CHAPTER 2

Canonical light front QCD

Investigations of Yang-Mills theory and QCD in the light front framework prior to the nineties can be found in Ref. [1]. For motivations for studying light front QCD, see Ref. [2]. For many examples of tree level and one loop calculations see Refs. [3, 4, 5]. In this section, starting from the QCD Lagrangian density, we give the explicit form of the canonical light front Hamiltonian for QCD in the two-component[6] formulation.

2.1 Canonical Hamiltonian

The QCD Lagrangian is

$$\begin{aligned}\mathcal{L} \quad &= -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \bar{\psi}(i\gamma_\mu D^\mu - m_F)\psi, \\ &= -\frac{1}{4}(F^{\mu\nu a})^2 + \bar{\psi}(i\gamma_\mu D^\mu - m_F)\psi. \end{aligned} \tag{2.1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$ are the gluon field strength tensors and $A^\mu = \sum_a A^\mu_a T^a$ are the 3×3 gluon field matrices, with T^a, the generators of $SU(3) = \frac{1}{2}\lambda_a$, with λ_a the Gell-Mann $SU(3)$ matrices. $[T^a, T^b] = if^{abc}T^c$ and $\mathrm{Tr}(T^aT^b) = (1/2)\delta_{ab}$. The field variable ψ describes quarks with three colors and N_f flavors, and $D^\mu = \partial^\mu - igA^\mu$ are the covariant derivatives, while m_F is a $N_f \times N_f$ diagonal quark mass matrix.

The Lagrange equations of motion are

$$(i\gamma_\mu\partial^\mu + g\gamma_\mu A^\mu - m)\psi = 0, \tag{2.2}$$

$$\partial_\mu F^{\mu\nu}_a + gf^{abc}A_{\mu b}F^{\mu\nu}_c + g\bar{\psi}\gamma^\nu T^a\psi = 0. \tag{2.3}$$

We choose the light-front gauge $A^+_a = 0$. From the fermion equation of motion, Eq. (2.2), multiplying from the left by γ^+, we arrive at

$$i\partial^+\psi^- = [\alpha^\perp \cdot (i\partial^\perp + gA^\perp) + \gamma^0 m]\psi^+ \tag{2.4}$$

which shows that ψ^- is a constrained variable. Taking the plus component of the equation of motion of the gauge field, Eq. (2.3), we have,

$$-\frac{1}{2}(\partial^+)^2 A^{-a} = -\partial^+\partial^i A^{ia} - gf^{abc}A^i_b\partial^+ A^i_c - 2g{\psi^+}^\dagger T^a\psi^+ \tag{2.5}$$

which shows that A^{-a} is a constrained variable.

The Lagrangian density can be rewritten as

$$\begin{aligned}
\mathcal{L} &= i\psi^{+\dagger}\partial^-\psi^+ + g\psi^{+\dagger}A^-\psi^+ + i\psi^{-\dagger}\partial^+\psi^- \\
&\quad - i\overline{\psi}\gamma^\perp.\partial^\perp\psi + ig\overline{\psi}\gamma^\perp.A^\perp - m\overline{\psi}\psi \\
&\quad + \frac{1}{8}(\partial^+A^{-a})^2 - \frac{1}{2}\partial^+A^{ia}\Big(\partial^iA^{-a} - \partial^-A^{ia} + gf^{abc}A^{ib}A^{-c}\Big) \\
&\quad - \frac{1}{4}\Big(\partial^iA^{ja} - \partial^jA^{ia} + g\ f^{abc}A^{ib}A^{jc}\Big)^2
\end{aligned} \tag{2.6}$$

We note that only ψ^+ and A^i have time derivatives. Define the conjugate momenta

$$\Pi_{\psi^+} = \frac{\partial\mathcal{L}}{\partial\partial^-\psi^+} = i\psi^{+\dagger}\ , \tag{2.7}$$

$$\Pi^{ia}_{A^a_i} = \frac{\partial\mathcal{L}}{\partial\partial^-A^a_i} = \frac{1}{2}\partial^+A^a_i = -\frac{1}{2}\partial^+A^{ia}\ . \tag{2.8}$$

The canonical Hamiltonian density

$$\mathcal{H} = \Pi_{\psi^+}\partial^-\psi^+ + \Pi^{ia}_{A^a_i}\partial^-A^a_i - \mathcal{L}. \tag{2.9}$$

In the two component representation introduced in the previous chapter,

$$\psi^+ = \begin{bmatrix} \eta \\ 0 \end{bmatrix}\quad,\quad \psi^- = \begin{bmatrix} 0 \\ \left(\frac{1}{i\partial^+}\right)[\hat{\sigma}^i(i\partial^i + gA^i) + im]\eta \end{bmatrix}. \tag{2.10}$$

This immediately shows that ψ^+ and ψ^- are two-component fermion fields. Hereafter, we shall simply let η represent the light-front quark field.

After a lot of algebra, one arrives at the canonical light front QCD Hamiltonian

$$H = \int dx^- d^2x_\perp(\mathcal{H}_0 + \mathcal{H}_{int}). \tag{2.11}$$

$$\mathcal{H}_0 = \frac{1}{2}(\partial^iA^j_a)(\partial^iA^j_a) + \eta^\dagger\left(\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\right)\eta, \tag{2.12}$$

$$\mathcal{H}_{int} = \mathcal{H}_{qqg} + \mathcal{H}_{ggg} + \mathcal{H}_{qqgg} + \mathcal{H}_{qqqq} + \mathcal{H}_{gggg}, \tag{2.13}$$

and

$$\begin{aligned}
\mathcal{H}_{qqg} &= g\ \eta^\dagger\Big\{ - 2\left(\frac{1}{\partial^+}\right)(\partial_\perp\cdot A_\perp) \\
&\quad + \hat{\sigma}_\perp\cdot A_\perp\left(\frac{1}{\partial^+}\right)(\hat{\sigma}_\perp\cdot\partial_\perp + m) + \left(\frac{1}{\partial^+}\right)(\hat{\sigma}_\perp\cdot\partial_\perp - m)\hat{\sigma}_\perp\cdot A_\perp\Big\}\eta,
\end{aligned} \tag{2.14}$$

$$\mathcal{H}_{ggg} = gf^{abc}\Big\{\partial^iA^j_aA^i_bA^j_c + (\partial^iA^i_a)\left(\frac{1}{\partial^+}\right)(A^j_b\partial^+A^j_c)\Big\}, \tag{2.15}$$

$$
\begin{aligned}
\mathcal{H}_{qqgg} \quad &= g^2\Big\{\eta^\dagger\hat{\sigma}_\perp\cdot A_\perp\left(\frac{1}{i\partial^+}\right)\hat{\sigma}_\perp\cdot A_\perp\eta \\
&\qquad\qquad + 2\left(\frac{1}{\partial^+}\right)(f^{abc}A_b^i\partial^+A_c^i)\left(\frac{1}{\partial^+}\right)(\eta^\dagger T^a\eta)\Big\}, \\
&= \mathcal{H}_{qqgg1}+\mathcal{H}_{qqgg2}, \qquad (2.16) \\
\mathcal{H}_{qqqq} \quad &= 2g^2\left\{\left(\frac{1}{\partial^+}\right)(\eta^\dagger T^a\eta)\left(\frac{1}{\partial^+}\right)(\eta^\dagger T^a\eta)\right\}, \qquad (2.17) \\
\mathcal{H}_{gggg} \quad &= \frac{g^2}{4}f^{abc}f^{ade}\left\{A_b^iA_c^jA_d^iA_e^j+2\left(\frac{1}{\partial^+}\right)(A_b^i\partial^+A_c^i)\left(\frac{1}{\partial^+}\right)(A_d^j\partial^+A_e^j)\right\}, \\
&= \mathcal{H}_{gggg1}+\mathcal{H}_{gggg2}. \qquad (2.18)
\end{aligned}
$$

For the two-component quark field variables $\eta(x)$ (the non-vanishing upper components of $\psi^+(x)$) we have the canonical commutation relations

$$\{\eta_i(x),\eta_j^\dagger(y)\}_{x^+=y^+}=\delta^3(x-y)\delta_{ij}, \qquad (2.19)$$

with

$$\{b(k,\lambda),b^\dagger(k',\lambda')\}=\{d(k,\lambda),d^\dagger(k',\lambda')\}=2(2\pi)^3k^+\delta^3(k-k'). \qquad (2.20)$$

The canonical equal x^+ commutation relations for the dynamical gauge field components are

$$[A_a^i(x),A_b^j(y)]_{x^+=y^+}=-\frac{i}{4}\epsilon(x^- - y^-)\delta^2(x_\perp-y_\perp)\delta_{ij}\delta_{ab} \qquad (2.21)$$

with

$$[a(k,\lambda),a^\dagger(k',\lambda')]=2(2\pi)^3k^+\delta^3(k-k'). \qquad (2.22)$$

$\lambda=\pm 1$.

We note the following features of the canonical light front QCD Hamiltonian:

- Presence of the inverse longitudinal derivatives $\frac{1}{\partial^+}$ and $(\frac{1}{\partial^+})^2$. They arise from the elimination of constrained quark and gluon variables and give rise to instantaneous fermion and gluon exchange vertices. The presence of inverse derivatives are usually forbidden in the equal-time Hamiltonians since derivatives carry mass dimensions in that case. On the light front, power counting is different[2] and ∂^+ does not carry mass dimensions. Hence their inverses are allowed in the Hamiltonian without spoiling the ultraviolet structure of the theory.

- Presence of quark masses: In the free part quark mass appears quadratically. Linear mass term appears in the quark-gluon vertex which is responsible for the helicity flip processes. Note that the free part is free from the presence of any σ matrices and helicity flip part contains $\hat{\sigma}^\perp$. Under light front chiral transformation $\psi^+\to e^{i\theta\gamma^5}\psi^+$, the free part remains invariant even though it contains fermion mass term. The linear mass term (helicity flip) violates light front chiral symmetry. This follows from the curious fact that on the light front, chirality coincides with helicity[2] even for massive fermions.

We also note that in 1+1 dimensions, the Hamiltonian density reduces to

$$\mathcal{H} = \eta^\dagger \left(\frac{+m^2}{i\partial^+}\right) \eta + 2g^2 \left\{ \left(\frac{1}{\partial^+}\right) (\eta^\dagger T^a \eta) \left(\frac{1}{\partial^+}\right) (\eta^\dagger T^a \eta) \right\} \tag{2.23}$$

which contains only the instantaneous four fermion interaction apart from the free fermion kinetic energy.

2.2 Poincare generators in light front QCD

In this section we derive the complete set of Poincare generators in light front QCD starting from the gauge invariant symmetric energy momentum tensor $\Theta^{\mu\nu}$. To begin with, $\Theta^{\mu\nu}$ is interaction dependent. In the gauge fixed theory we find that the seven kinematical generators are manifestly independent of the interaction.

We shall work in the gauge $A^+ = 0$. Note that this condition does not completely fix the gauge since x^- independent gauge transformations are still allowed. We further impose the condition $\psi^+, A^i \to 0$ as $x^- \to \pm\infty$ so that residual gauge freedom is also fixed. Subsequently we ignore all surface terms. Thus we are working in the completely gauge fixed sector of the theory.

The symmetric, gauge invariant energy momentum tensor

$$\Theta^{\mu\nu} = \frac{1}{2}\overline{\psi}\Big[\gamma^\mu iD^\nu + \gamma^\nu iD^\mu\Big]\psi - F^{\mu\lambda a}F^{\nu a}_{\lambda} - g^{\mu\nu}\Big[-\frac{1}{4}(F_{\lambda\sigma a})^2 + \overline{\psi}(\gamma^\lambda iD_\lambda - m)\psi\Big], \tag{2.24}$$

where

$$\begin{aligned} iD^\mu &= \frac{1}{2}\, i\overleftrightarrow{\partial^\mu} + gA^\mu, \\ F^{\mu\lambda a} &= \partial^\mu A^{\lambda a} - \partial^\lambda A^{\mu a} + gf^{abc}A^{\mu b}A^{\lambda c}, \\ F^{\nu a}_{\lambda} &= \partial^\nu A^a_\lambda - \partial_\lambda A^{\nu a} + gf^{abc}A^{\nu b}A^c_\lambda. \end{aligned} \tag{2.25}$$

We have the equations of constraint

$$i\partial^+\psi^- = [\alpha^\perp \cdot (i\partial^\perp + gA^\perp) + \gamma^0 m]\psi^+, \tag{2.26}$$

and

$$\frac{1}{2}\partial^+ A^{-a} = \partial^i A^{ia} + gf^{abc}\frac{1}{\partial^+}(A^{ib}\partial^+ A^{ic}) + 2g\frac{1}{\partial^+}\Big(\xi^\dagger T^a \xi\Big), \tag{2.27}$$

and the equation of motion

$$i\partial^-\psi^+ = -gA^-\psi^+ + [\alpha^\perp \cdot (i\partial^\perp + gA^\perp) + \gamma^0 m]\frac{1}{i\partial^+}[\alpha^\perp \cdot (i\partial^\perp + gA^\perp) + \gamma^0 m]\psi^+. \tag{2.28}$$

In terms of the gauge invariant, symmetric energy momentum tensor $\Theta^{\mu\nu}$ the four-vector P^μ and the tensor $M^{\mu\nu}$ are given by

$$P^\mu = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+\mu}, \tag{2.29}$$

$$M^{\mu\nu} = \frac{1}{2}\int dx^- d^2x^\perp \left[x^\mu\Theta^{+\nu} - x^\nu\Theta^{+\mu}\right]. \tag{2.30}$$

The boost operators are $M^{+-} = 2K^3$ and $M^{+i} = E^i$. The rotation operators are $M^{12} = J^3$ and $M^{-i} = F^i$. We expect the Hamiltonian P^- and the transverse rotation operators F^i to be dynamical (depend on the interaction) while other seven operators to be kinematical.

2.2.1 Kinematic Generators

At $x^+ = 0$, the operators K^3 and E^i depend only on the density Θ^{++}. A straightforward calculation leads to

$$\Theta^{++} = \psi^{+\dagger}\, i\overleftrightarrow{\partial^+}\, \psi^+ + \partial^+ A^i \partial^+ A^i. \tag{2.31}$$

Then, longitudinal momentum operator,

$$\begin{aligned} P^+ &= \frac{1}{2}\int dx^- d^2x^\perp \Theta^{++} \\ &= \frac{1}{2}\int dx^- d^2x^\perp \left[2\eta^\dagger i\partial^+\eta + \partial^+ A^j \partial^+ A^j\right]. \end{aligned} \tag{2.32}$$

Generator of longitudinal scaling,

$$\begin{aligned} K^3 &= -\frac{1}{4}\int dx^- d^2x^\perp x^- \Theta^{++}, \\ &= -\frac{1}{4}\int dx^- d^2x^\perp x^- \left[\eta^\dagger\, i\overleftrightarrow{\partial^+}\, \eta + \partial^+ A^j \partial^+ A^j\right]. \end{aligned} \tag{2.33}$$

Transverse boost generators,

$$\begin{aligned} E^i &= -\frac{1}{2}\int dx^- d^2x^\perp x^i \Theta^{++}, \\ &= -\frac{1}{2}\int dx^- d^2x^\perp x^i \left[2\eta^\dagger i\partial^+\eta + \partial^+ A^j \partial^+ A^j\right]. \end{aligned} \tag{2.34}$$

The transverse momentum operator

$$P^i = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+i} \tag{2.35}$$

which appears to have explicit interaction dependence. Using the constraint equations for ψ^- and A^-, we have

$$\Theta^{+i} = \Theta^{+i}_F + \Theta^{+i}_G,$$

$$\Theta^{+i}_F = 2\psi^{+\dagger} i\partial^i\psi^+ + 2g\psi^{+\dagger}A^i\psi^+, \tag{2.36}$$

$$\Theta^{+i}_G = \partial^+ A^j \partial^i A^j - \partial^+ A^j \partial^j A^i + \partial^+ A^i \partial^j A^j - 2g\psi^{+\dagger}A^i\psi^+. \tag{2.37}$$

Thus

$$P^i = \frac{1}{2}\int dx^- d^2x^\perp \Big[2\eta^\dagger i\partial^i\eta + \partial^+ A^j\partial^i A^j - \partial^+ A^j\partial^j A^i + \partial^+ A^i\partial^j A^j\Big]. \qquad (2.38)$$

The generator of rotations in the transverse plane

$$J^3 = \frac{1}{2}\int dx^- d^2x^\perp[x^1\Theta^{+2} - x^2\Theta^{+1}]. \qquad (2.39)$$

After using the equations of constraint, we arrive at[7]

$$J^3 = J^3_{f(o)} + J^3_{f(i)} + J^3_{g(o)} + J^3_{g(o)} \qquad (2.40)$$

with

$$\begin{aligned}
J^3_{f(o)} &= \int dx^- d^2x^\perp \eta^\dagger i(x^1\partial^2 - x^2\partial^1)\eta, \\
J^3_{f(i)} &= \frac{1}{2}\int dx^- d^2x^\perp \eta^\dagger\Sigma^3\eta, \\
J^3_{g(o)} &= \frac{1}{2}\int dx^- d^2x^\perp \Big\{x^1[\partial^+ A^1\partial^2 A^1 + \partial^+ A^2\partial^2 A^2] - x^2[\partial^+ A^1\partial^1 A^1 + \partial^+ A^2\partial^1 A^2]\Big\}, \\
J^3_{g(i)} &= \frac{1}{2}\int dx^- d^2x^\perp[A^1\partial^+ A^2 - A^2\partial^+ A^1]. \qquad (2.41)
\end{aligned}$$

Thus we indeed verify that all the kinematical operators are explicitly independent of interactions.

2.2.2 Dynamic Generators

Next we look at the dynamical operators[8]. The transverse rotation operator

$$F^i = \frac{1}{2}\int dx^- d^2x^\perp\Big[x^-\Theta^{+i} - x^i\Theta^{+-}\Big]. \qquad (2.42)$$

After a tedious and long calculation, the full transverse rotation operator in QCD can be written as,

$$F^2 = F^2_I + F^2_{II} + F^2_{III} \qquad (2.43)$$

where

$$\begin{aligned}
F^2_I &= \frac{1}{2}\int dx^- d^2x^\perp[x^-\mathcal{P}^2_0 - x^2(\mathcal{H}_0 + \mathcal{V})], \qquad (2.44) \\
F^2_{II} &= \frac{1}{2}\int dx^- d^2x^\perp\Bigg[\eta^\dagger\Big[\sigma^3\partial^1 + i\partial^2\Big]\frac{1}{\partial^+}\eta + \Big[\frac{1}{\partial^+}(\partial^1\eta^\dagger\sigma^3 - i\partial^2\eta^\dagger)\Big]\eta\Bigg] \\
&\quad + \frac{1}{2}\int dx^- d^2x^\perp m\Bigg[\eta^\dagger\Big[\frac{\sigma^1}{i\partial^+}\eta\Big] - \Big[\frac{1}{i\partial^+}\eta^\dagger\sigma^1\Big]\eta\Bigg] \\
&\quad + \frac{1}{2}\int dx^- d^2x^\perp g\Bigg[\eta^\dagger\frac{1}{\partial^+}[(-i\sigma^3 A^1 + A^2)\eta] + \frac{1}{\partial^+}[\eta^\dagger(i\sigma^3 A^1 + A^2)]\eta\Bigg], \qquad (2.45) \\
F^2_{III} &= -\frac{1}{2}\int dx^- d^2x^\perp 2(\partial^1 A^1)A^2 \\
&\quad - \frac{1}{2}\int dx^- d^2x^\perp g\frac{4}{\partial^+}(\eta^\dagger T^a\eta)A^{2a} - \frac{1}{2}\int dx^- d^2x^\perp g f^{abc}\frac{2}{\partial^+}(A^{ib}\partial^+ A^{ic})A^{2a} \\
&\qquad (2.46)
\end{aligned}$$

where $\mathcal{P}_0^i$ is the free momentum density, $\mathcal{H}_o$ is the free Hamiltonian density and $\mathcal{V}$ are the interaction terms in the Hamiltonian. The operators F_{II}^2 and F_{III}^2 which do not explicitly depend upon coordinates arise from the fermionic and bosonic parts respectively of the gauge invariant, symmetric, energy momentum tensor in QCD. Lastly, the Hamiltonian operator written in the manifestly Hermitian form is,

$$P^- = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+-} = \frac{1}{2}\int dx^- d^2x^\perp (\mathcal{H}_0 + \mathcal{H}_{int}). \tag{2.47}$$

$\mathcal{H}_0$ is the free part given by,

$$\mathcal{H}_0 = (\partial^i A_a^j)(\partial^i A_a^j) + \eta^\dagger \Big[\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\Big]\eta - \Big[\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\eta^\dagger\Big]\eta\ . \tag{2.48}$$

The interaction terms are given by,

$$\mathcal{H}_{int} = \mathcal{H}_{qqg} + \mathcal{H}_{ggg} + \mathcal{H}_{qqgg} + \mathcal{H}_{qqqq} + \mathcal{H}_{gggg}, \tag{2.49}$$

where,

$$\begin{aligned}
\mathcal{H}_{qqg} \quad &= -4g\eta^\dagger \frac{1}{\partial^+}(\partial^\perp.A^\perp)\eta + g\frac{\partial^\perp}{\partial^+}[\eta^\dagger(\tilde{\sigma}^\perp \cdot A^\perp)]\tilde{\sigma}^\perp\eta + g\eta^\dagger(\tilde{\sigma}^\perp \cdot A^\perp)\frac{1}{\partial^+}(\tilde{\sigma}^\perp \cdot \partial^\perp)\eta \\
&\quad + g(\frac{\partial^\perp}{\partial^+}\eta^\dagger)\tilde{\sigma}^\perp(\tilde{\sigma}^\perp \cdot A^\perp)\eta + g\eta^\dagger\frac{1}{\partial^+}(\tilde{\sigma}^\perp \cdot \partial^\perp)(\tilde{\sigma}^\perp \cdot A^\perp)\eta \\
&\quad - mg\frac{1}{\partial^+}[\eta^\dagger(\tilde{\sigma}^\perp \cdot A^\perp)]\eta + mg\eta^\dagger(\tilde{\sigma}^\perp \cdot A^\perp)\frac{1}{\partial^+}\eta, \\
&\quad + mg(\frac{1}{\partial^+}\eta^\dagger)(\tilde{\sigma}^\perp \cdot A^\perp)\eta - mg\eta^\dagger\frac{1}{\partial^+}[(\tilde{\sigma}^\perp \cdot A^\perp)\eta] &(2.50)\\
\mathcal{H}_{ggg} \quad &= 2gf^{abc}\Big[\partial^i A_a^j A_b^i A_c^j + (\partial^i A_a^i)\frac{1}{\partial^+}(A_b^j\partial^+ A_c^j)\Big], &(2.51)\\
\mathcal{H}_{qqgg} \quad &= g^2\Big[\eta^\dagger(\tilde{\sigma}^\perp.A^\perp)\frac{1}{i\partial^+}(\tilde{\sigma}^\perp.A^\perp)\eta - \frac{1}{i\partial^+}(\eta^\dagger\tilde{\sigma}^\perp.A^\perp)\tilde{\sigma}^\perp.A^\perp\eta \\
&\quad + 4\frac{1}{\partial^+}(f^{abc}A_b^i\partial^+ A_c^i)\frac{1}{\partial^+}(\eta^\dagger T^a\eta)\Big], &(2.52)\\
\mathcal{H}_{qqqq} \quad &= 4g^2\frac{1}{\partial^+}(\eta^\dagger T^a\eta)\frac{1}{\partial^+}(\eta^\dagger T^a\eta), &(2.53)\\
\mathcal{H}_{gggg} \quad &= \frac{g^2}{2}f^{abc}f^{ade}\Big[A_b^i A_c^j A_d^i A_e^j \\
&\quad + 2\frac{1}{\partial^+}(A_b^i\partial^+ A_c^i)\frac{1}{\partial^+}(A_d^j\partial^+ A_e^j)\Big]. &(2.54)
\end{aligned}$$

2.3 Spin operators

2.3.1 Massive case

The Pauli-Lubanski spin operator

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}M_{\nu\rho}P_\sigma \tag{2.55}$$

with $\epsilon^{+-12} = -2$. For a massive particle, the transverse spin operators[9] $\mathcal{J}^i$ in light front theory are given in terms of Poincare generators by

$$M\mathcal{J}^1 \quad = W^1 - P^1\mathcal{J}^3 = \frac{1}{2}F^2P^+ + K^3P^2 - \frac{1}{2}E^2P^- - P^1\mathcal{J}^3, \tag{2.56}$$
$$M\mathcal{J}^2 \quad = W^2 - P^2\mathcal{J}^3 = -\frac{1}{2}F^1P^+ - K^3P^1 + \frac{1}{2}E^1P^- - P^2\mathcal{J}^3. \tag{2.57}$$
$$\tag{2.58}$$

The first term in Eqs. (2.56) and (2.57) contains both center of mass motion and internal motion and the next three terms in these equations serve to remove the center of mass motion.

The helicity operator is given by

$$\mathcal{J}^3 \quad = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1P^2 - E^2P^1). \tag{2.59}$$

Here, J^3 contain both center of mass motion and internal motion and the other two terms serve to remove the center of mass motion. The operators $\mathcal{J}^i$ obey the angular momentum commutation relations

$$\left[\mathcal{J}^i, \mathcal{J}^j\right] = i\epsilon^{ijk}\mathcal{J}^k. \tag{2.60}$$

They commute with P^μ.

2.3.2 Massless case

Just as in the case of massive particle, we have the helicity operator for the massless particle,

$$\mathcal{J}^3 \quad = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1P^2 - E^2P^1). \tag{2.61}$$

In analogy with the transverse spin for massive particles, we define the transverse spin operators for massless particles as

$$\mathcal{J}^i = W^i - P^i\mathcal{J}^3. \tag{2.62}$$

They do satisfy

$$\begin{aligned} \mathcal{J}^i \mid k\lambda\rangle \quad &= 0, \\ \mathcal{J}^3 \mid k\lambda\rangle \quad &= \lambda \mid k\lambda\rangle, \end{aligned} \tag{2.63}$$

where k is an arbitrary momentum. The operators $\mathcal{J}^i$ and $\mathcal{J}^3$ obey the E_2-like algebra

$$\left[\mathcal{J}^1, \mathcal{J}^2\right] = 0, \ \left[\mathcal{J}^3, \mathcal{J}^1\right] = i\mathcal{J}^2, \ \left[\mathcal{J}^3, \mathcal{J}^2\right] = -i\mathcal{J}^1. \tag{2.64}$$

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