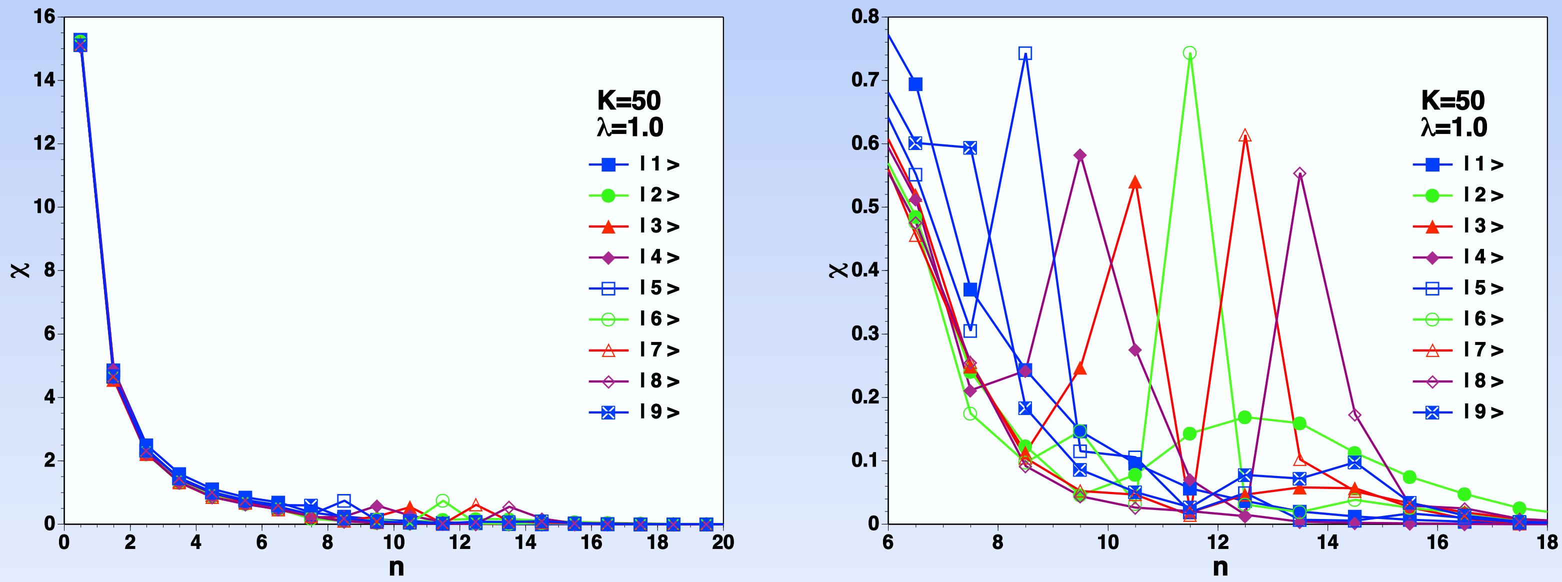
The next excitation is an excited kink for which also we expect and find a smooth distribution function which in addition to the sharp peak at lowest allowed momentum exhibits a broad and smooth peak.

The third and higher excitations are expected to be kink plus boson states. For a free kink plus boson state, we expect the minimum energy configuration with $x_{boson} = \frac{m_{boson}}{m_{kink}+m_{boson}}$.

Number Density of Excited States

a) χ versus n, the half-odd integer representing light front momentum with APBC, for the lowest nine excitations for $K = 50,\ \lambda = 1$. (b) Same as in (a) but showing the region from $n = 6$ to 18 in detail.

(a) (b)

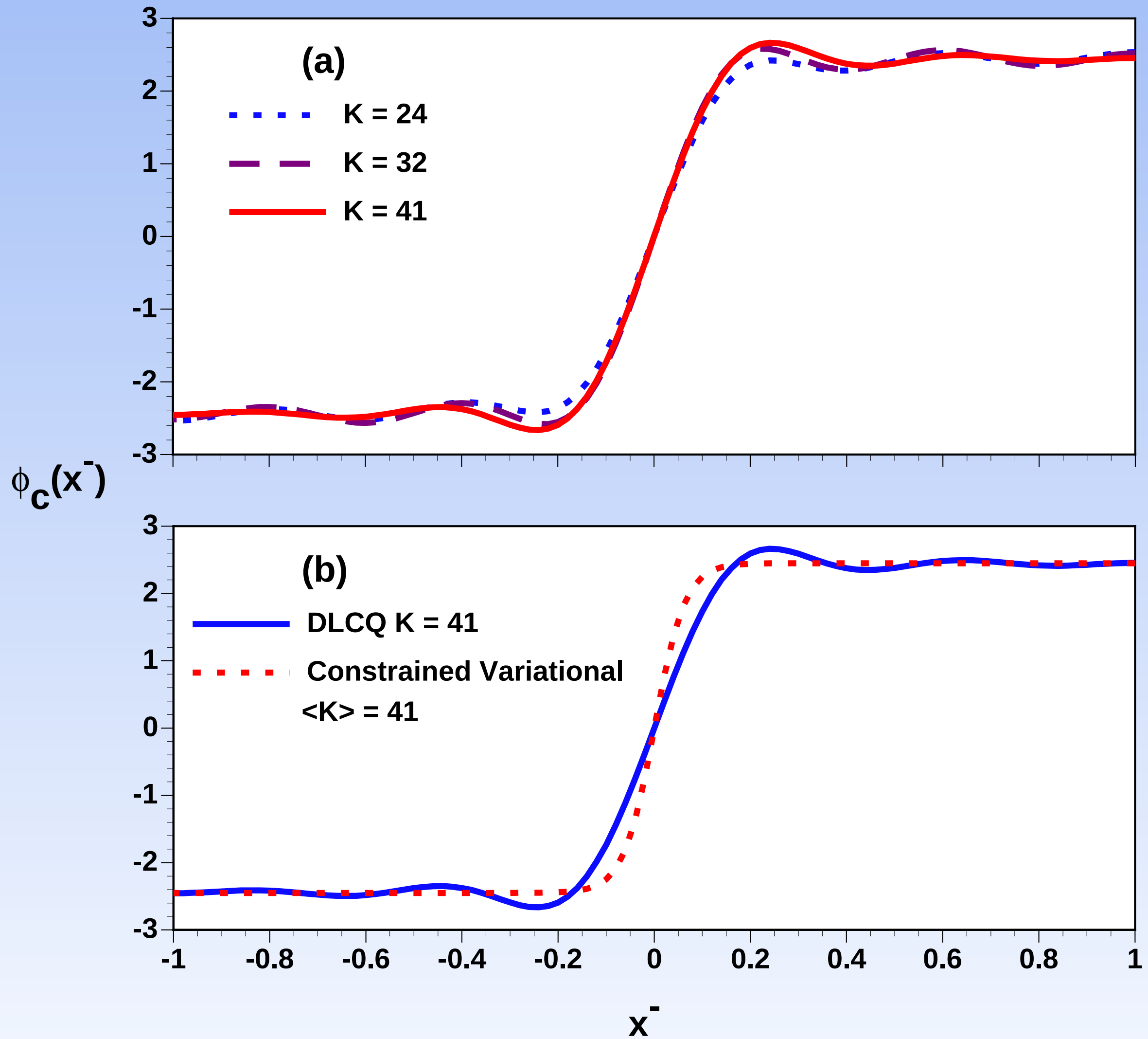


Fourier Transform of the Kink Form Factor

What is the observable in quantum field theory that is related to the classical kink solution?
According to Goldstone and Jackiw, the Fourier transform of the form factor of the lowest excitation, in the weak coupling (static) limit, corresponds to the classical kink solution. Let $\mid K\rangle$ and $\mid K'\rangle$ denote this state with momenta K and K'. In the continuum theory,

$$\int_{-\infty}^{+\infty} dq^+ exp\{-\tfrac{i}{2}q^+a\}\langle K' \mid \phi(x^-) \mid K\rangle = \phi_c(x^- - a).$$

In DLCQ, we diagonalize the Hamiltonian for a given $K = \frac{L}{2\pi}P^+$. For the computation of the form factor, we need the same state at different K values since $K' = K + q$.



(a)
K = 24
K = 32
K = 41
(b)
DLCQ K = 41
Constrained Variational
<K> = 41
$\phi_c(x^-)$
x^-

Extraction of the condensate $\langle | \phi | \rangle$:

How is the symmetry breakdown communicated to a system in the Infinite Momentum Frame if the vacuum and the system become decoupled?

"*Thus, however fast you run, you can't outrun the long arm of the vacuum*"

Kogut and Susskind, PR **8**, 75 (1973).

In our calculations, we haven't bothered about the vacuum state at all.

However, we can extract the value of the vacuum condensate. Drawing on the connection between the classical kink solution and the Fourier Transform of the kink form factor (kink profile), we extract $\langle | \phi | \rangle$ as the asymptotic ($x^- = \pm 1$ in units of L) intercept. For $\lambda = 1.0$, the intercept is 2.4 which is very close to the classical value.

Transition in the spectrum at strong coupling

At weak coupling, the values of observables extracted are close to classical or semi-classical results. As we increase the coupling, we observe drastic departure from semi-classical results. The lowest two energy levels cross each other.

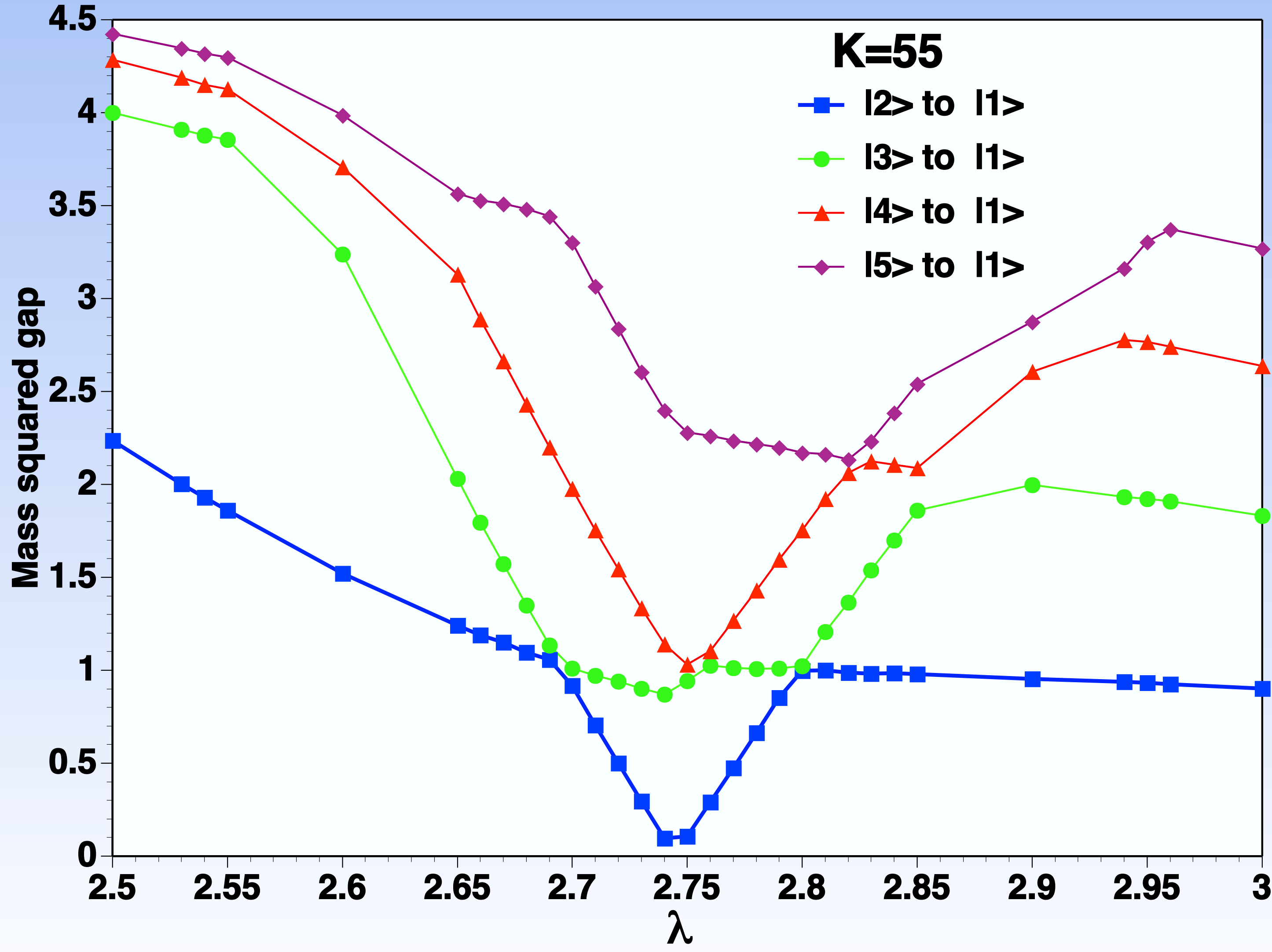
To gain further understanding of the nature of levels that cross, we examine the behaviour of two other observables:

1) $\langle | \int dx^- \; : \phi^2(x^-) : | \rangle$

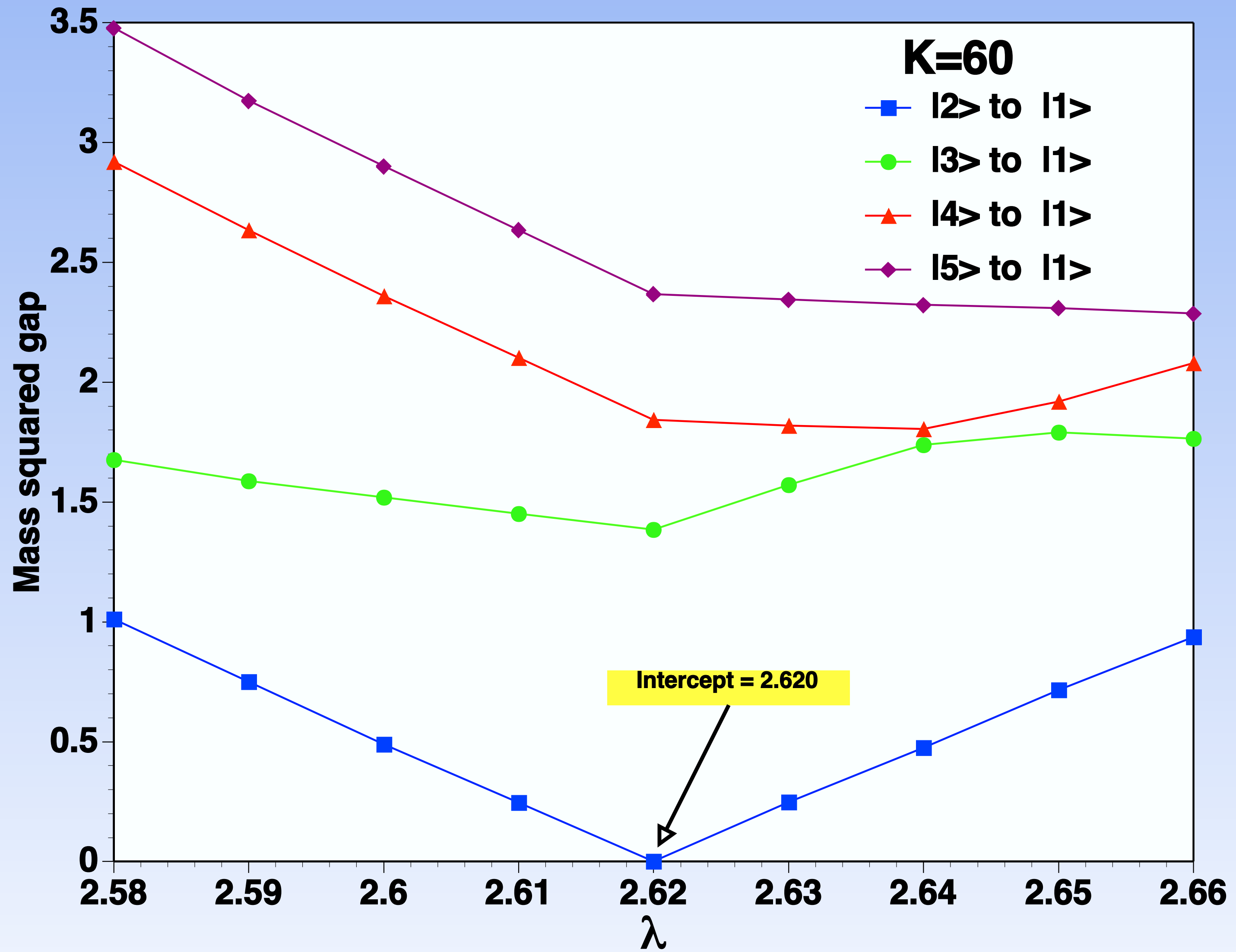
2) The number density

3) Fourier Transform of the form factor.

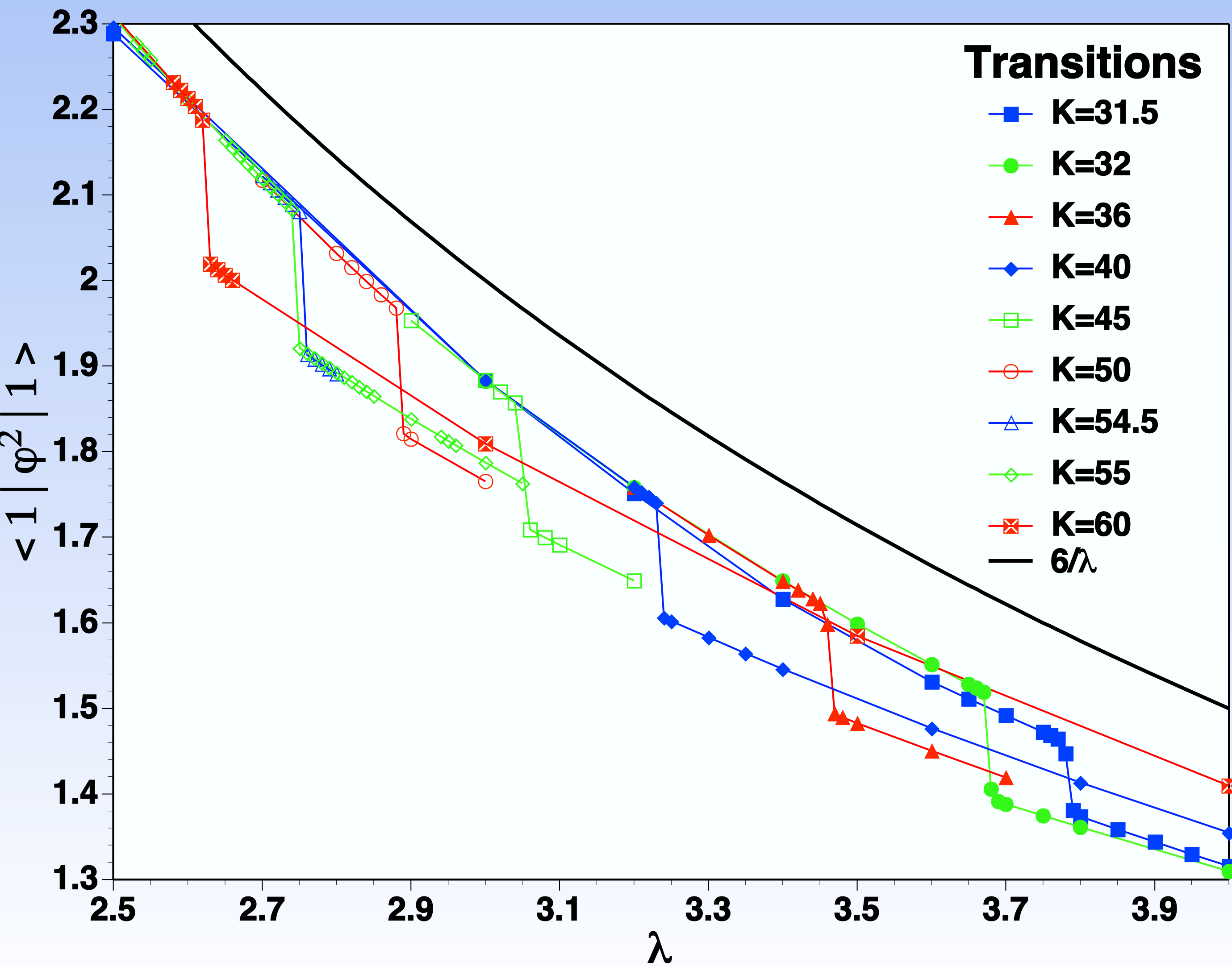
Mass^2 gap as a function of λ for K=55. All calculated results are connected by straight line segments to guide the eye.



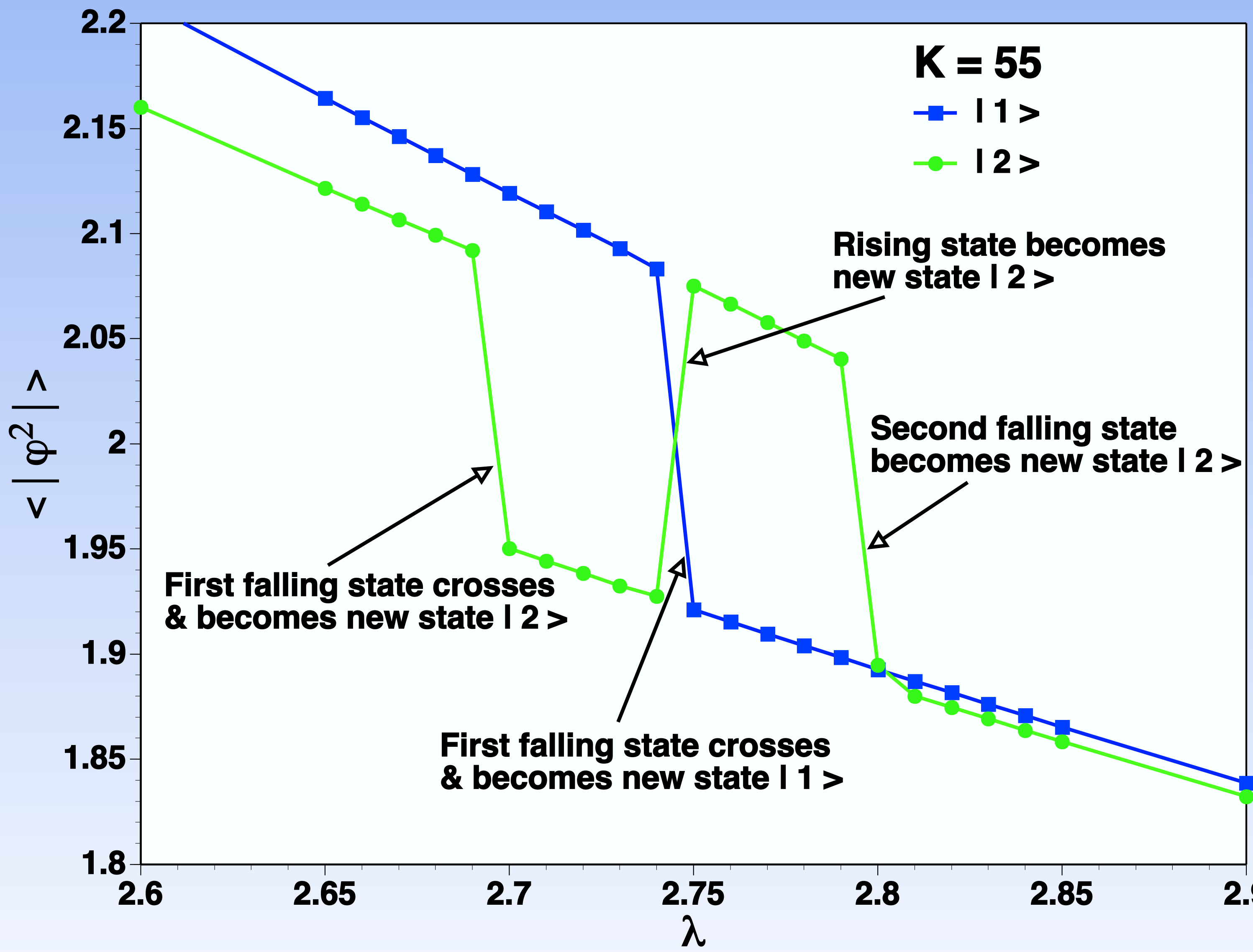
Mass-squared gaps for $K = 60$.



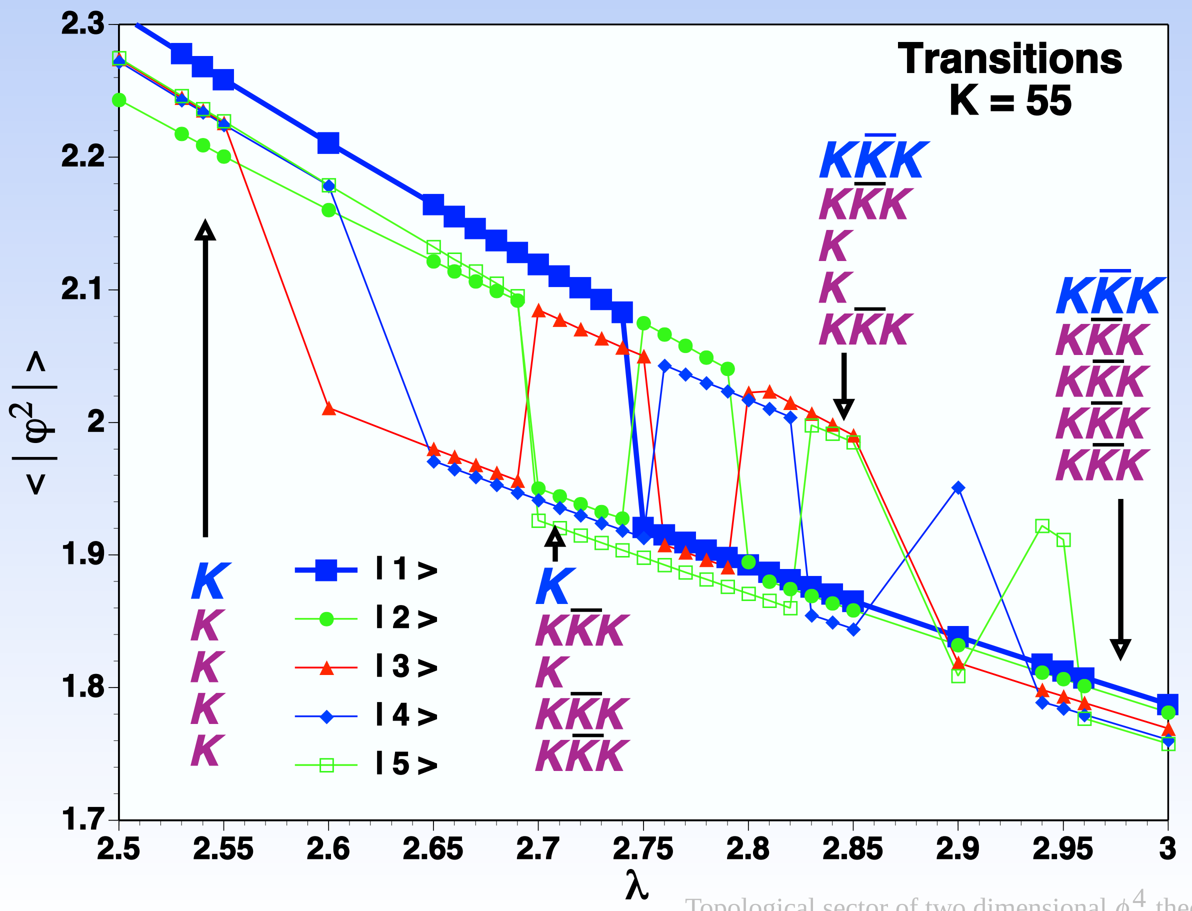
$\langle 1 \mid \phi^2 \mid 1 \rangle$ (short hand notation for the expectation value of the integral of the normal ordered ϕ^2 operator) as a function of λ and selected K values. For comparison we have also shown $\phi^2_{classical} = 6\frac{\mu^2}{\lambda}$ with $\mu^2 = 1$.



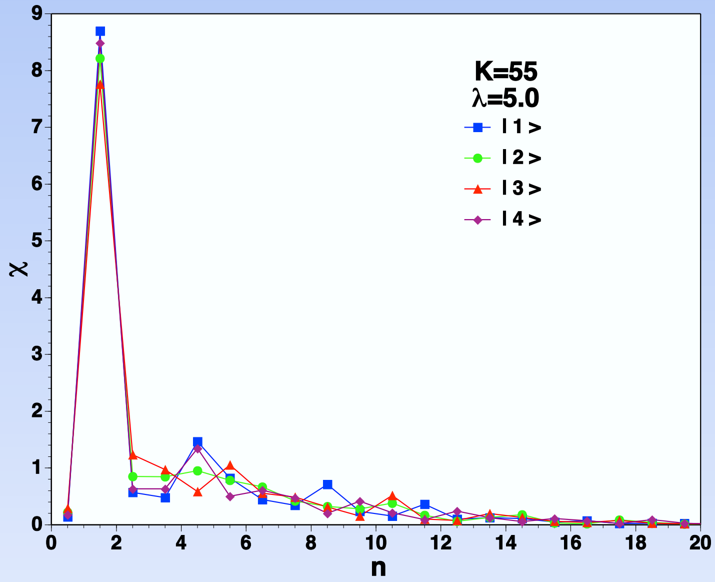
$\langle | \phi^2 | \rangle$ as a function of λ for K=55 for the lowest two excitations.

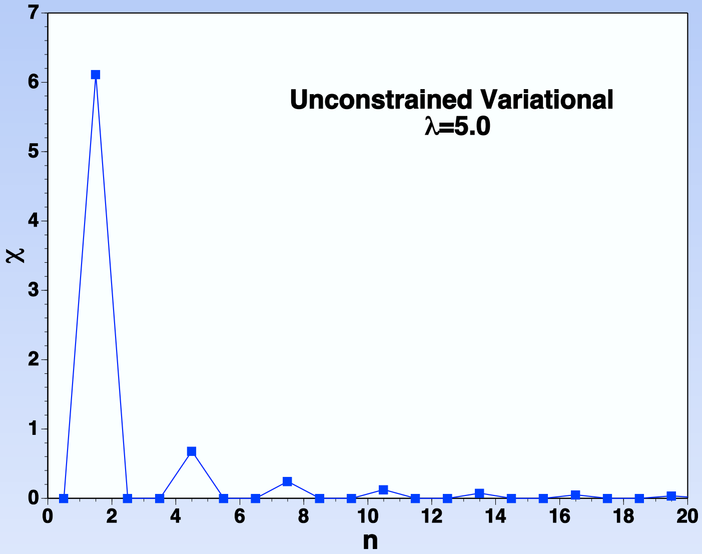


$\langle | \phi^2 | \rangle$ as a function of λ for K=55 for the lowest five excitations. The pattern of transitions correspond to 5 states falling with increasing λ and crossing the 5 lowest states, thus replacing them and becoming the new 5 lowest states. At selected values of λ, the character of the lowest states is indicated on the figure with the top level of each column signifying the nature of the lowest state. Successive excited states are signified by the labels proceeding down the column. The letter ‘K" represents ‘kink" while ‘KAK" represents ‘kink-antikink-kink".



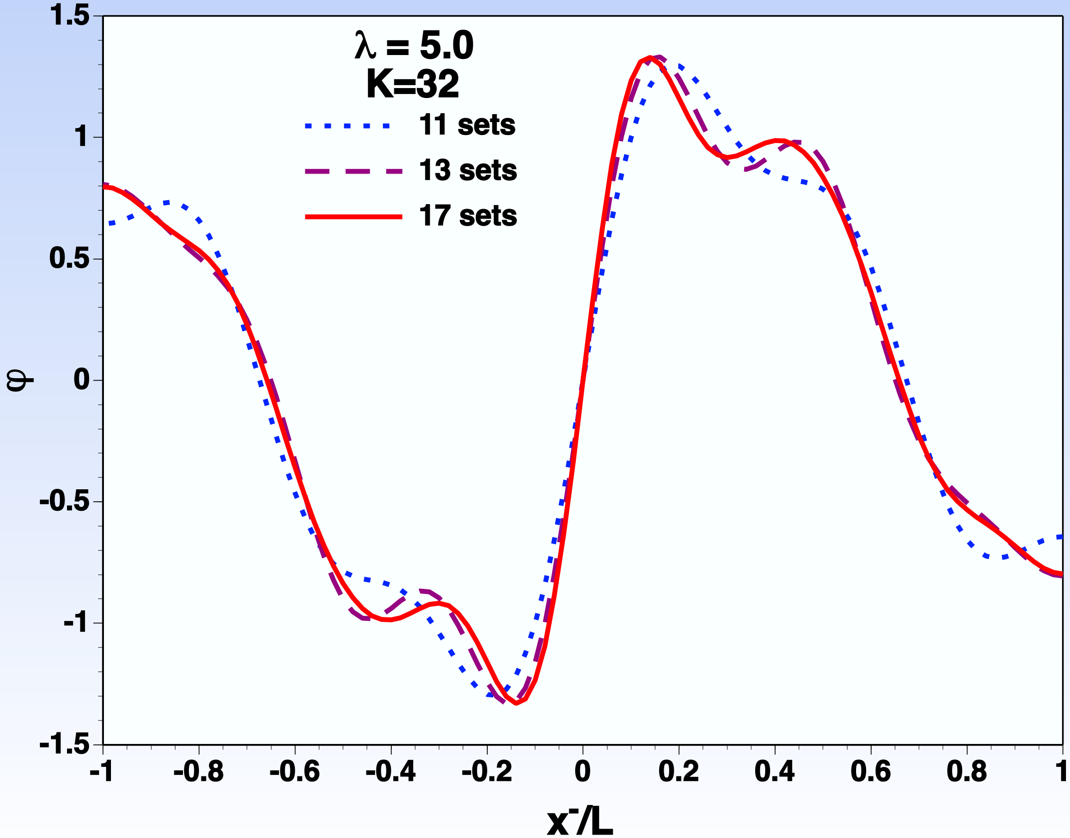
(a) χ versus n, the half-odd integer representing light front momentum with APBC, for the lowest four excitations for $K = 55$, $\lambda = 5$. (b) Kink-antikink-kink parton density in unconstrained variational calculation for λ=5. (a) (b)





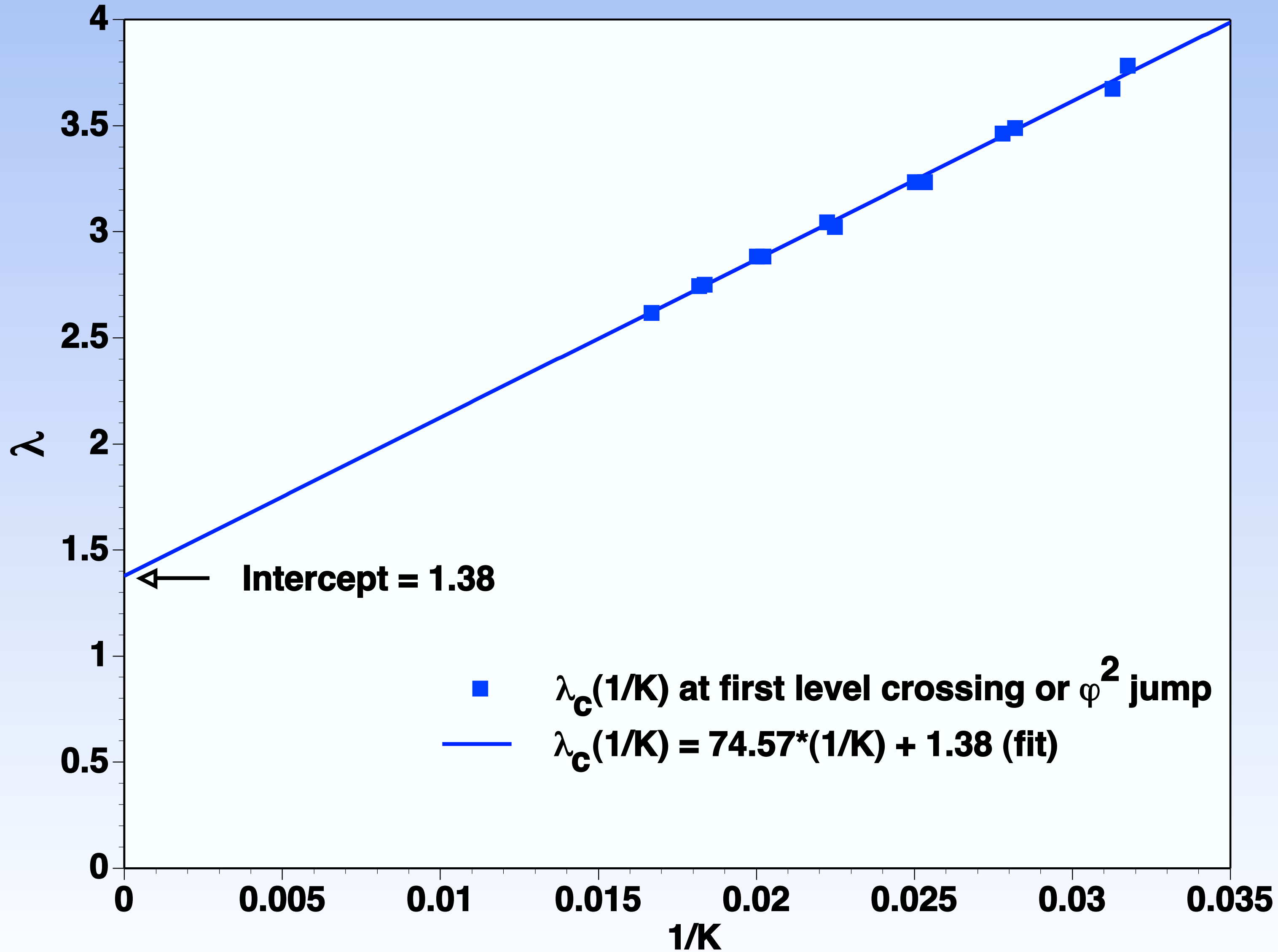
Kink-antikink-kink Profiles

Fourier Transform of the form factor of the lowest excitation at λ=5, $K = 32$. The figure legend indicates the number of adjoining momentum transfer terms (sets) included in the summation.



Critical Coupling:

Critical coupling for level crossing as a function of $\frac{1}{K}$, and an indication of the critical coupling in the continuum limit



Physical Implication of the Transition:

In the two-dimensional Ising model, physical mechanism for the symmetry restoring phase transition is the phenomena of kink condensation.

It is known that at strong coupling, the ϕ_2^4 theory undergoes a symmetry restoring phase transition.

We have demonstrated that in this theory, at strong coupling, it is energetically favourable for a dominantly kink-antikink-kink configuration to be the lowest excitation rather than a kink configuration.

At still higher coupling we have observed additional level crossings for the lowest state for both PBC and APBC.

In the light of all our observations, we interpret the observed level crossing presented here as the onset of kink condensation which leads to the restoration of symmetry.

Summary

- Ab initio results for kink mass and other observables: number density, Fourier Transform of the form factor, vacuum condensate (without the vacuum state).
- Signals for a transition in the spectrum of lowest excitations.
- Physical implication –
  Onset of kink condensation?
- We have given one more demonstration of the utility of light front Hamiltonian approach to Quantum Field Theory. (Meson and glueball spectra in transverse lattice QCD, Supersymmetric DLCQ, etc etc.)