Testing Contagion in Multivariate Financial Time Series

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Mar 14th, 2014

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Outlines

1. Introduction
2. Model Description
3. Model Validation
4. Simulation
5. Discussion

What is Contagion?

- According to Webster's dictionary, contagion is defined as "a disease that can be communicated rapidly through direct or indirect contact."
- Financial contagion refers to the transmission of a financial shock in one entity to other interdependent entities.
- 1994 Mexican"Tequila crisis": it was caused by the sudden devaluation of the Mexican peso in December 1994. And the impact had spread on to the Southern Cone and Brazil.
- 1997 "Asian flu": it started at Thailand July 1997 because of the adverse currency and stock market shock, and raised fears of a worldwide economic meltdown.
- Also, 1998 "Russian Virus", "US Subprime Crisis": **Contagion**, a crisis in one country (or region) can affect markets of very different sizes and structures around the globe.

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About exchange rates and stocks

- Multivariate exchange rates and stocks often exhibit effects of contagion, particularly when all the markets are going through extreme high or low economy phases.
- While the study of causes and prevention of contagion is very popular among economists, there has been no quantitative study on how to detect (hypothesis testing) and measure (estimate) contagion.

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Drawbacks of some common methods

- Methods of multivariate time series just concentrate on the joint behavior during stable and stationary periods.
- Methods in multivariate extreme value theory about tail dependence need IID assumption and the marginal extremes do NOT necessarily occur at the same time.

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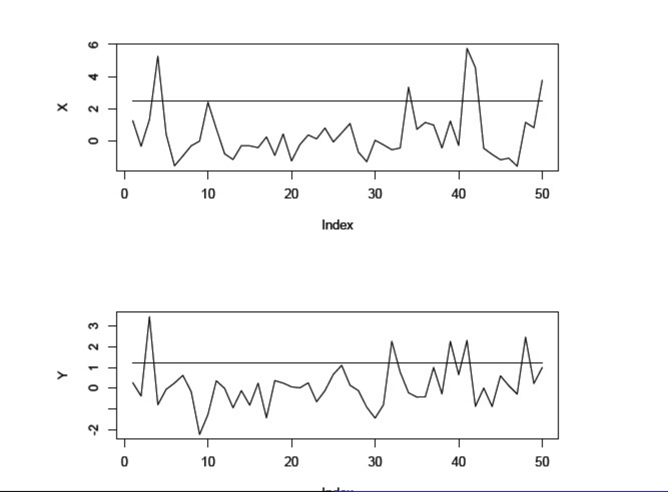
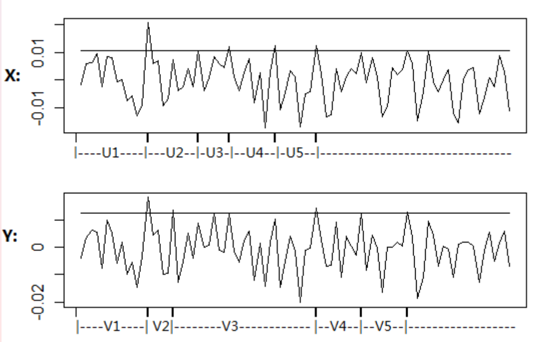
Motivation

Dependent extremes with independent non-extremes

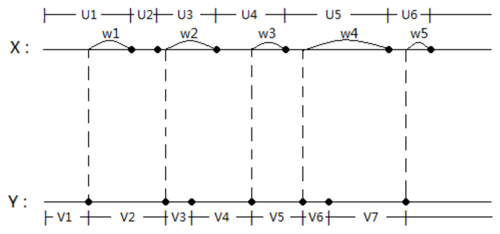


Some Definitions (Recurrence Times)

Series $\{X\} \sim F, \{Y\} \sim G$:
Define:
Recurrence time: $U_1, U_2, \ldots, U_m$ (interarrival times for exceedences in X), and $V_1, V_2, \ldots, V_n$ (interarrival times for exceedences in Y).

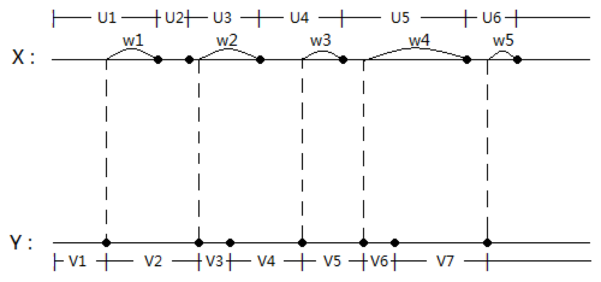


Some Definitions (Residual Times)



Residual time $(U|V)$: $W_1, W_2, \ldots, W_k$.

Intuitive idea from Geometric distribution



For iid series X, Y, if X and Y are independent, then

$U_i \sim$ iid Geometric

$V_i \sim$ iid Geometric

Intuitive idea from Geometric distribution

Testing Hypothesis:

- If there is no contagion transimition from Y to X, $U_1, U_2, \ldots, U_m$ and $W_1, W_2, \ldots, W_k$ are from the same distribution.
- If there is contagion transimition from Y to X, $U_1, U_2, \ldots, U_m$ and $W_1, W_2, \ldots, W_k$ are from different distributions.

Permutation Test can be applied to test the equality of two integer-valued series.

Degree of Contagion

To measure the extreme dependence (or tail-dependence), define *Degree of Contagion* (η_0):

$$\eta_0 := \frac{\mathrm{E}(U) - \mathrm{E}(W)}{\mathrm{E}(U) - 1}$$

1. If $\eta_0 = 0$, there is no contagion effect from Y to X (extreme independent).
2. If η_0 is significantly greater than 0, we could say that there exist contagion effects.
3. If $\eta_0 = 1$, co-movement.

Model Validation: Exceedances of IID series

The recurrence times (inter-arrival times) $\{U_i\}$ (or $\{V_i\}$) of an IID series $\{X_i\}$ (or $\{Y_i\}$) to hit its p^{th} percentile follow a geometric distribution with $p_1 = \mathrm{P}(X \geq p^{th} \text{ percentile of } X)$. That is,

$$\{U_i\} \text{ iid} \sim \text{Geometric}(p_1)$$
$$\{V_i\} \text{ iid} \sim \text{Geometric}(p_2)$$

Also, it can be proved that, provided that X and Y are independent

$$\{W_i\} \text{ iid} \sim \text{Geometric}(p_1)$$

Model Validation: Exceedances of M IID Observations

Let $\{U_i\}$ ($\{V_i\}$) be the recurrence times (inter-arrival times) of M IID observations $\{X_i\}_{i=1}^M$ ($\{Y_i\}_{i=1}^M$) to hit its p^{th} **Sample** percentile and $\{W_i\}$ are the corresponding residual times. Then, provided that X and Y are independent

$$\{W_i\} \text{ asymptotically iid} \sim \text{Geometric}(p_1), \text{ as } M \to \infty$$

Model Validation: Extreme Exceedances of ARCH(1)

Theorem (The extremes of an ARCH(1) process)

Let (X_t) be a stationary ARCH(1) process. For $x > 0$ let

$$N_n(\cdot) = \sum_{i=1}^{n} \epsilon_{n^{-1}i}(\cdot) I_{\{X_i > xn^{1/(2\kappa)}\}}$$

be the point process of exceedances of the threshold $xn^{1/(2\kappa)}$ by $X_1, \ldots, X_n$.
Then

$$N_n \xrightarrow{d} N, \quad n \to \infty,$$

in $M_p((0, 1])$,

Model Validation: Extreme Exceedances of ARCH(1)

Theorem (continued)

where N is a compound Poisson process (CPP) with intensity $c\theta^{-2\kappa}$ and cluster probabilities

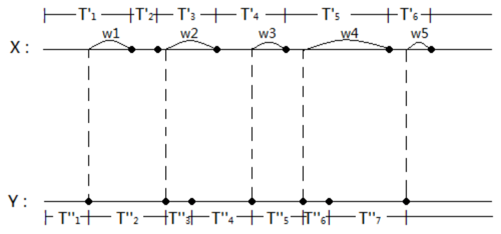
$$\pi_k = (1 - \Pi^{(2)}(0.5))^{-1} \sum_{m=k}^{\infty} \binom{m}{k} \pi_m^{(2)} 2^{-m}, \quad k \in \mathbb{N}_0.$$

where $\theta = 2\theta^{(2)}(1 - \Pi^{(2)}(0.5))$, $\Pi^{(2)}(u) = \sum_{k=1}^{\infty} \pi_k^{(2)} u^k$,
$\pi_k^{(2)} = \frac{\theta_k^{(2)} - \theta_{k+1}^{(2)}}{\theta^{(2)}}$,
$\theta_k^{(2)} = k \int_1^{\infty} P(card\{n \in \mathbb{N} : \Pi_{t=1}^{n}(\lambda Z_t^2) > y^{-1}\} = k - 1) y^{-\kappa-1} dy$,
$\theta_1^{(2)} = \theta^{(2)}$.

Remark: Similar result holds for GARCH(1,1).

Model Validation: residual time for two independent CPP

Two compound poisson processes X and Y on $[0, 1]$ with intensity λ_1 and λ_2, respectively. Let T_i' and T_j'' are the interarrival times for X and Y, respectively. Then, $T_i' \overset{iid}{\sim} \exp(\lambda_1)$ and $T_j'' \overset{iid}{\sim} \exp(\lambda_2)$.



Then, $W_i \overset{iid}{\sim} \exp(\lambda_1)$, provided independence between X and Y.

Other candidates

- Censored Likelihood Method (CLM) of Ledford and Tawn (1996)
- Extremogram of Davis and Mikosch (2009)
- Methods in testing tail independence following Falk and Michael (2006)

The last two assume extremes happen at the same point of time in each series.
The first is likelihood based and hence often computationally infeasible.

Simulation: IID Normal

- Size: Simulate two series of IID normal samples with $\sigma_1 = 1, \sigma_2 = 10$, with sample size 1000.
- Power: simulate a series of IID multivariate normal random vector with mean (0,0), covariance matrix positive correlated: $\begin{pmatrix} 10 & 2 \\ 2 & 3 \end{pmatrix}$, and negetive correlated: $\begin{pmatrix} 10 & -2 \\ -2 & 3 \end{pmatrix}$.

We compare our results to the Censored Likelihood Method of Ledford and Tawn (1996).

Simulation: IID Normal

Table: Size, Upper vs Upper

Threshold	0.70	0.75	0.80	0.85	0.90
Size(RRT)	0.045	0.044	0.047	0.039	0.042
Size(CLM)	0.077 (0.587)	0.077 (0.442)	0.072 (0.318)	0.061 (0.251)	0.051 (0.176)

Note: Significant level 0.05 is used. The value corresponding to Threshold equal to 0.9 in the table is the size of the test when we define the values above 90th percentile for X as extreme events and above 90th percentile for Y as extreme events. Figure in brackets is the percentage of occurrences of bad results (e.g., warning messages, errors and NaN results) in the likelihood computation.

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NP	Fish	KS	ChiSq
0.068	0.048	0.035	0.032

Simulation: IID Normal

Table: Power, Positive Correlated, Upper vs Upper

Threshold	0.70	0.75	0.80	0.85	0.90
Power(RRT)	1.000	1.000	1.000	0.998	0.954
Power(CLM)	1.000 (0.57)	1.000 (0.443)	1.000 (0.301)	1.000 (0.233)	1.000 (0.185)

NP	Fish	KS	ChiSq
0.870	0.144	0.467	0.329

Simulation: GARCH

- Size: Use log returns of weekly indices for Mexico and USA from May 2003 to May 2007 (about 4 years weekly data) to fit two unitvariate GARCH models (GARCH(1,1)), then use the fitted models to simulate two series of length 1000.
- Power:
  Use the same data to fit a bivariate GARCH model (BEKK(1,1)), then use the fitted model to simulate 1000 log returns.
- Data shows contagion effects from USA to Mexico in the Lower vs Lower case, but not in the Upper vs Lower case.

Simulation: GARCH

Table: Size and Power, Mex and USA weekly indices, Lower vs Lower

Method	RRT	CLM	NP	Fish	KS	ChiSq
Size	0.038	0.066(0.119)	0.067	0.050	0.060	0.055
Power (LowvsLow)	0.911	1.000(0.153)	1.000	0.169	0.876	0.785
Power (HighvsLow)	0.195	0.823(0.156)	0.924	0.111	0.533	0.419

Large values in row 2 and small values in row 3 are desirable.

Empirical Study: Indices Series

Pairwise P-values (0.1 vs 0.1 threshold):

	Arg	Bra	Chile	Colombia	Mexico	Peru	China	India	Indonesia	Korea	Malaysia	Phili	Taiwan	Thailand	Europe	USA	Japan
Arg	NA	0.055	0.320	**0.040**	0.095	**0.005**	0.300	0.180	0.105	0.250	0.595	0.050	0.130	0.105	0.180	0.985	0.310
Bra	0.060	NA	0.380	**0.020**	**0.030**	**0.000**	0.220	0.100	0.140	0.135	0.335	**0.045**	0.060	**0.010**	0.225	0.500	0.255
Chile	0.070	0.125	NA	0.130	0.150	**0.005**	0.435	0.215	0.320	0.520	0.650	0.100	0.110	0.070	0.180	0.840	0.220
Colombia	0.175	**0.010**	0.840	NA	0.080	**0.000**	0.415	0.360	0.185	0.530	0.630	0.100	0.150	**0.025**	0.345	0.910	0.990
Mexico	0.085	0.065	0.655	0.075	NA	**0.000**	0.930	0.335	0.365	0.550	0.975	0.385	0.345	0.100	0.055	0.275	0.240
Peru	**0.015**	**0.000**	0.230	**0.020**	0.015	NA	0.390	0.165	0.145	0.080	0.915	0.020	0.180	0.010	0.285	0.835	0.245
China	0.125	0.030	0.415	0.235	0.225	**0.010**	NA	0.240	0.130	0.185	0.385	**0.015**	0.070	0.025	0.555	0.795	0.295
India	0.175	0.055	0.710	0.050	0.240	**0.035**	0.295	NA	0.210	0.380	0.390	0.170	0.270	0.120	0.385	0.560	0.615
Indonesia	0.300	**0.045**	0.695	0.115	0.200	**0.030**	0.095	0.170	NA	0.510	0.410	0.060	0.060	**0.010**	0.405	0.895	0.415
Korea	0.170	0.210	0.240	0.380	0.200	**0.045**	0.430	0.445	0.365	NA	0.430	0.095	0.070	0.155	0.240	0.685	0.220
Malaysia	0.130	0.290	0.390	0.230	0.225	0.060	0.165	0.245	0.300	0.350	NA	0.090	0.115	0.100	0.415	0.930	0.200
Phili	0.195	**0.045**	0.375	**0.025**	0.125	**0.005**	0.330	0.240	0.070	0.285	0.800	NA	0.050	**0.030**	0.430	0.915	0.305
Taiwan	0.185	0.065	0.460	0.125	0.210	**0.025**	0.170	0.220	0.120	0.210	0.425	**0.045**	NA	**0.015**	0.230	0.780	0.090
Thailand	0.395	**0.005**	0.980	**0.015**	**0.035**	**0.000**	0.690	0.405	0.215	0.200	0.870	0.415	0.175	NA	0.370	0.910	0.190
Europe	0.145	0.240	0.355	0.165	0.160	**0.040**	0.615	0.240	0.170	0.690	0.475	0.300	0.160	0.135	NA	0.390	0.170
USA	0.115	0.295	0.660	0.285	0.340	0.130	0.525	0.540	0.470	0.610	0.705	0.280	0.545	0.545	0.185	NA	0.550
Japan	0.170	0.170	0.695	0.145	0.250	0.075	0.825	0.430	0.280	0.990	0.625	0.730	0.190	0.100	0.240	0.795	NA

Empirical Study: Indices Series

Pairwise P-values (0.1 vs 0.1 threshold) Partial table:

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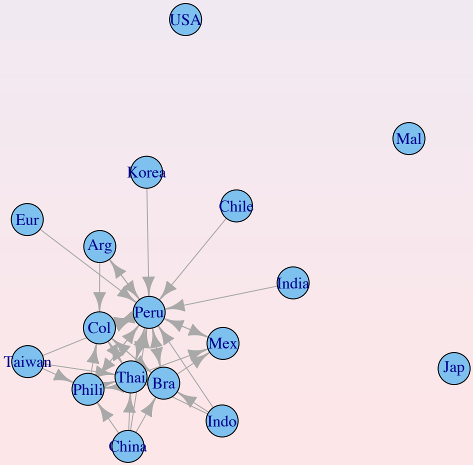
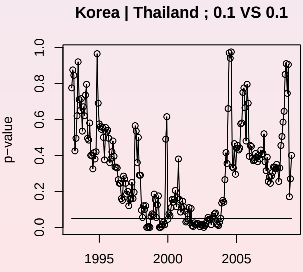
Empirical Study: Indices Series

Contagion Plot:

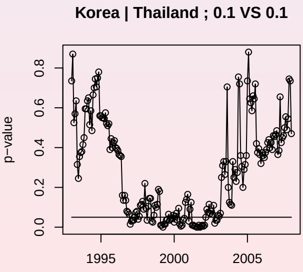


Empirical Study: Asian Flu

The crisis that has generated in 1997 following the devaluation of Thai Baht exerts its impact on Korea and the impact lasts until end 1999. The contagion effect is stronger until 2004.



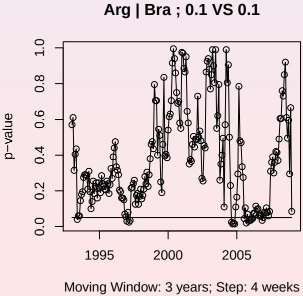
Moving Window: 3 years; Step: 4 weeks

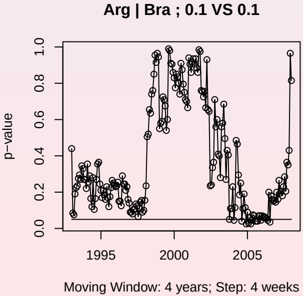


Moving Window: 4 years; Step: 4 weeks

Empirical Study: Brazilian Crisis

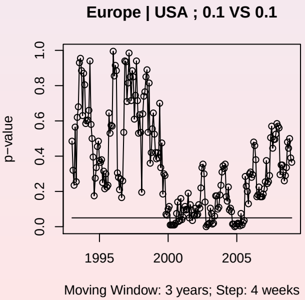
Argentina is the largest partner of Brazil. The impact was strong during 1998-1999, the years of the crisis, and then again from late 2004.

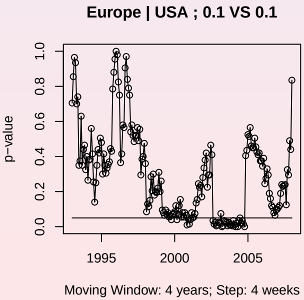




Empirical Study: Subprime crisis

The contagion effect from USA to Europe after year 2000 becomes more significant. This provides an evidence of continuous interaction between USA and European countries for the recent decade.





Discussion

Some future work:

- Extend the bivariate method to multivariate and detect if there are some series (say, indices series) that drive the other markets.
- How to choose good thresholds and how would thresholds affect the method.
- Establish a method of estimation of the time it takes for transmission of a shock from one market to another.
- How to deal with clusters of exceedances and strong serial dependence in the same series.

Discussion

Some wider applications:

- Analysis of spatial patterns of disease and spatial spread of epidemics.
- Analysis of spread of social influences
- Transmission of neural signals on the onset of epilepsy with irregular behavior of one neuron triggering that of others and resulting in everything breaking down.

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Thank You!