CHAPTER 1

Introduction

1.1 Motivation

In relativistic quantum field theories, Hamiltonian methods are very seldom used. As was observed some time ago [1] "This is a pity". This is because most of our intuitions about spectra and wave functions in quantum mechanics come from solving Hamiltonians and a variety of methods exist in quantum mechanics to analyze Hamiltonians. One would like to make use of the intuition and experience in solving non-relativistic systems in devising methods to solve relativistic quantum field theories.

BUT, the ground state (vacuum) in quantum field theory, especially Quantum Chromo Dynamics (QCD), is generally believed to be very complicated, making any simple minded use of Hamiltonian almost useless.

A proposed solution: Go to the light front!

Triviality of the vacuum:

In light front theory, Fock vacuum $\mid 0\rangle$ is an eigenstate of the full (free plus interaction parts) Hamiltonian. (*Look for fine print in the discussion in the following subsections*). To build the hadron, we need not worry about the ground state of the theory.

With a trivial vacuum (pairs popping out the vacuum being forbidden), "constituent picture of the hadron" may make sense. Let us give the operational meaning of the constituent picture: Instantaneous distribution (wave function) of constituents present at any time in the system. This requires probing the system over a time duration small on the time scale of the internal motion of the constituents. This is natural for non-relativistic systems. But hadron is a relativistic system! This implies that only a fuzzy picture of the instantaneous state of the hadron could be obtained. In this situation Feynman[2] came to the rescue.

Wavefunction of a relativistic system has non-trivial frame dependence. This is thought to be bad news. Turn it into a virtue: Certain properties of the system may be more easily seen from the wavefunction in one frame than in another. According to Feynman: "Study hadron in a frame of reference in which it moves with nearly the speed of light". Due to time dilation, internal motion of the constituents slow down! This is the celebrated ($P \rightarrow \infty$) infinite momentum frame. In such a frame the wave function is expressed in terms of the longitudinal momenta $k_L(= \eta P)$ and the transverse momenta k_T of the constituents. η is the longitudinal momentum fraction and the constituents of the hadron are called partons (quarks and gluons in the modern language).

Light front quantization provides a field theoretic realization of the Feynman's infinite momentum frame based parton picture of the hadron.

Confinement and chiral symmetry breaking:

We still do not have a clear physical physical picture of confinement and chiral symmetry breaking in QCD. They have to do with long distance physics and they are attributed to the properties of a complicated vacuum structure of QCD.

Consider the light front dispersion relation for a free massive particle: "energy" $k^- = \frac{(k^\perp)^2+m^2}{k^+}$ where $k^\perp$ is the transverse momentum and k^+ is the longitudinal momentum. We put a cutoff on the longitudinal momentum $k^+ > \epsilon$. When k^+ is near ϵ, long longitudinal distances along the light cone are probed. However, they give rise to large energy. This offers a possibility to address long distance (nonperturbative) effects through renormalization.

Deep Inelastic Scattering:

Lastly let us recall that historically interest in light front dynamics arose because of deep inelastic scattering phenomena. Bjorken [3] predicted scaling of structure functions using current algebra in the infinite momentum frame. After the discovery of scaling, Feynman's parton model came into being which is formulated in the infinite momentum frame. It is well-known that deep inelastic scattering is a light cone dominated process. Various distribution functions that appear in the description of deep inelastic scattering cross sections can be represented as Fourier transforms (along the light front longitudinal direction) of equal light front time (x^+) correlation functions.

1.2 Brief introduction to Light Front Dynamics

1.2.1 Preliminaries

According to Dirac [4], " . . . the three dimensional surface in space-time formed by a plane wave front advancing with the velocity of light. Such a surface will be called *front* for brevity". An example of a light front is given by $x^0 + x^3 = 0$.

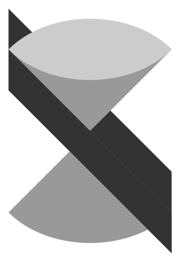


Fig. 1.1

Define $x^\pm = x^0 \pm x^3$, $x^\perp = (x^1, x^2)$. x^+ is the light front time, x^- is the longitudinal coordinate, and $x^\perp$ are the transverse coordinates. Light front quantum field theory results from the quantization on the light front surface. In the literature, same is referred to as light cone field theory, null plane field theory and field theory in the infinite momentum frame.

We have,

$$x.y = \frac{1}{2}x^+y^- + \frac{1}{2}x^-y^+ - x^\perp \cdot y^\perp. \tag{1.1}$$

Metric tensor $g^{+-} = g^{-+} = 2$, $g^{11} = g^{22} = -1$. Further, $g_{+-} = g_{-+} = \frac{1}{2}$. We have, $x_- = \frac{1}{2}x^+$, $x_+ = \frac{1}{2}x^-$.

Since covariant derivate $\partial_\mu = \frac{\partial}{\partial x^\mu}$ we have

$$\partial_+ = \frac{\partial}{\partial x^+} = \frac{1}{2}\partial^- \tag{1.2}$$

and

$$\partial_- = \frac{\partial}{\partial x^-} = \frac{1}{2}\partial^+. \tag{1.3}$$

Thus $\partial^+ = 2\frac{\partial}{\partial x^-}$ and $\partial^- = 2\frac{\partial}{\partial x^+}$.

For future purposes, we also need the following definitions. Step function

$$\theta(x) = 0, \quad \text{for} \quad x < 0, \quad = 1, \quad \text{for} \quad x > 0. \tag{1.4}$$

Antisymmetric step function

$$\epsilon(x) = \theta(x) - \theta(-x). \tag{1.5}$$

We have $\frac{\partial \epsilon(x)}{\partial x} = 2\delta(x)$ and $\mid x \mid = x\epsilon(x)$. We define the integral operators

$$\frac{1}{\partial^+} f(x^-) \quad = \frac{1}{4}\int dy^- \epsilon(x^- - y^-) f(y^-), \tag{1.6}$$

$$\frac{1}{(\partial^+)^2} f(x^-) \quad = \frac{1}{8}\int dy^- \mid x^- - y^- \mid f(y^-). \tag{1.7}$$

Note that in defining these integral operators we have taken antisymmetric boundary conditions.

1.2.2 Dispersion Relation

Since

$$k.x = \frac{1}{2}k^+x^- + \frac{1}{2}k^-x^+ - k^\perp \cdot x^\perp, \tag{1.8}$$

k^- which is conjugate to x^+, the time, is the energy and k^+ which is conjugate to x^-, the longitudinal coordinate, is the longitudinal momentum. By definition $k^\pm = k^0 \pm k^3$. For an *on-mass shell particle*, $k^0 > k^3$ implies that $k^+ \geq 0$. Further, for an on-mass shell particle, $k^2 = k^+k^- - (k^\perp)^2 = m^2$ leads to the light front dispersion relation

$$k^- = \frac{(k^\perp)^2 + m^2}{k^+}. \tag{1.9}$$

This dispersion relation has the following interesting features:

- Even though a relativistic dispersion relation, no square root factor.
- Dependence of energy on transverse momentum is just like the nonrelativistic dispersion relation, namely, $E = \frac{k^2}{2m}$.
- Energy can be large for $k^\perp$ large and/or k^+ small.
- Dependence of energy on $k^\perp$ and k^+ is multiplicative. This implies nonlocal divergences and a complicated renormalization process. Contrast this with the usual relativistic dispersion relation $E = \sqrt{(k^1)^2 + (k^2)^2 + (k^3)^2 + m^2}$ which implies local divergences and simple renormalization.

1.2.3 Hamiltonian Picture

Both the relations $k^+ \geq 0$ and $k^- = \frac{(k^\perp)^2 + m^2}{k^+}$ are true only for on mass-shell particles ($k^2 = m^2$). Does this mean that our arguments so far hold only at the tree level of field theory since there are virtual processes! In the familiar covariant perturbation theory, at any loop, particles are off mass-shell. This simply means that we can't use covariant perturbation theory to take advantage of the special features of light front dynamics. Fortunately, there is an alternative, namely, old-fashioned noncovariant perturbation theory with transition matrix elements and energy denominators.

For illustrative purposes let us consider the self-energy process in $\lambda\phi^3$ theory. In covariant perturbation theory, we have four momentum conservation at the vertex and off mass-shell particle propagation. For the Feynman amplitude, we have (Fig. 1.2),

$$-i\Sigma(p^2) = \frac{1}{2}\frac{(-i\lambda)^2}{(2\pi)^4}\int d^4k \frac{i}{k^2 - m^2 + i\epsilon}\frac{i}{(p+k)^2 - m^2 + i\epsilon}. \tag{1.10}$$

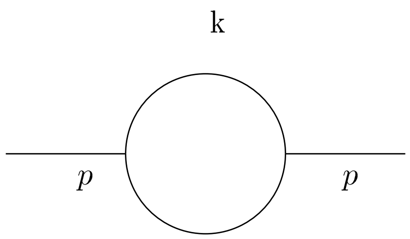
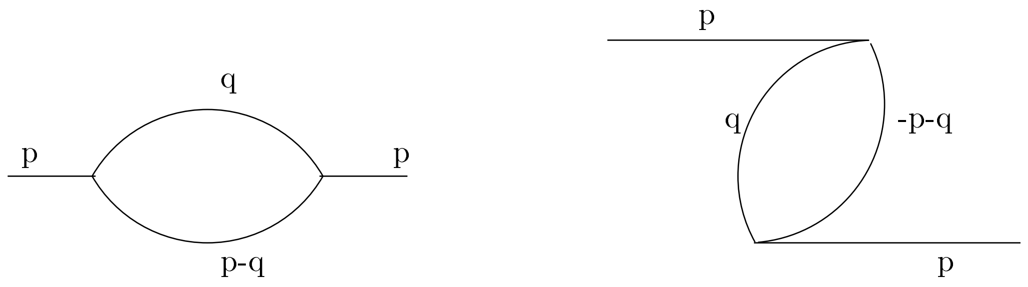


Fig. 1.2

In old fashioned perturbation theory, on the other hand, we have to consider all possible time ordered diagrams. Three momentum is conserved at the vertex and we have on mass-shell particles in initial and intermediate states. However, energy of the intermediate state is different from the energy of the initial state the difference of which appears as the energy denominator. For the ϕ^3 self energy, (Fig. 1.3), using the rules of old fashioned perturbation theory, we obtain

$$\begin{aligned}\Sigma(p^2) &= \frac{1}{2}\lambda^2\int_{-\infty}^{+\infty}\frac{d^3q}{(2\pi)^3}\frac{1}{2E_q}\frac{1}{2E_{p-q}}\Big(\frac{1}{E_p-E_q-E_{p-q}+i\epsilon} \\ &\qquad -\frac{1}{E_p+E_q+E_{-p-q}-i\epsilon}\Big). \qquad (1.11)\\ &= \Sigma_I(p^2)+\Sigma_{II}(p^2). \qquad (1.12)\end{aligned}$$

Here $E_p = \sqrt{p^2+(p^\perp)^2+m^2}$. The two contributions correspond to two different time orderings in old fashioned perturbation theory.



(a) (b)

Fig. 1.3

1.2.4 Infinite Momentum Limit

This is a convenient place to discuss the concept of infinite momentum limit. To facilitate the infinite momentum limit, we parameterize the internal momenta as follows: $\mathbf{q} = (xp, q^\perp)$, $\mathbf{p}-\mathbf{q} = ((1-x)p, p^\perp - q^\perp)$. It is important to note that the range of x is $-\infty < x < +\infty$. Now $\frac{d^3q}{2E_q} = \frac{pdxd^2q^\perp}{2E_q}$. Let us now take the infinite momentum, $p\to\infty$ limit of these expressions. It follows that $\frac{pdx}{2E_q}\to\frac{1}{2}\frac{dx}{|x|}$, $E_q \to\mid x\mid p+\frac{m^2+(q^\perp)^2}{2|x|p}$.

First consider $\Sigma_I(p^2)$. We get

$$\begin{aligned}\Sigma_I(p^2) &= \frac{1}{2}\frac{\lambda^2}{(2\pi)^3}\int_{-\infty}^{+\infty}dxd^2q^\perp\,\frac{1}{2x}\,\frac{1}{2\mid 1-x\mid p} \\ &\quad \frac{1}{p(1-\mid x\mid-\mid 1-x\mid)+\frac{m^2+(p^\perp)^2}{2p}-\frac{m^2+(q^\perp)^2}{2|x|p}-\frac{m^2+(p^\perp-q^\perp)^2}{2|1-x|p}}. \qquad (1.13)\end{aligned}$$

Now we have to distinguish various regions. For $x \geq 0$, $1-x \geq 0$, we get

$$\Sigma_I(p^2) = \frac{1}{2}\frac{\lambda^2}{(2\pi)^3}\int_0^1 dx \frac{1}{\frac{m^2+(p^\perp)^2}{2} - \frac{m^2+(q^\perp)^2}{2x} - \frac{m^2+(p^\perp-q^\perp)^2}{2(1-x)}} \tag{1.14}$$

which agrees with the light front answer. For $x > 0$, $1-x < 0$, the amplitude scales as $\frac{1}{p^2}$ which vanishes as $p \rightarrow \infty$. For $x < 0$, $1-x > 0$ the amplitude again scales as $\frac{1}{p^2}$ and vanishes in the limit.

Next we consider $\Sigma_{II}(p^2)$. In the limit $p \rightarrow \infty$, we get

$$\begin{aligned}\Sigma_{II}(p^2) \quad &= -\frac{1}{2}\frac{\lambda^2}{(2\pi)^3}\int_{-\infty}^{+\infty} dx\ \frac{1}{2\mid x\mid}\ \frac{1}{2\mid 1-x\mid p} \\ &\quad \frac{1}{\left(p(1+\mid x\mid+\mid 1-x\mid) + \frac{m^2+(p^\perp)^2}{2p} + \frac{m^2+(q^\perp)^2}{2|x|p} + \frac{m^2+(p^\perp-q^\perp)^2}{2|1-x|p}\right)}.\end{aligned} \tag{1.15}$$

For the three cases namely, (a) $x > 0, 1-x > 0$, (b) $x > 0, 1-x < 0$, and (c) $x < 0, 1-x > 0$, we find that $\Sigma_{II}(p^2)$ scale as $\frac{1}{p^2}$ which vanishes in the limit.

Thus in old fashioned perturbation theory in the infinite momentum limit ($p \rightarrow \infty$), the "backward going diagram" vanishes as $\frac{1}{p^2}$ in accordance with Weinberg's results[5].

1.2.5 Triviality of the Vacuum

We have, for any set of particles, $k_i^+ \geq 0$. Introduce a cutoff ϵ, $k_i^+ \geq \epsilon$. Consider the vacuum diagram in equal time theory (Fig. 1.4).

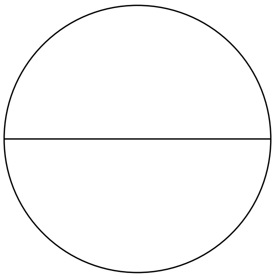


Fig. 1.4

This is possible because $-\infty \leq k_i \leq +\infty$. The vacuum diagram arises because the Hamiltonian of ϕ^3 theory has interaction terms of the form aaa and $a^\dagger a^\dagger a^\dagger$. As a consequence, bare vacuum cannot be an eigenstate of the interaction Hamiltonian. Contrast this case with light front theory with $k_i^+ > \epsilon$. Now the vacuum diagram is not possible simply because $\sum_i k_i^+ \neq 0$. Light front Hamiltonian of ϕ^3 theory has no terms of the form aaa and $a^\dagger a^\dagger a^\dagger$. Bare vacuum is an eigenstate of the Hamiltonian after trivial normal ordering with eigenvalue zero. Thus vacuum is trivial in (cutoff) light front theory.

1.2.6 Boost Symmetry

In light front theory (see appendix for details), longitudinal boost of equal time theory becomes longitudinal scaling,

$$x^{\pm} \rightarrow x^{\pm'} = e^{\pm\phi} x^{\pm} \tag{1.16}$$

and longitudinal momenta

$$k_i^{+} \rightarrow k_i^{+'} = e^{+\phi} k_i^{+}. \tag{1.17}$$

Consider an n-body system with total longitudinal momentum P^+. The ratios $x_i = \frac{k_i^+}{P^+}$ remain invariant under scaling.

Under infinitesimal transverse boosts, $x^+ \rightarrow x^+$ and $x^i \rightarrow x^{i'} = x^i - \phi x^+$. This is exactly like the Galilean boosts under which $t' = t$ and $x^{i'} = x^i - \delta v t$. Thus under transverse boosts, $k^i \rightarrow k^{i'} = k^i - \phi k^+$.

This implies that in light front theory, for a relativistic many body system, center of mass motion can be decoupled from the internal motion just like in non-relativistic many body system.

1.2.7 Free scalar field

The Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2}\mu^2\phi^2 = \frac{1}{2}\partial^+ \phi \partial^- \phi - \frac{1}{2}\partial^\perp \phi \cdot \partial^\perp \phi - \frac{1}{2}\mu^2 \phi^2. \tag{1.18}$$

The equation of motion is

$$\partial^+ \partial^- \phi - (\partial^\perp)^2 \phi + \mu^2 \phi = 0 \tag{1.19}$$

which can be rewritten as

$$i\partial^- \phi = \frac{-(\partial^\perp)^2 + \mu^2}{i\partial^+} \phi \tag{1.20}$$

which is first order in the "time" derivative.

From the Lagrangian density, conjugate momentum

$$\Pi_\phi = \frac{\partial \mathcal{L}}{\partial \partial^- \phi} = \frac{1}{2}\partial^+ \phi \tag{1.21}$$

which does not involve any time derivate and hence is constrained.

Hamiltonian density

$$\mathcal{P}^- = \Pi_\phi \partial^- \phi - \mathcal{L} = \frac{1}{2}\partial^\perp \phi \cdot \partial^\perp \phi + \frac{1}{2}\mu^2 \phi^2 \tag{1.22}$$

and the Hamiltonian

$$P^- = \frac{1}{2}\int dx^- d^2x^\perp [\partial^\perp\phi \cdot \partial^\perp\phi + \mu^2\phi^2]. \tag{1.23}$$

Since the conjugate momentum is not dynamical, the naive canonical quantization prescription, $[q_i, p_j] = i\delta_{ij}$ fails. Different methods due to Dirac[6], Schwinger[7], and Faddeev and Jackiw[8] are available in the literature for quantization. Instead of using these elegant and elaborate techniques, we here follow a cheap and dirty way: make use of something that we already know.

In free field theory, the commutator of $\phi(x)$ and $\phi(y)$ is known for arbitrary x and y (see, for example, Bjorken and Drell, Vol. II):

$$[\phi(x), \phi(y)] = i\Delta(x-y) = \int \frac{d^4k}{(2\pi)^4} 2\pi\delta(k^2-\mu^2)\epsilon(k^0)e^{-ik.(x-y)}. \tag{1.24}$$

For an on mass-shell particle, $\epsilon(k^0) = \epsilon(k^+)$, and we arrive at

$$[\phi(x), \phi(y)]_{x^+=y^+} = -\frac{i}{4}\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp). \tag{1.25}$$

Contrast this with the equal time commutator $[\phi(x), \phi(y)]_{x^0=y^0} = 0$.

The Fock expansion is given by

$$\phi(x) = \int_0^\infty \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}[a(k)e^{-ik.x} + a^\dagger(k)e^{ik.x}] \tag{1.26}$$

with

$$[a(k), a^\dagger(k')] = 2(2\pi)^3 k^+\delta(k-k'),\ [a(k), a(k')] = 0,\ [a^\dagger(k), a^\dagger(k')] = 0. \tag{1.27}$$

1.2.8 Free fermion field

The Lagrangian densityis

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi \tag{1.28}$$

and the equation of motion is

$$(i\gamma^\mu\partial_\mu - m)\psi = 0. \tag{1.29}$$

Introduce $\gamma^\pm = \gamma^0 \pm \gamma^3$. We have, $(\gamma^\pm)^2 = 0$. Introduce projection operators $\Lambda^\pm = \frac{1}{4}\gamma^\mp\gamma^\pm$. $(\Lambda^\pm)^2 = \Lambda^\pm$, $\Lambda^\pm\Lambda^\mp = 0$. Also we have $\gamma^0\gamma^\pm = 2\Lambda^\pm$.

Introduce $\psi^\pm = \Lambda^\pm\psi$.

The equation of motion in light front variables is

$$\left[\frac{i}{2}\gamma^+\partial^- + \frac{i}{2}\gamma^-\partial^+ - i\gamma^\perp\cdot\partial^\perp - m\right]\psi = 0. \tag{1.30}$$

We can simply get rid of the time derivative ∂^- by hitting with a γ^+ from the left. Then we arrive at

$$2i\partial^+\psi^- = [i\gamma^+\gamma^\perp \cdot \partial^\perp + m\gamma^+]\psi. \tag{1.31}$$

Using $\gamma^+\psi = 2\gamma^0\psi^+$ and $i\gamma^+\gamma^\perp \cdot \partial^\perp\psi = 2i\alpha^\perp \cdot \partial^\perp\psi^+$ we arrive at the constraint equation

$$i\partial^+\psi^- = [\alpha^\perp \cdot i\partial^\perp + \gamma^0 m]\psi^+. \tag{1.32}$$

At any x^+, time evolution of ψ^- is determined by that of ψ^+. This means that ψ^- is a constrained field.

Hitting the equation of motion from the left by γ^-, we get,

$$i\partial^-\psi^+ \quad = [\alpha^\perp \cdot i\partial^\perp + \gamma^0 m]\psi^- = [\alpha^\perp \cdot i\partial^\perp + \gamma^0 m]\frac{1}{i\partial^+}[\alpha^\perp \cdot i\partial^\perp + \gamma^0 m]\psi^+ \tag{1.33}$$

$$= \frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\psi^+. \tag{1.34}$$

This equation of motion for the dynamical fermion field has the following characteristics:

- nonlocality in x^-.
- fermion mass appears quadratically.
- No gamma matrices are present.

The Lagrangian density can be rewritten as

$$\mathcal{L} = i\psi^{+\dagger}\partial^-\psi^+ + i\psi^{-\dagger}\partial^+\psi^- - i\psi^\dagger\alpha^\perp \cdot \partial^\perp\psi - m\overline{\psi}\psi. \tag{1.35}$$

We note that onle ψ^+ has time derivative. The conjugate momentum

$$\Pi_{\psi^+} = \frac{\partial\mathcal{L}}{\partial\partial^-\psi^+} = i\psi^{+\dagger}. \tag{1.36}$$

The Hamiltonian density

$$\begin{aligned} \mathcal{P}^- \quad &= \Pi_{\psi^+}\partial^-\psi^+ - \mathcal{L} \\ &= i\psi^{+\dagger}\partial^-\psi^+ = \psi^{+\dagger}\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\psi^+. \end{aligned} \tag{1.37}$$

The Hamiltonian

$$P^- = \int dx^- d^2x^\perp \psi^{+\dagger}\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\psi^+. \tag{1.38}$$

To find the equal x^+ anti-commutation relation between dynamical fields, we borrow from what we already know.

In free field theory

$$\{\psi(x), \psi^\dagger(y)\} = [i\gamma^\mu \partial^x_\mu + m]\gamma^0 i\Delta(x-y). \tag{1.39}$$

Hitting from the left and the right by Λ^+, we get,

$$\{\psi^+(x), {\psi^+}^\dagger(y)\}_{x^+=y^+} = \Lambda^+\delta^3(x-y). \tag{1.40}$$

To proceed, it is convenient to choose the following representation of γ matrices [9]:

$$\gamma^0 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \;,\; \gamma^3 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \;,\; \gamma^i = \begin{bmatrix} -i\hat{\sigma}^i & 0 \\ 0 & i\hat{\sigma}^i \end{bmatrix} \;,\; \gamma^5 = \begin{bmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{bmatrix}. \tag{1.41}$$

Here $\hat{\sigma}^1 = \sigma^2$ and $\hat{\sigma}^2 = -\sigma^1$. Then we have,

$$\gamma^+ = \begin{bmatrix} 0 & 0 \\ 2i & 0 \end{bmatrix} \;,\; \gamma^- = \begin{bmatrix} 0 & -2i \\ 0 & 0 \end{bmatrix}. \tag{1.42}$$

Then the projection operators $\Lambda_\pm$ become

$$\Lambda^+ = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \;,\; \Lambda^- = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{1.43}$$

The field components are given by

$$\psi^+ = \begin{bmatrix} \eta \\ 0 \end{bmatrix} \;,\; \psi^- = \begin{bmatrix} 0 \\ \left(\frac{1}{i\partial^+}\right)[\hat{\sigma}^i i\partial^i + im]\eta \end{bmatrix}. \tag{1.44}$$

This immediately shows that ψ^+ and ψ^- are two-component fermion fields. Hereafter, we shall simply let η represent the light-front quark field. The anti-commutator is

$$\{\eta(x), \eta^\dagger(y)\}_{x^+=y^+} = I\delta^3(x-y) \tag{1.45}$$

where I is the 2X2 unit matrix.

The Fock expansion is given by

$$\eta(x) = \sum_\lambda \chi_\lambda \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3\sqrt{k^+}}[b(k,\lambda)e^{-ik.x} + d^\dagger(k,-\lambda)e^{ik.x}] \tag{1.46}$$

with

$$\{b(k,\lambda), b^\dagger(k',\lambda')\} = \{d(k,\lambda), d^\dagger(k',\lambda')\} = 2(2\pi)^3 k^+\delta^3(k-k'). \tag{1.47}$$

Meaning of λ: the light front helicity $\lambda = \pm 1$ for the fermions. Why call it light front helicity, not just spin projection? In equal time case, for a massive fermion we have the spinors, $u(p,s)$ and $v(p,s)$ where s denotes the spin projection in the rest frame. In light front case, p^+ can never be zero even though $p^\perp$ can be zero. (One can never localize a particle in x^-, there is no x^- position operator). In the $p^\perp = 0$ frame, λ measures the angular momentum along the direction of motion - helicity. From the Pauli-Lubansky operator W^μ, we can construct the helicity operator $\mathcal{J}^3 (= \frac{W^+}{P^+})$ whose eigenvalue is $\frac{1}{2}\lambda$ in an arbitrary reference frame. Furthermore, for both $M > 0$ and $M = 0$ we find $\mathcal{J}^3 \mid p, \lambda\rangle = \frac{1}{2}\lambda \mid p\lambda\rangle$. However, in such a frame $\frac{1}{2}\lambda$ is not the spin projection along the direction of motion.

1.2.9 Free massless vector field

With $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, the equation of motion is

$$\partial_\mu F^{\mu\nu} = 0 = [\partial^+\partial^- - (\partial^\perp)^2]A^\nu - \partial^\nu[\frac{1}{2}\partial^+ A^- + \frac{1}{2}\partial^- A^+ - \partial^\perp \cdot A^\perp] = 0. \tag{1.48}$$

Choose the gauge $A^+ = 0$. For $\nu = +$, we get

$$\frac{1}{2}(\partial^+)^2 A^- = \partial^+(\partial^\perp \cdot A^\perp). \tag{1.49}$$

Thus we have the equation of constraint: $\partial^+ A^- = 2\partial^\perp \cdot A^\perp + F(x^+, x^\perp)$. We shall choose $F = 0$. Then for $\nu = i$, we get,

$$[\partial^+\partial^- - (\partial^\perp)^2]A^i = 0 \tag{1.50}$$

just like the free massless scalar field. The Hamiltonian

$$P^- = \frac{1}{2}\int dx^- d^2x^\perp \sum_{ij}(\partial^i A^j)^2. \tag{1.51}$$

Equal x^+ commutation relation

$$[A^i(x), A^j(y)]_{x^+=y^+} = \delta_{ij}(-)\frac{i}{4}\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp). \tag{1.52}$$

The Fock expansion is

$$A^\perp(x) = \sum_\lambda \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}[a(k,\lambda)\epsilon^\perp_\lambda e^{-ik.x} + a^\dagger(k,\lambda)(\epsilon^\perp_\lambda)^* e^{ik.x}] \tag{1.53}$$

with $\lambda = \pm 1$ and $\epsilon^\perp_1 = \frac{1}{\sqrt{2}}(1, i)$ and $\epsilon^\perp_2 = \frac{1}{\sqrt{2}}(1, -i)$

BIBLIOGRAPHY

[1] K.G. Wilson, Phys. Rev. **140**, B445 (1965).

[2] R. P. Feynman, *Photon-Hadron Interactions*, (Addison-Wesley Publishing Company, INC., California 1972).

[3] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

[4] P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[5] S. Weinberg, Phys. Rev. **150**, 1313 (1966).

[6] Dirac quantization procedure is described in the following books: *Lectures on Quantum Mechanics*, P.A.M. Dirac, (Belfer Graduate School of Science, Yeshiva University, New York 1964); *Constrained Hamiltonian Systems*, A. Hanson, T. Regge and C. Teitelboim, (Academia Nationale Dei Linci, Roma 1976); *Constrained Dynamics*, K. Sundermeyer, (Springer-Verlag, New York 1982); *Quantization of Gauge Systems*, M. Henneaux and C. Teitelboim, (Princeton University Press, Princeton, New Jersey 1992).

[7] The Schwinger quantization procedure is described in *Introduction to Quantum Field Theory*, P. Roman, (John Wiley and Sons, Inc. New York); Application to light front field theory can be found in S.J. Chang, R.G. Root and T.M. Yan, Phys. Rev. D**7**, 1133 (1973).

[8] L. Faddeev and R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988); R. Jackiw, *(Constrained) Quantization Without Tears*, hep-th/9306075.

[9] W.-M. Zhang, Phys. Rev. D**56**, 1528 (1997). Note that γ^i in this representation is different from the one given in W.-M. Zhang and A. Harindranath, Phys. Rev. D**48**, 4881 (1993).