Towards Understanding Strong Interactions

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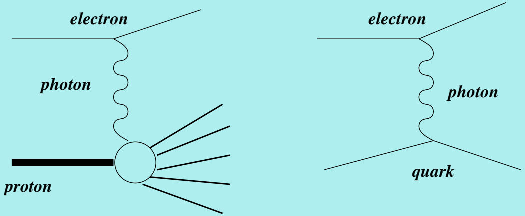
- Theoretical understanding of polarized deep inelastic scattering
- Kink in discrete light front quantization
- Topological susceptibility and pion properties in Lattice QCD with Wilson quarks.

Theoretical understanding of polarized deep inelastic scattering

An important tool to probe strong interaction physics: Deep inelastic scattering



P Target four momentum
q Virtual photon four momentum
$q = (q^0, \mathbf{q}), \quad q^2 < 0$
Let $Q^2 = -q^2$, $\nu = P.q$
Deep inelastic limit: $Q^2, \nu \to \infty$ such that $\frac{Q^2}{2\nu}$ is finite
Define $x = \frac{Q^2}{2\nu}$, Bjorken Scaling Variable $(0 < x < 1)$.
Differential Cross Section $\propto W(x, Q^2$, proton structure function.
Extracted from experimental data.
Tells us about the structure of the proton in detail.

Unpolarized case: Structure functions F_2 and F_L. F_2 dominant, leading part of F_L power suppressed.
Polarized case: Longitudinally polarized lepton

Longitudinally polarized target: Longitudinal spin asymmetry	Transversely polarized target: Transverse spin asymmetry
$g_1(x, Q^2)$:	$g_T(x, Q^2)$:
Longitudinal polarized structure function	Transverse polarized structure function

Measurement of F_2 structure function in the late sixties lead to the discovery of quarks and eventually lead to the discovery of asymptotic freedom and the establishment of Quantum Chromodynamics as the underlying theory of strong interactions.

Measurement of the g_1 structure function in the late eighties lead to the "proton spin crisis (mystery, puzzle)".

Phys.Lett. B206 (1988) 364

A recent news:
Scientific American, Jul 21, 2014
Proton Spin Mystery Gains a New Clue

Recent Review:
The angular momentum controversy: What's it all about and does it matter? E. Leader and C. Lorce, Phys. Rept. 541 (2014) 163-248.

Theoretical Tool I: Appropriate formalism

Nucleon is a relativistic system. What are the possible forms of relativistic dynamics?

Forms of Relativistic Dynamics

P. A. M. Dirac, RMP (1949)

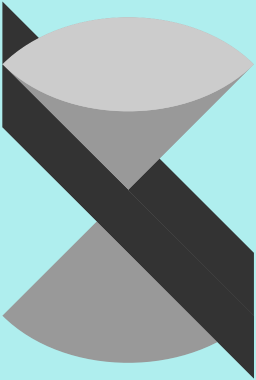
"The theory of a dynamical system is built up in terms of ten dynamical variables each of which is defined with respect to a system of coordinates in space-time."

"Dynamical variables change when the system of coordinates with respect to which they are defined changes ... ."

The ten variables are Hamiltonian, Momenta (3), Rotations (3) and Boosts (3).

In the familiar form of dynamics, Momenta and Rotations are kinematical whereas Hamiltonian and Boosts are dynamical (depends on the interaction).

Light Front Dynamics Dirac, RMP (!949)



$x^\mu = (x^0, x^1, x^2, x^3)$ with
$x^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$.
$x^\pm = x^0 \pm x^3,\ x^\perp = (x^1, x^2)$
Define x^+ "time",
x^- longitudinal coordinate
$x^2 = x^+ x^- - (x^\perp)^2$
$k.x = \frac{1}{2}k^+ x^- + \frac{1}{2}k^- x^+ - k^\perp \cdot x^\perp$
Longitudinal momentum
$k^+ = k^0 + k^3$

For an on mass shell particle $k^2 = m^2 \rightarrow$ energy $k^- = \frac{(k^\perp)^2 + m^2}{k^+}$.
Dependence on $k^\perp$ just like in the nonrelativistic dispersion relation (no square root)!!

Longitudinal boost becomes scaling, transverse boosts become Galilean boosts. Thus boost become kinematical (non-relativistic).

In a relativistic theory, internal motion and the motion associated with center of mass can be separated $\Rightarrow$ Can construct boost invariant wave functions.

Furthermore, the relativistic fermion is described by a two component field.

We start seeing the "remarkably non-relativistic character of the extreme relativistic limit" (Bjorken)

Theoretical Tool II: Study of the high energy ($q^- \to \infty$) limit of the appropriate scattering amplitude

Detailed calculations $\Rightarrow$ structure functions in deep inelastic scattering are equal light front time correlation functions in relativistic physics just as structure functions in quasi elastic scattering are equal time correlation functions in non-relativistic physics.

Structure functions in deep inelastic scattering as equal light front time correlation functions

$$\frac{F_2(x)}{x} = \frac{1}{4\pi P^+} \int d\eta \; e^{-i\eta x} \langle PS | \left[\overline{\psi}(\xi^-)\gamma^+ \psi(0) + \mathrm{h.c.} \right] | PS \rangle$$

$$F_L(x) = \frac{P^+}{4\pi} \left(\frac{2x}{Q}\right)^2 \int d\eta \; e^{-i\eta x} \langle PS | \left[\overline{\psi}(\xi^-)\gamma^- \psi(0) + h.c. \right] | PS \rangle$$

$$g_1(x) = \frac{1}{8\pi S^+} \int d\eta \; e^{-i\eta x} \langle PS | \left[\overline{\psi}(\xi^-)\gamma^+ \gamma_5 \psi(0) + \mathrm{h.c.} \right] | PS \rangle$$

$$g_T(x) = \frac{1}{8\pi M} \int d\eta \; e^{-i\eta x} \times \langle PS^1 | \left[\overline{\psi}(\xi^-)\gamma^1 \gamma_5 \psi(0) + \; \mathrm{h.c.} \right] | PS^1 \rangle$$

Here $\eta = \frac{1}{2} P^+ \xi^-$, P^μ and S^μ are the four momentum and the polarization vector of the target.

(1) Deep Inelastic Structure Functions in Light-Front QCD: A Unified Description of Perturbative and Nonperturbative Dynamics, A. H., Rajen Kundu, and Wei-Min Zhang, Phys. Rev. D 59 , 094012 (1999). (2) Deep Inelastic Structure Functions in Light-Front QCD: Radiative Corrections, A. H., Rajen Kundu, and Wei-Min Zhang, Phys. Rev. D 59, 094013 (1999).

Resolving the proton spin puzzle and more . . .

1) *On Orbital Angular Momentum in Deep Inelastic Scattering*, A. H and Rajen Kundu, Phys. Rev. D 59, 116013 (1999).

2) *Transverse Spin in QCD and Transverse Polarized Deep Inelastic Scattering*, A. H., Asmita Mukherjee and Raghunath Ratabole, Phys. Lett. B 476, 471 (2000).

3) *Transverse Spin in QCD: Radiative Corrections*, A. H., Asmita Mukherjee, Raghunath Ratabole, Phys. Rev. D 63 045006 (2001).

Postcript:

4) *Comment on "Proton Spin Structure from Measurable Parton Distributions"*, A. H., Rajen Kundu, Asmita Mukherjee and Raghunath Ratabole, Phys. Rev. Lett. 111 039102 (2013)

5) *On transverse spin sum rules*, A. H., Rajen Kundu and Asmita Mukherjee, Phys. Lett. B728 63 (2014)

Theoretical Tool III: Poincare generators in light front QCD

In terms of the symmetric, gauge invariant energy momentum tensor $\Theta^{\mu\nu} = \frac{1}{2}\overline{\psi}\Big[\gamma^\mu iD^\nu + \gamma^\nu iD^\mu\Big]\psi - F^{\mu\lambda a}F^{\nu a}_{\lambda}$
$- g^{\mu\nu}\Big[-\frac{1}{4}(F_{\lambda\sigma a})^2 + \overline{\psi}(\gamma^\lambda iD_\lambda - m)\psi\Big]$, with
$iD^\mu = \frac{1}{2}\, i\overleftrightarrow{\partial^\mu} + gA^\mu$, $F^{\mu\lambda a} = \partial^\mu A^{\lambda a} - \partial^\lambda A^{\mu a} + gf^{abc}A^{\mu b}A^{\lambda c}$,

Momenta $P^\mu = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+\mu}$,

Generalized angular momenta
$M^{\mu\nu} = \frac{1}{2}\int dx^- d^2x^\perp \left[x^\mu \Theta^{+\nu} - x^\nu \Theta^{+\mu}\right].$

P^- is the Hamiltonian, P^+ longitudinal momentum, P^i transverse Momenta, $M^{+-} = 2K^3$ and $M^{+i} = E^i$ are boost operators and $M^{12} = J^3$ and $M^{-i} = F^i$ are the rotation operators. Gauge of choice: Light front gauge $A^+ = 0$.

Spin Operators

By means of the Pauli-Lubanski spin operators $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}M_{\nu\rho}P_\sigma$ with $\epsilon^{+-12} = -2$, one constructs the longitudinal and transverse spin operators $\mathcal{J}^3$ and $\mathcal{J}^i$.

The helicity operator $\mathcal{J}^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1P^2 - E^2P^1)$

For a massive particle, the transverse spin operators $\mathcal{J}^i$ in light front theory are given in terms of Poincare generators by

$$M\mathcal{J}^i = W^i - P^i\mathcal{J}^3 \quad (i = 1, 2)$$
$$= \epsilon^{ij}\left(\frac{1}{2}F^jP^+ - \frac{1}{2}E^jP^- + K^3P^j\right) - P^i\mathcal{J}^3.$$

The interaction dependence of $\mathcal{J}^i$ arises from F^i which depends on both center of mass and internal variables. The rest of the terms in $\mathcal{J}^i$ serves to remove the center of mass motion effects from F^i.

Relevance to Deep inelastic scattering (Resolution of the proton spin puzzle):

After all the dust settles, the longitudinal spin operator $\mathcal{J}^3 = \mathcal{J}^3_{q(i)} + \mathcal{J}^3_{q(o)} + \mathcal{J}^3_{g(i)} + \mathcal{J}^3_{g(o)}$. The matrix element in a longitudinally polarized nucleon state gives rise to the nucleon helicity sum rule. Integrals of quark (g_1) and gluon distribution functions that appear in longitudinally polarized deep inelastic scattering are proportional to the matrix elements of $\mathcal{J}^3_{q(i)}$ and $\mathcal{J}^3_{g(i)}$ in a longitudinally polarized nucleon state. [Scale Dependence]

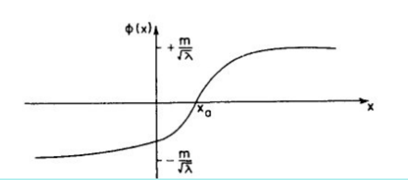
Transverse spin operators $\mathcal{J}^i$, $(i = 1, 2)$ can also be written as the sum of three parts, $\mathcal{J}^i_I$ which arises from the fermionic part, and $\mathcal{J}^i_{II}$ which arises from the bosonic part of the energy momentum tensor and $\mathcal{J}^i_{III}$ whose integrand has explicit coordinate dependence.

Integrals of quark (g_T) and gluon distribution functions that appear in transversely polarized deep inelastic scattering are proportional to the matrix elements of $\mathcal{J}^i_{II}$ and $\mathcal{J}^i_{III}$ in a transversely polarized nucleon state.

Kink in discrete light front quantization

Kink solution in two dimensional ϕ^4 theory

R. Rajaraman, Solitons and Instantons
Lagrangian density $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$.
Nonperturbative classical solution (kink)
$\phi(x) = \pm\sqrt{\frac{m}{\lambda}}\,\tanh[(\frac{m}{\sqrt{2}})(x - x_0)]$



Topological charge $Q = \frac{\sqrt{\lambda}}{2m}[\phi(x = +\infty) - \phi(x = -\infty)]$ associated with the conserved current $j^\mu = \frac{\sqrt{\lambda}}{2m}\epsilon^{\mu\nu}\partial_\nu\phi$ where $\epsilon^{\mu\nu}$ is the antisymmetric tensor.
All finite energy solutions fall into four topological sectors:
$(-\sqrt{\frac{m}{\lambda}}, \sqrt{\frac{m}{\lambda}}), (\sqrt{\frac{m}{\lambda}} - \sqrt{\frac{m}{\lambda}}), (\sqrt{\frac{m}{\lambda}}, \sqrt{\frac{m}{\lambda}})$ and $(-\sqrt{\frac{m}{\lambda}}, -\sqrt{\frac{m}{\lambda}})$.

Topological Sector of Two Dimensional ϕ^4 in Discrete Light Front Quantization (DLFQ)

Using DLFQ, masses of the lowest few excitations, parton distribution functions and Fourier transforms of the form factor are calculated. Also vacuum energy density and the value of the condensate.

Dipankar Chakrabarti, AH, Lubomir Martinovic and James P. Vary, Kinks in Discrete Light Cone Quantization, Phys. Lett. **B 582, 196** (2004).

Lagrangian density $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$.
The longitudinal momentum operator
$P^+ = \frac{1}{2}\int_{-L}^{+L} dx^- \partial^+\phi\partial^+\phi \equiv \frac{2\pi}{L}K$ where K is the dimensionless longitudinal momentum operator and

Hamiltonian $P^- = \int dx^- \left\{\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4\right\} \equiv \frac{L}{2\pi}H$ where L defines our compact domain $-L \leq x^- \leq +L$.

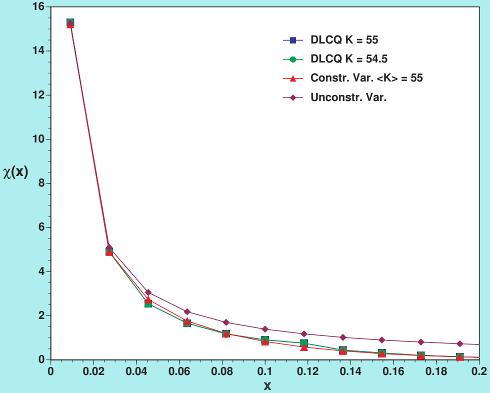
Mass operator $M^2 = P^+P^- = KH$

odd sector		even sector	
K	dimension	K	dimension
15.5	295	16	336
31.5	12839	32	14219
39.5	61316	40	67243
44.5	151518	45	165498
49.5	358000	50	389253
54.5	813177	55	880962

Dimensionality of the Hamiltonian matrix in odd and even particle sectors. (Anti periodic boundary condition)

The number density of bosons with momentum fraction $x = j/K$ (the parton distribution function)

$\chi(x) = \langle kink \mid aj^{\dagger} a_j \mid kink \rangle$

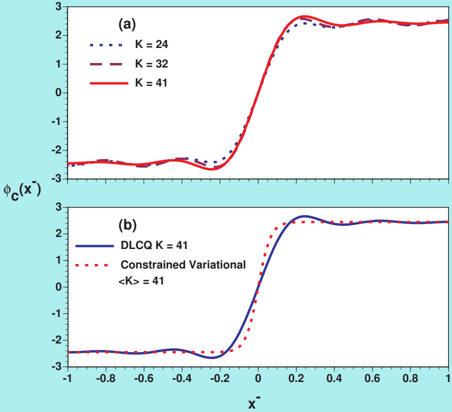


The parton distribution function $\chi(x)$ for even $(K = 55.0)$ and odd $(K = 54.5)$ sectors for $\mu^2 = -1$, $\lambda = 1$.

J. Goldstone and R. Jackiw, Phys. Rev. D 11, 1486 (1975).

$\int_{-\infty}^{+\infty} dq^{+} \exp\{-\frac{i}{2}q^{+}a\}\langle K' \mid \Phi(x^{-}) \mid K\rangle = \phi_c(x^{-} - a).$

$q^{+} = K' - K.$

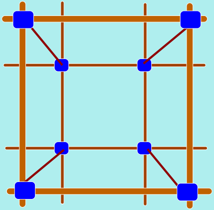


Fourier Transform of the kink form factor at $\mu^2 = -1$, λ=1; (a) results for $K = 24, 32$, and 41. (b) comparison of DLFQ profile at K=41 with variational result.

Topological susceptibility and pion properties in Lattice QCD with Wilson quarks

Lattice Quantum Chromodynamics

K.G. Wilson, Confinement of Quarks, Phys. Rev. D (1974)
A.M. Polyakov, Compact Gauge Fields and the Infrared Catastrophe, Phys. Lett. B (1975)
Jan Smit (Unpublished)



Discrete Space-Time Euclidean Lattice
Short distance (UV) Cut-off (lattice spacing a)
Long distance (IR) Cut-off (box length L)

Path integral quantization $\rightarrow$ finite dimensional integrals over the space of fields

$$\langle O \rangle \; = \; \frac{1}{Z} \; \int \mathcal{D}\phi \, \exp\left[-S[\phi]/\hbar\right] \; O$$
$$Z \; = \; \int \mathcal{D}\phi \, \exp\left[-S[\phi]/\hbar\right]$$

Formally equivalent to Classical Statistical Mechanics in 4 dimensions: $T \rightarrow \hbar$ (Thermal fluctuations $\leftrightarrow$ Quantum fluctuations)

Free Fermions on a Lattice

Free fermion Dirac action in continuum Euclidean space (Hermitian γ matrices)

$S_F^{cont} \;=\; \int d^4x\; \overline{\psi}(x)\,(\gamma_\mu \partial_\mu + m)\,\psi(x)$

Transcribe it on to the discrete space-time lattice

$S_F \;=\; \frac{1}{2}a^4\; \sum_{x,\mu}\; \overline{\psi}_x \gamma_\mu \left[\frac{\psi_{x+\mu} - \psi_{x-\mu}}{a}\right] \;+\; a^4\;\; m\; \sum_x\; \overline{\psi}_x \psi_x$

16 degenerate fermions: Fermion Doubling

Simplest remedy for fermion doubling: Wilson Fermions

To the original action, add momentum dependent irrelevant mass term:

$S_W = -\frac{r}{2}\, a^4\; \sum_x a\, \overline{\psi}_x \Box \psi_x \;=\; \frac{r}{2}\, a^4\; \sum_{x,\mu}\; a\; \frac{2\overline{\psi}_x \psi_x - \overline{\psi}_x \psi_{x+\mu} - \overline{\psi}_x \psi_{x-\mu}}{a^2}$

Doubler masses $\to \infty$ as $a \to 0$

S_W violates chiral symmetry

In interacting theory, fermion mass receives additive renormalization

Introducing gauge fields on the lattice

Products of matter fields $\overline{\psi}_x \psi_{x+\mu}$ not gauge invariant

because $\psi_x \to g_x \psi_x, \quad \overline{\psi}_x \to \overline{\psi}_x g_x^\dagger$

$g_x^\dagger g_{x+\mu} \neq 1, \quad g_x \in G$

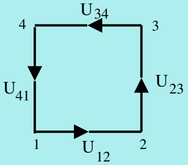
Needs objects which span finite distances

Lattice gauge fields (link variables) $U_{x\mu}$ placed on the link between the sites of matter fields $\quad U_{x\mu} \in G$

Gauge Transformation: $\quad U_{x\mu} \to g_x U_{x\mu} g_{x+\mu}^\dagger$

Now, $\overline{\psi}_x U_{x\mu} \psi_{x+\mu}$ Gauge invariant.

Kinetic term for the gauge fields: Gauge invariant objects constructed out of link fields



$\text{Trace } \mathcal{U}_P \;=\; \text{Trace } (U_{12}U_{23}U_{34}U_{41})$

Gauge Invariant

Wilson action for Lattice QCD

$S_{QCD} = S_G[U] + S_F[\psi, \overline{\psi}, U]$ where,
$S_G[U] \;=\; \beta \;\sum \left[1 - \frac{1}{3} \text{ Re Trace } U_P\right]$
and $S_F[\psi, \overline{\psi}, U] = \sum_{x,y} \overline{\psi}_x M_{xy} \psi_y$ with
$M_{xy} = \delta_{xy} - \kappa \left[(r - \gamma_\mu)\, U_{x,\mu} + (r + \gamma_\mu)\, U^\dagger_{x-\mu,\mu} \right]$

The parameters of the theory are the gauge coupling, $\beta = 6/g^2$ and the Wilson hopping coefficient $\kappa = \frac{1}{2(\overline{m}+4)}$ with $r = 1$ and $\overline{m} = am$ is the dimensionless bare quark mass.

Fermion field has been re-scaled: $\frac{a^{3/2}}{\sqrt{2\kappa}} \;\psi \to \psi$

After rescaling the dimensionful parameters and fields with appropriate powers of the lattice spacing a, lattice spacing disappears from the lattice action.

Parameter in the lattice gauge action $\beta = \frac{6}{g^2}$ where g is the continuum gauge coupling, $g = g(a)$. Due to asymptotic freedom, $g(a) \to 0$ as $a \to 0$. $\to \beta$ needs to be as large as possible.
Ideal Lattice scale hierarchy:

$L^{-1} \ll m_q \ll m_{\rm hadron} \ll a^{-1}$
Mass scale in the real world: $m_q = m_u \sim m_d \sim 5$ MeV
$m_{nucleon} \sim 1000$ MeV
$\to$ beyond the scope of most of lattice simulation
Extrapolation of Lattice data to the chiral region necessary
For pure gauge:
β=6.21, 24X24X24X48 lattice, a = 0.067 fm, 1/a = 2.9 GeV, size of a gauge configuration 380 MB
β=6.71, 64X64X64X128 lattice, a = 0.034 fm, 1/a = 5.8 GeV, size of a gauge configuration 19 GB.

In the early days of QCD, it was realized that the noninvariance of the QCD action under the chiral transformations $\psi(x) \to \psi'(x) = e^{i\gamma_5 \alpha(x)}\psi(x),\ \ \overline{\psi}(x) \to \overline{\psi}'(x) = \overline{\psi}(x)e^{i\gamma_5 \alpha(x)}$ lead to an anomalous Ward Identity

$\partial_\mu J_{5\mu}(x) = 2m\overline{\psi}(x)\gamma_5\psi(x) - \frac{1}{16\pi^2}\ \epsilon_{\mu\nu\rho\lambda}\ \mathrm{trace}\ F_{\mu\nu}(x)F_{\rho\lambda}(x)$.

In those days it was also realized that Yang-Mills theory in Euclidean space possess classical solutions that have finite action, which were called instantons. An instanton carries a conserved topological charge

$Q = \int d^4x\ q(x) = \frac{1}{32\pi^2}\ \int d^4x\ \epsilon_{\mu\nu\rho\lambda}\ \mathrm{trace}\ F_{\mu\nu}(x)F_{\rho\lambda}(x).$

Q is also known as the winding number.

It is not possible to continuously deform a field configuration with one winding number in to one of a different winding number while maintaining the finiteness of the action.

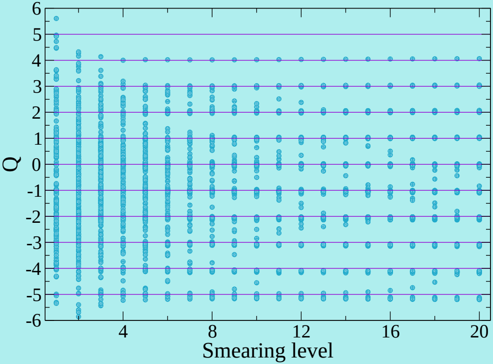
However, Q is not an observable and the topological structure of the gauge fields is quantitatively summarized by the **Topological Susceptibility** $\chi = \int d^4x\ \langle q(x)q(0)\rangle$. According to a chiral Ward Identity, in QCD χ vanishes with vanishing quark mass.

Also, the low energy effective theory of QCD, namely chiral perturbation theory, predicts the behaviour of the square of the pion mass with respect to the quark mass.

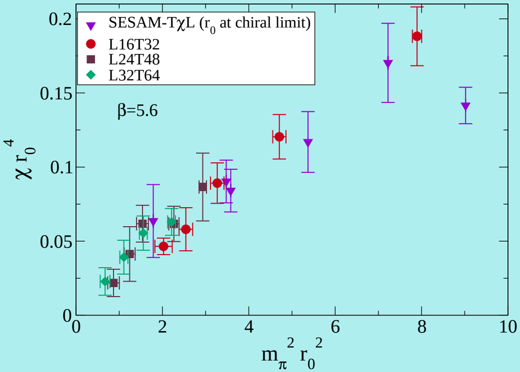
Can Wilson fermion, with modest (human and computational) resources, fulfill the expectations of continuum QCD, regarding the chiral behaviour?

Topological susceptibility in Lattice QCD with unimproved Wilson fermions, Abhishek Chowdhury, Asit K. De, Sangita De Sarkar, A. H., Jyotirmoy Maiti, Santanu Mondal, Anwesa Sarkar, Phys. Lett. B707 228 (2012).

Pion and nucleon in two flavour QCD with unimproved Wilson fermion, Abhishek Chowdhury, Asit K. De, Sangita De Sarkar, A. H., Jyotirmoy Maiti, Santanu Mondal and Anwesa Sarkar, Nucl. Phys. B 871 (2013) 82.

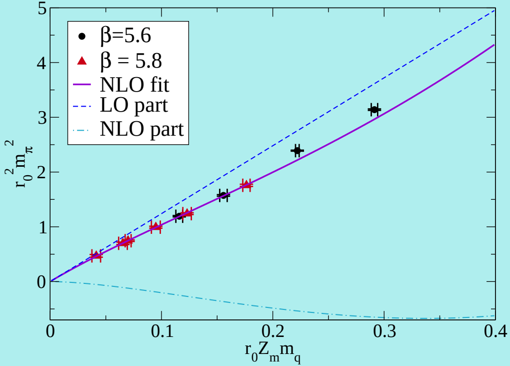


Topological charge of gauge configurations versus HYP smearing steps for $\beta = 5.8$ (a=0.057 fm) and $\kappa = 0.15475$ ($m_\pi \sim 300\mathrm{MeV}$) at lattice volume $32^3 \times 64$.



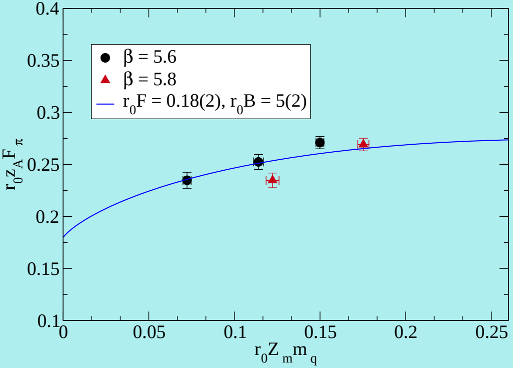
Highlighted in the 2014 International Lattice Conference:
Recent results on topology on the lattice (in memory of Pierre van Baal), M. Muller-Preussker, PoS LATTICE2014 (2015) 003

Behaviour of the pion mass squared versus the quark mass



In SU(2) χPT at NLO, the quark mass $(r_0 m_q)$ dependence of $(r_0 m_\pi)^2$ is given by $(r_0 m_\pi)^2 = 2 r_0^2 m_q B \left[1 - \frac{m_q B}{16\pi^2 F^2} \ln \frac{\Lambda_3^2}{2 m_q B}\right]$ where F is the chiral limit of the pion decay constant and B and Λ_3 are low energy constants.

Behaviour of the pion decay constant versus the quark mass
$\langle 0 \mid A_\mu(0) \mid \pi(p)\rangle = \sqrt{2}\ F_\pi p_\mu$ where $A_\mu = \overline{q}\gamma_\mu\gamma_5 q$.



The quark mass dependence of the pion decay constant $r_0 F_\pi$ in NLO χPT is given by $r_0 F_\pi = r_0 F \left[1 + \frac{m_q B}{8\pi^2 F^2} \ln \frac{\Lambda_4^2}{2 m_q B}\right]$ where Λ_4 is another low energy constant.

To summarize, We have shown

- Quantum Field Theoretical tools based on Light Front Dynamics can be used to understand the polarized deep inelastic scattering structure functions in the context of the proton spin mystery.
- Properties of Kink, especially the parton distribution and the elastic form factor can be calculated in discrete light front quantization
- Expected behaviours of topological susceptibility and pion properties can be reproduced in Lattice QCD with unimproved Wilson quarks.

A big **Thank You** to all my collaborators and You, the audience!