Network Analysis of Crisis in the Global Financial Network

Sunil Kumar

Department of Physics, Ramjas College
University of Delhi, Delhi-110007

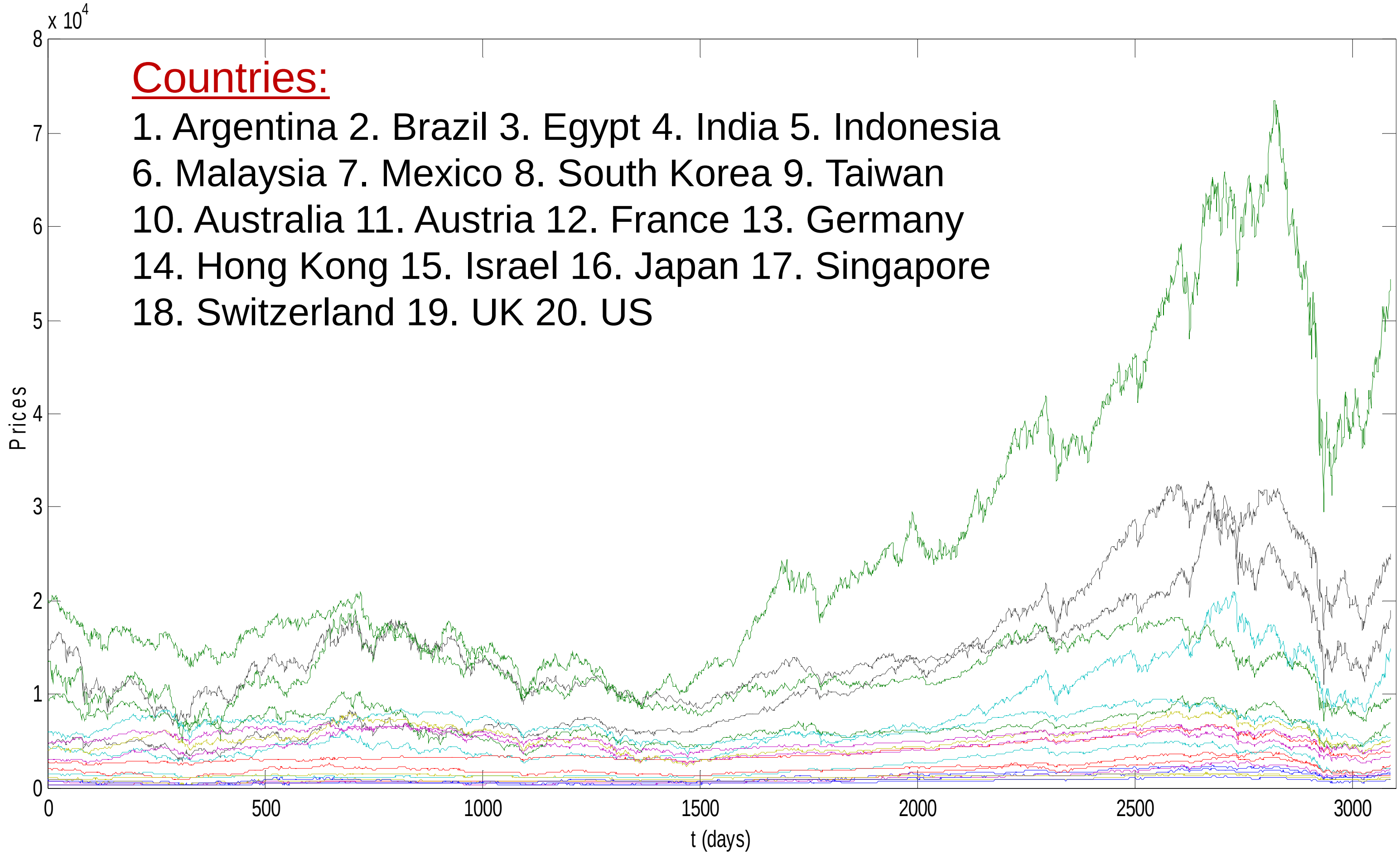
Collaborator: Dr. Nivedita Deo
Department of Physics & Astrophysics
University of Delhi, Delhi-110007

Contents

- Introduction
- Empirical Analysis of 20 global financial indices
- Global Financial Crisis of 2008
- Random matrix theory approach to crisis
- Complex Network Analysis of crisis:
    - (i) Correlation threshold method
    - (ii) Hierarchical methods:
        - a) Minimum Spanning Tree
        - b) Hierarchical Clustering
- Conclusion

Empirical Analysis of 20 Global Financial Indices

Daily closing prices (July, 1997 to June, 2009)

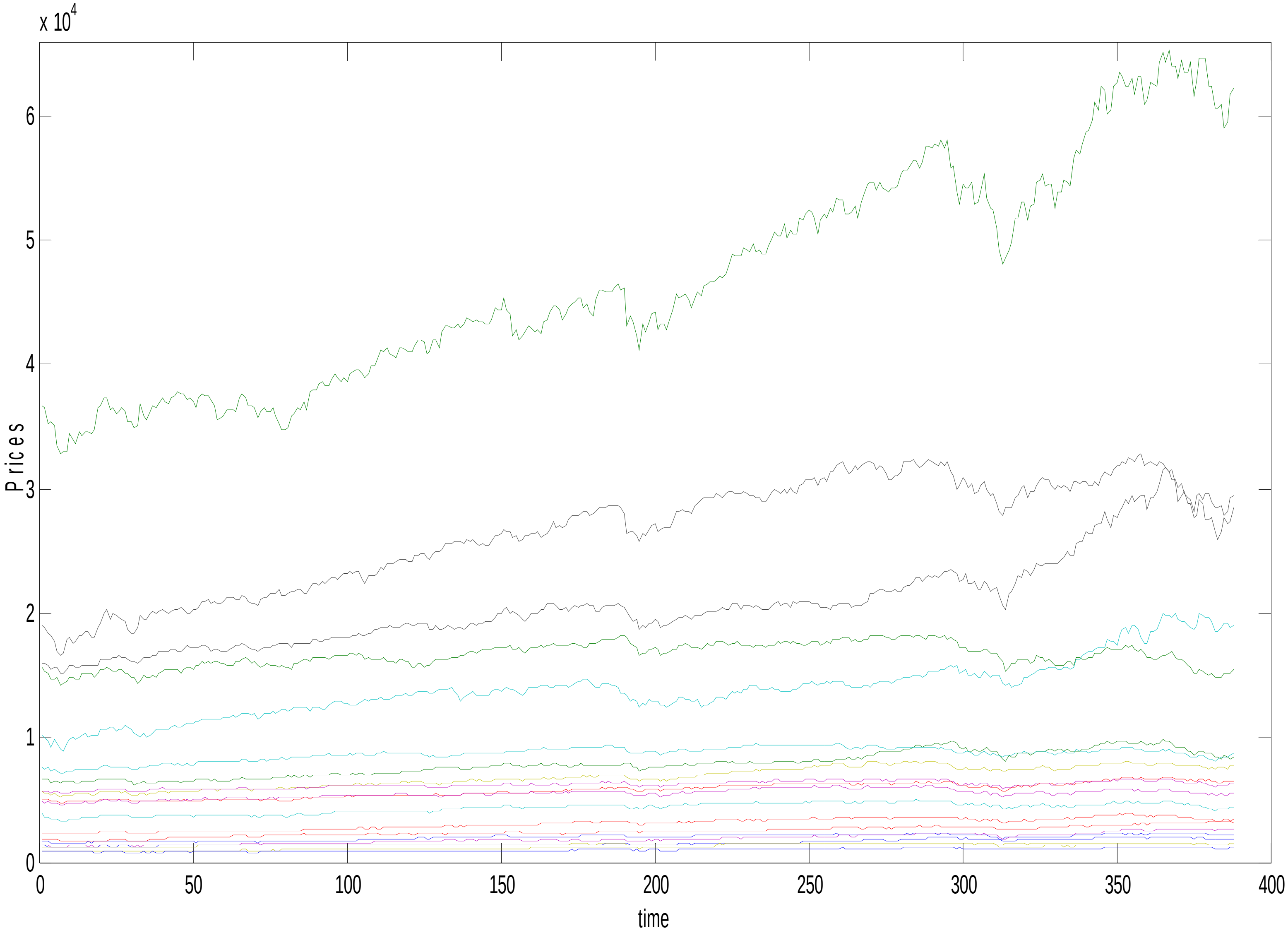


There are Differences in public holidays or weekends among countries (so we shifted the data according to the rule that) when more than 30 % of markets did not open on a particular day, we remove that day from the data, and when it is below 30 %, we kept existing indices and inserted the last closing price for each of the remaining indices.

Global Financial Crisis of 2008

Closing Prices: Before the Crisis

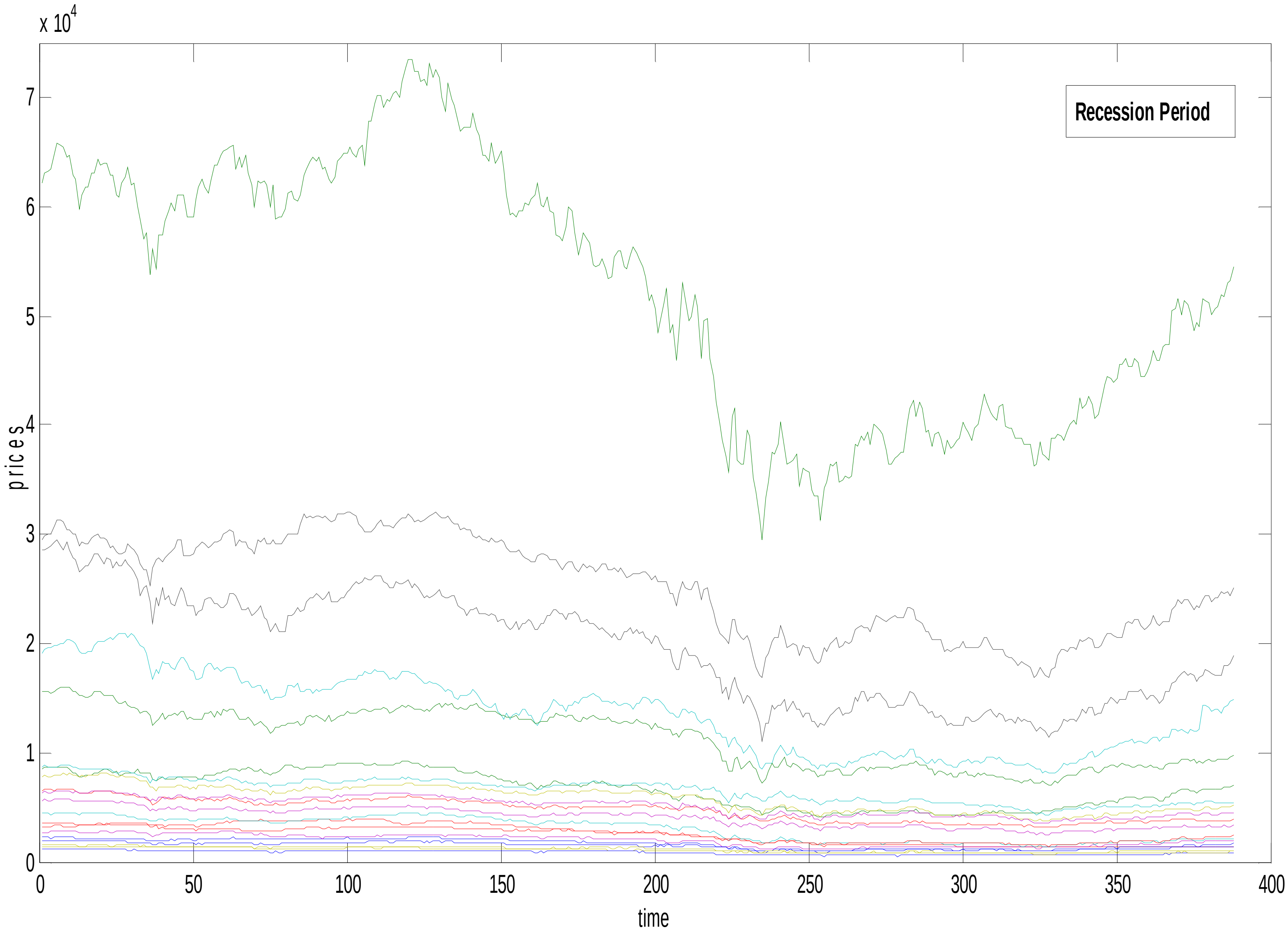
(June, 2006 to November, 2007)



Global Financial Crisis of 2008

Closing Prices: During the Crisis

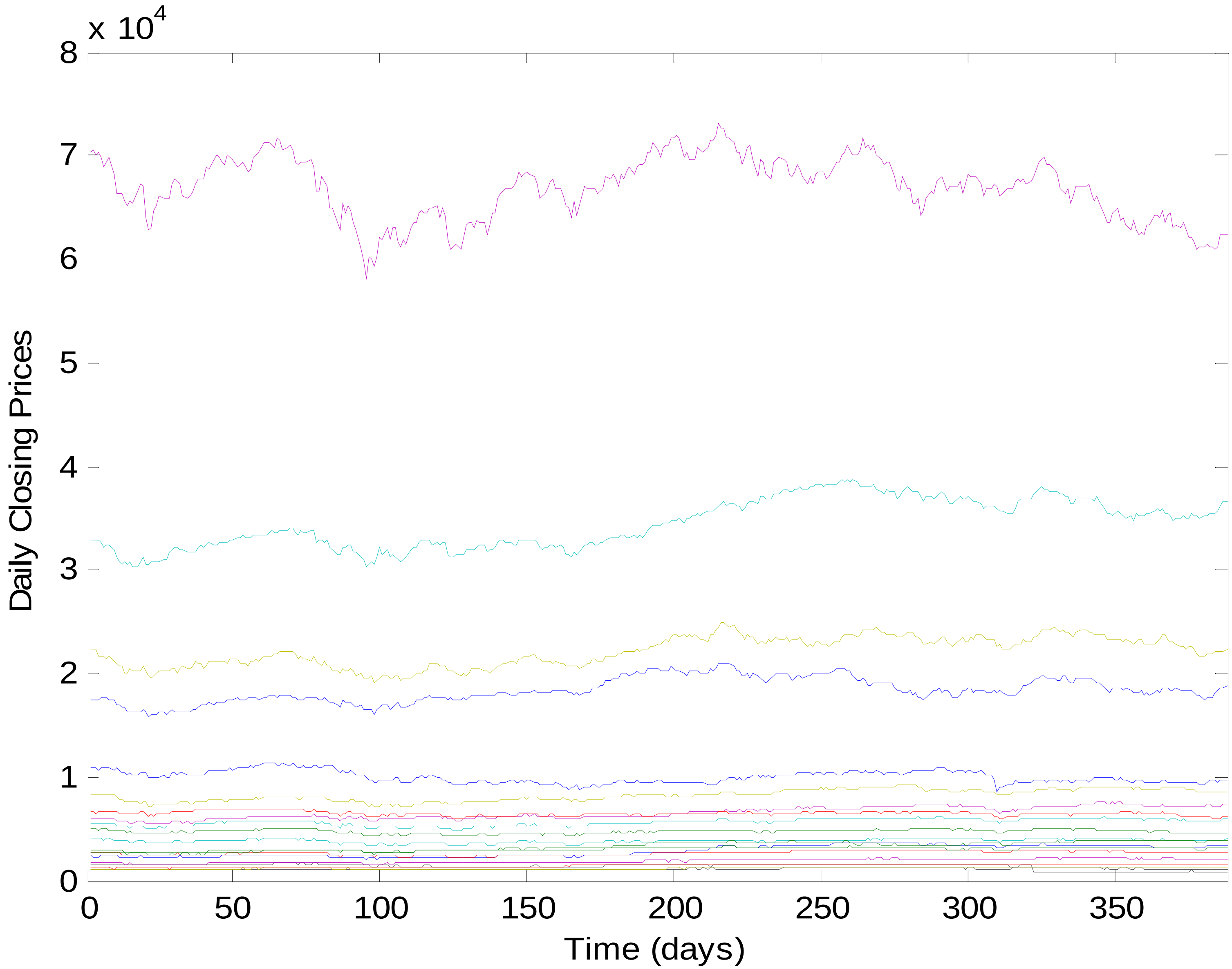
(December, 2007 to June, 2009)



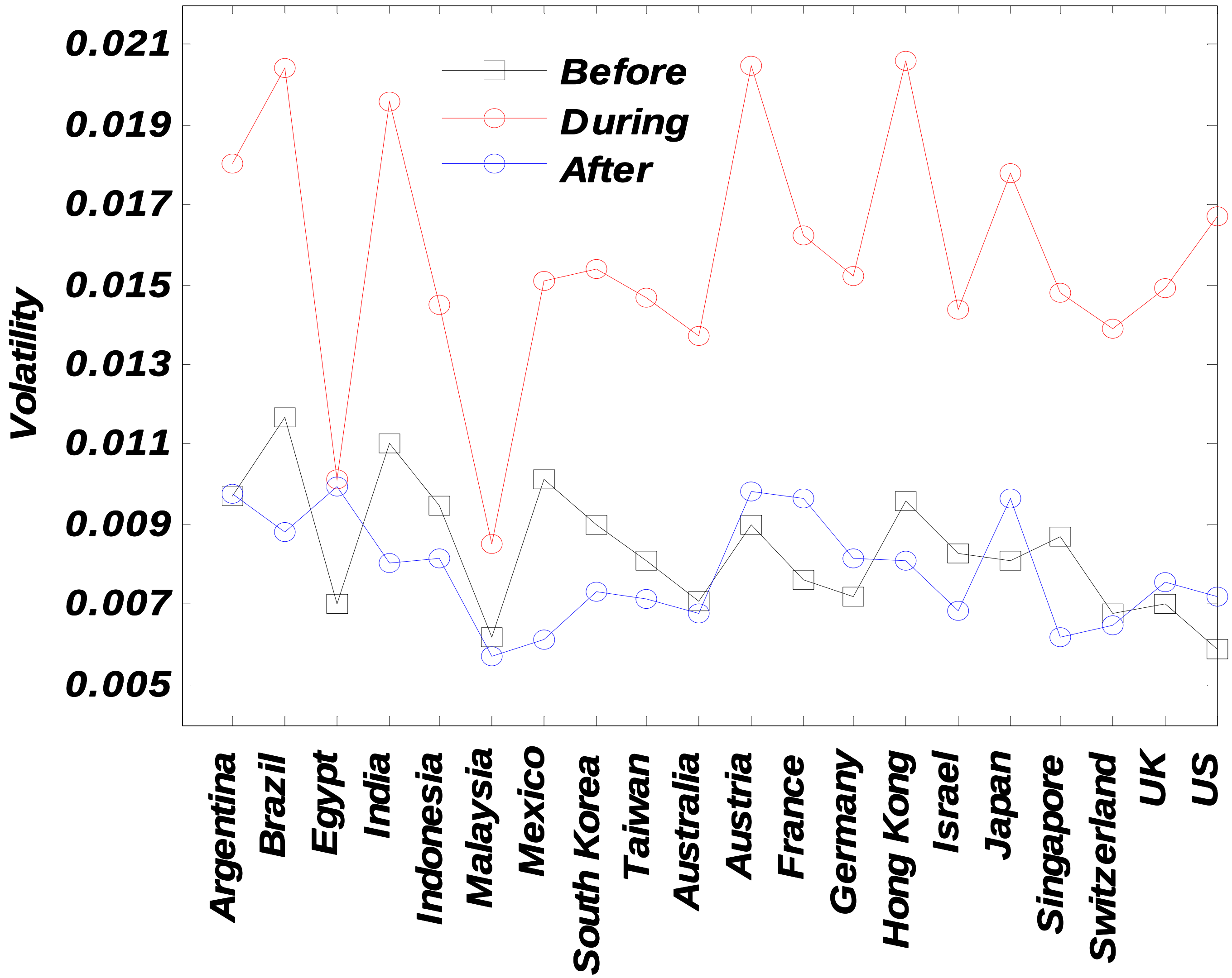
Global Financial Crisis of 2008

Closing Prices: After the Crisis

(January, 2010 to June, 2011)



Volatility: Measure of fluctuations (global financial crisis of 2008)



$$v(t) = \sum_{t=1}^{T-1} \frac{|G(t)|}{T-1}$$

T = 387 days

Random Matrix Theory Approach

$P_i(t) \longrightarrow$ Daily closing prices of financial indices i (i = 1,..,N) at time t (t = 1,..,T)

Log-returns: $\boxed{R_i(t) \equiv \ln P_i(t+1) - \ln P_i(t)}$

Normalized returns: $\boxed{r_i(t) \equiv \dfrac{R_i(t) - \langle R_i \rangle}{\sigma_i}}$ where $\sigma_i \equiv \sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2}$

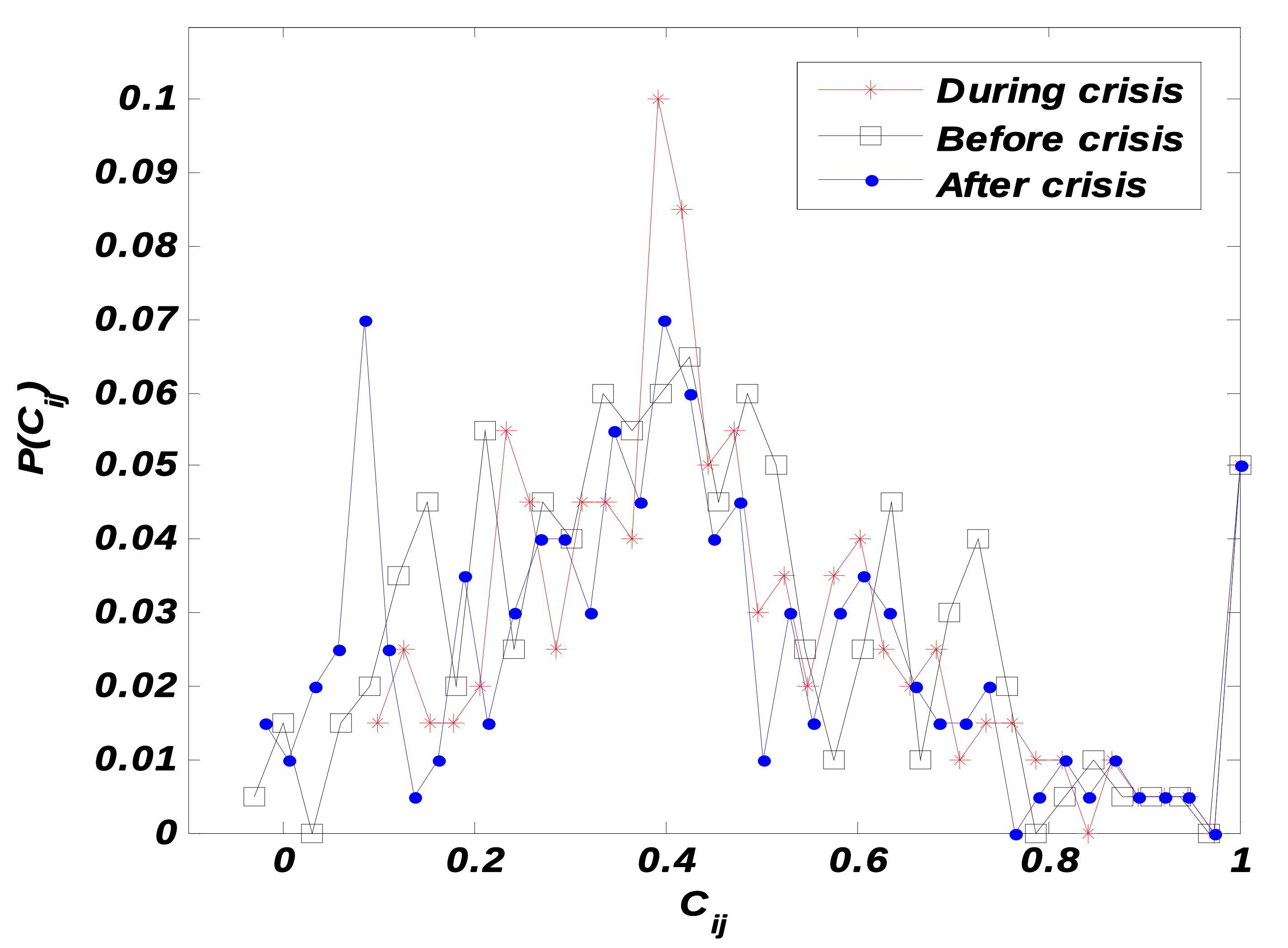
Cross-correlation matrix (C) is computed with elements,

$$\boxed{C_{ij} \equiv \langle r_i(t) r_j(t) \rangle}$$

which are limited to the domain [-1,1]

$C_{ij} = 1 \Longrightarrow$ Perfect correlation
$C_{ij} = -1 \Longrightarrow$ Perfect anti-correlation
$C_{ij} = 0 \Longrightarrow$ No correlation

Distribution of correlation coefficients (C_{ij})

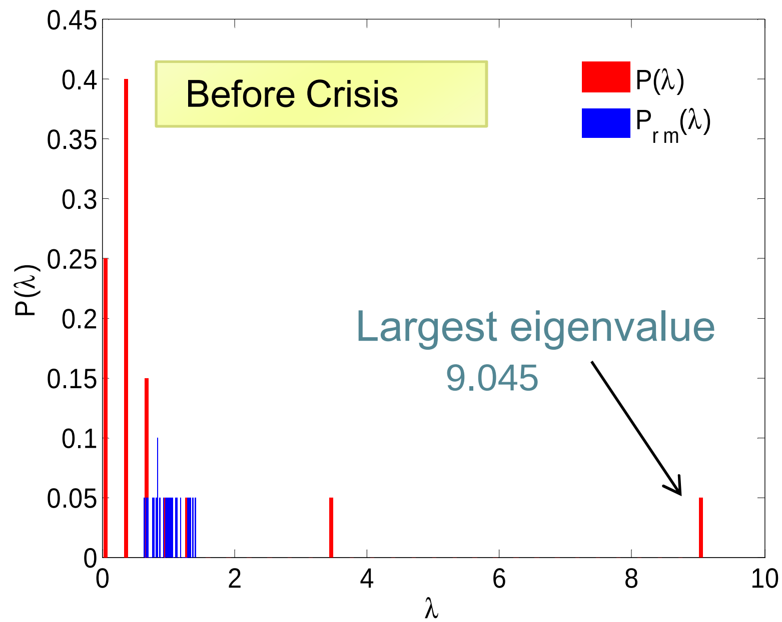


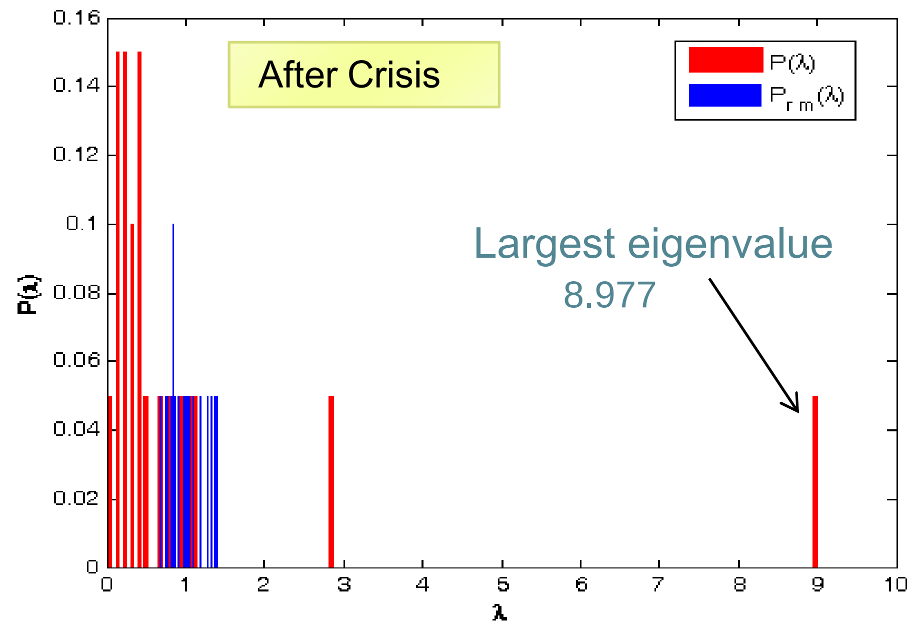
Average magnitude of correlation, <|C|> = 0.435 (before the crisis)

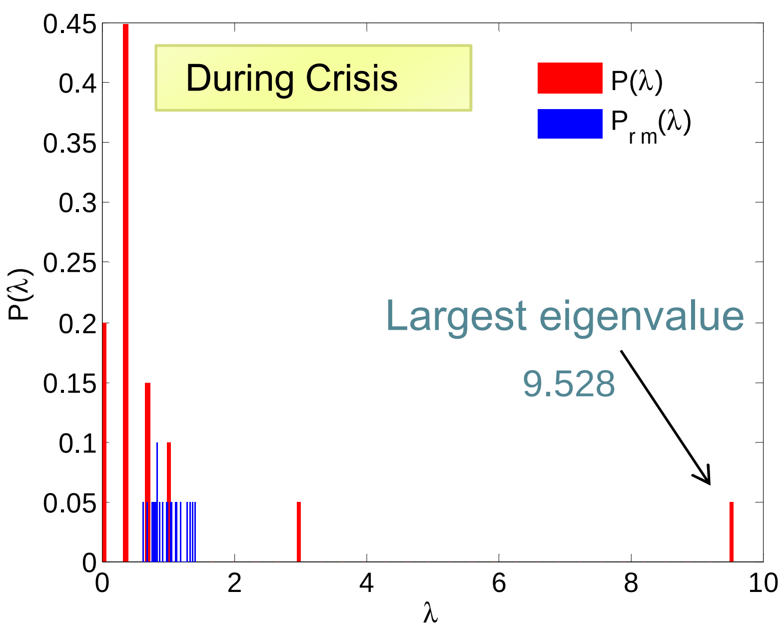
<|C|> = 0.463 (during the crisis)

<|C|> = 0.415 (after the crisis)

Eigenvalue distribution: N = 20 indices, T = 387 days, Q=19.35







Random Matrix Theory Prediction:

$\lambda^{rm}_{min} = 0.597$ and $\lambda^{rm}_{max} = 1.506$

Eigenvalue distribution: N = 20 indices, T = 387 days, Q=19.35

Random Matrix Theory Prediction $\longrightarrow$ $\lambda^{rm}_{min} = 0.597$ and $\lambda^{rm}_{max} = 1.506$

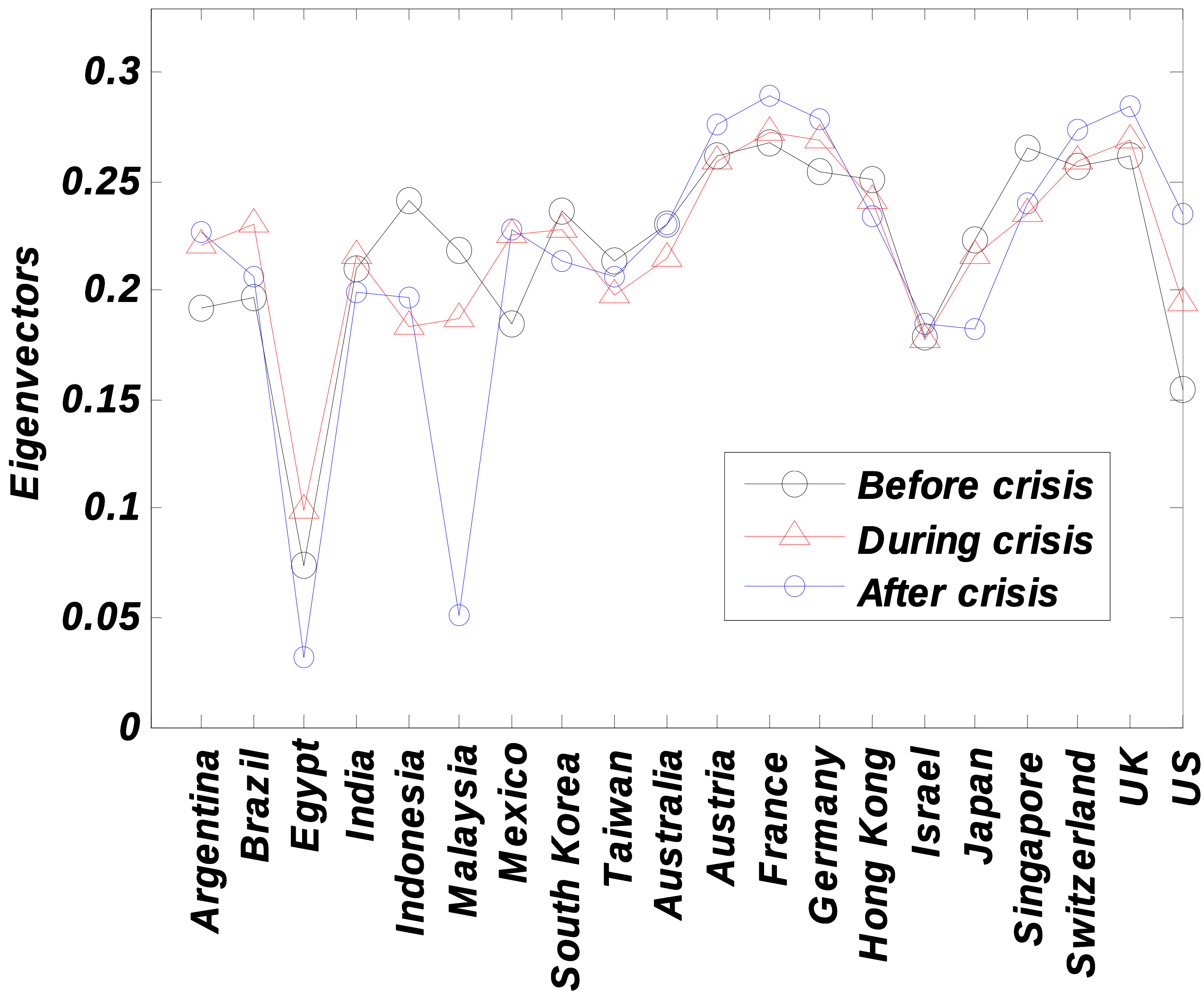
Experimentally: Global financial indices

Before the crisis $\lambda_{min} = 0.0527$ and $\lambda_{max} = 9.045$
During the crisis $\lambda_{min} = 0.0388$ and $\lambda_{max} = 9.528$
After the crisis $\lambda_{min} = 0.0505$ and $\lambda_{max} = 8.977$

If there is no correlation in financial indices $\Longrightarrow$ Eigenvalues should be bounded between RMT predictions.

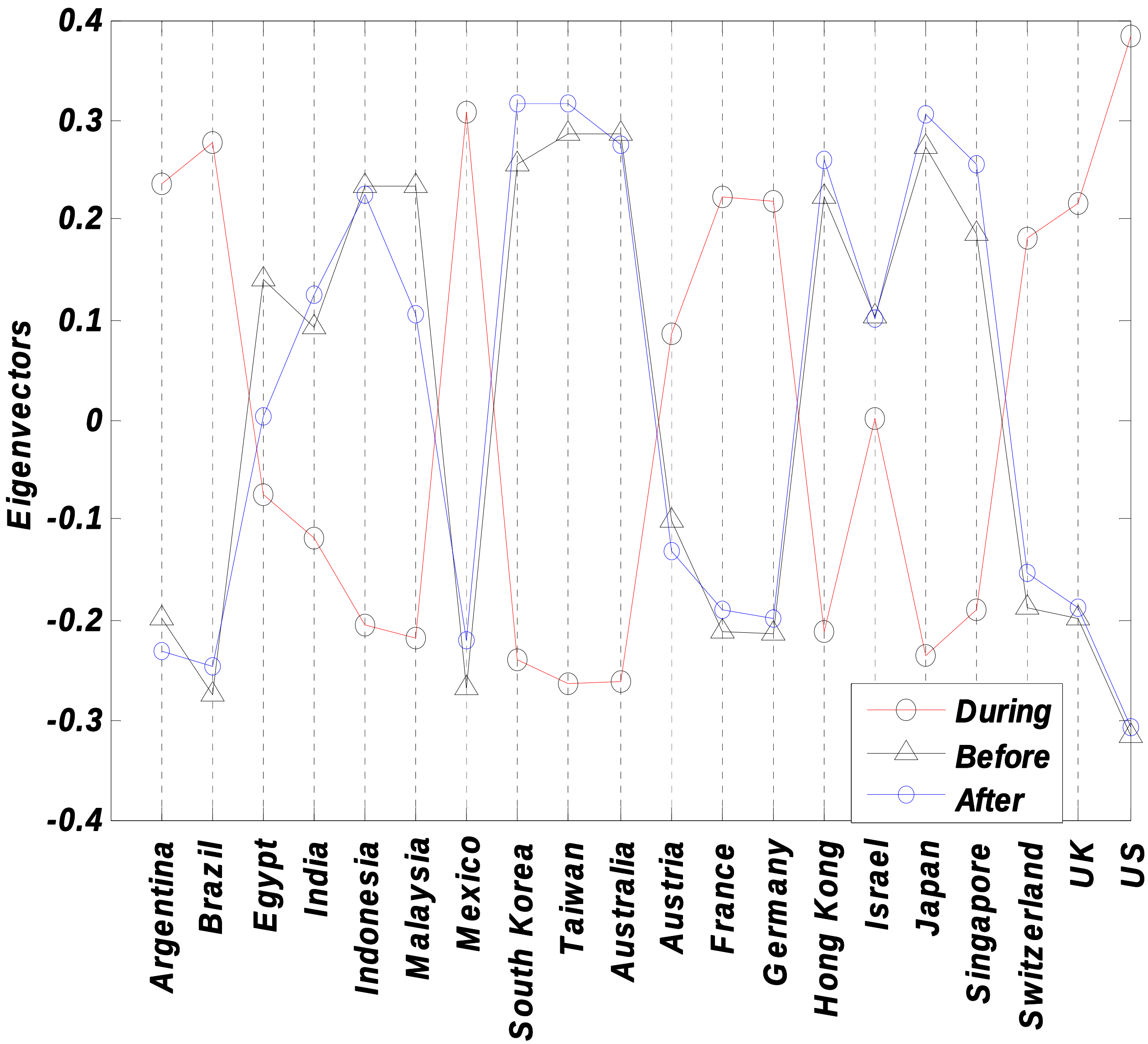
Significant deviation in eigenvalues from upper bound $\Longrightarrow$ Strong correlation in global financial indices.

Components of eigenvectors corresponding to First largest eigenvalue



All eigenvector components are positive which reflects a common global financial market mode

Components of eigenvectors corresponding to Second largest eigenvalue



Global financial indices form two clusters in positive and negative directions.

The **positive** significant contributions of components are associated with the cluster of **American**(Argentina, Brazil Mexico, United States) and **European**(Austria, France, Germany, Switzerland) indices.

The **negative** significant contributions are associated with cluster of indices corresponding to **Asia-Pacific**(Egypt, India, Indonesia, Malaysia, South Korea, Taiwan, Australia, Hong Kong, Japan, Singapore)

The components of these two clusters switch in opposite direction during the crisis period.

Conclusion (RMT analysis)

- First few largest eigenvalues deviates significantly from the RMT prediction and these deviation changes before, during, and after the crisis of 2008.

- The largest eigenvalue represent the collective information about the correlation between different indices and its trend is dependent on the market conditions.

- Components of eigenvectors corresponding to second largest eigenvalue form two clusters of indices in the positive and negative directions. The components of these two clusters switch in opposite directions during the financial crisis 2008.

- We find that RMT analysis of correlation matrices provides some information about the formation of clusters of indices.

- We use the techniques of network analysis to study these clusters clearly.

Construction of networks

(i) **Correlation threshold Method:**

Let set of financial indices defines the set of vertices of the network.

Specify a certain threshold θ ($-1 \leq \theta \leq 1$) and add an undirected edge connecting the vertices i and j if C_{ij} is greater than or equal to θ.

The edges (E) in graph G = (V,E) are defined by

$$E = \begin{cases} e_{ij} = 1, & i \neq j \text{ and } C_{ij} \geq \theta \\ e_{ij} = 0, & i = j \end{cases}$$

Thus, different values of θ generate networks with ***same set of vertices, but different set of edges.***

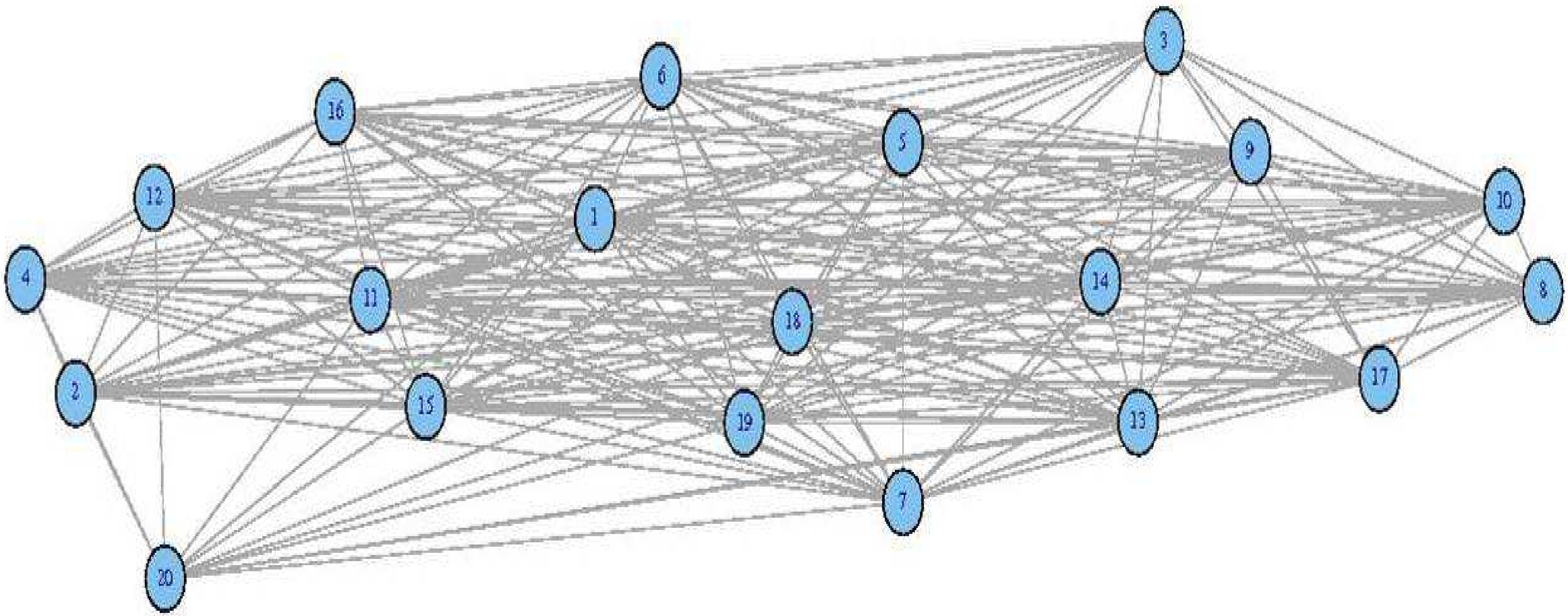
We have used the Fruchterman-Reingold layout to analyze the cluster structure in complex networks.

Correlation network of global financial indices

Threshold (θ) = 0.1 Before the crisis

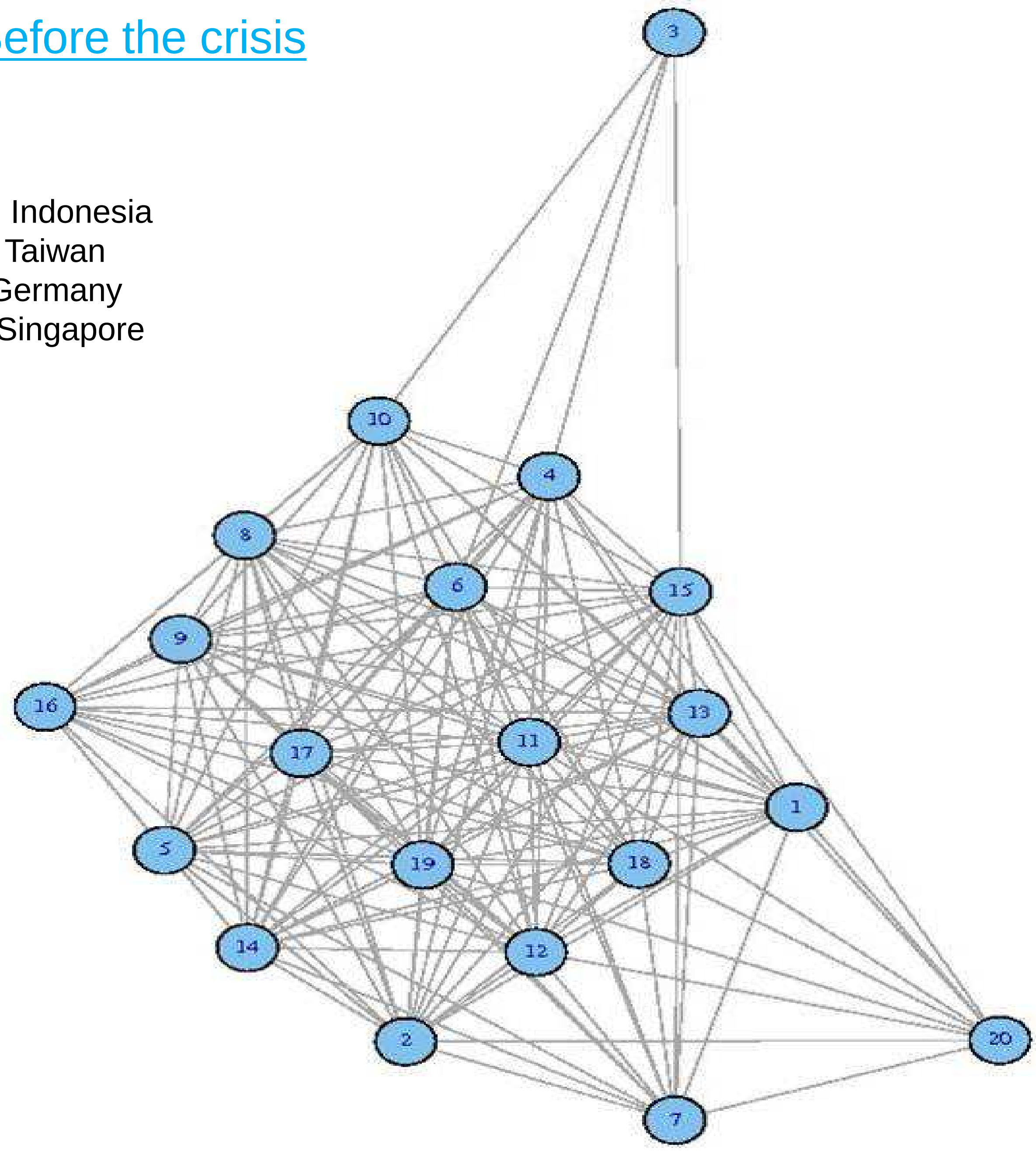
Countries:

1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia 6. Malaysia 7. Mexico 8. South Korea 9. Taiwan 10. Australia 11. Austria 12. France 13. Germany 14. Hong Kong 15. Israel 16. Japan 17. Singapore 18. Switzerland 19. UK 20. US



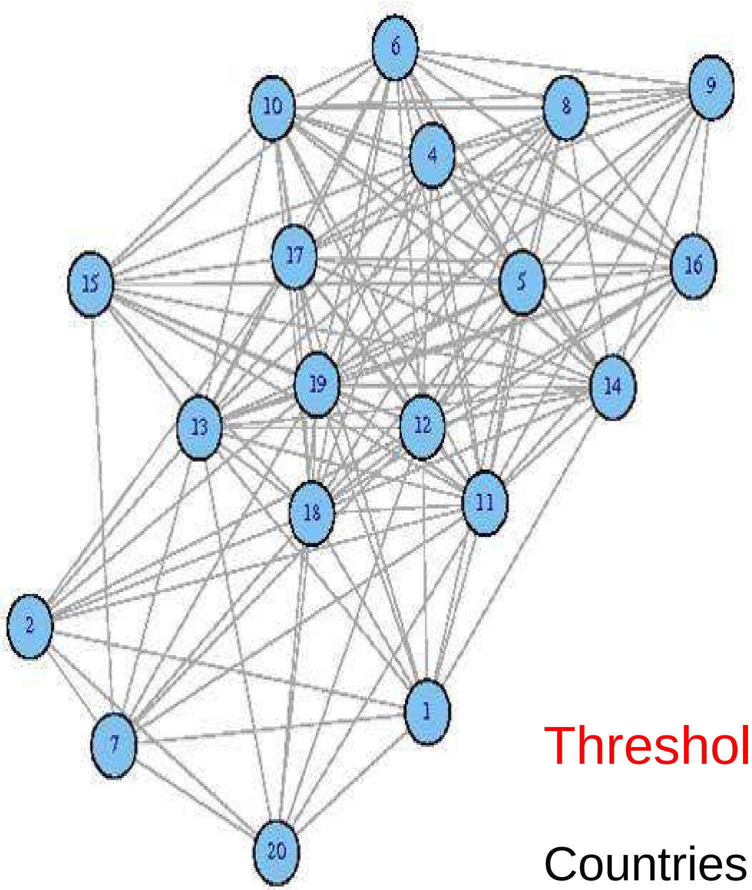
Threshold (θ) = 0.2

Before the crisis



Countries:

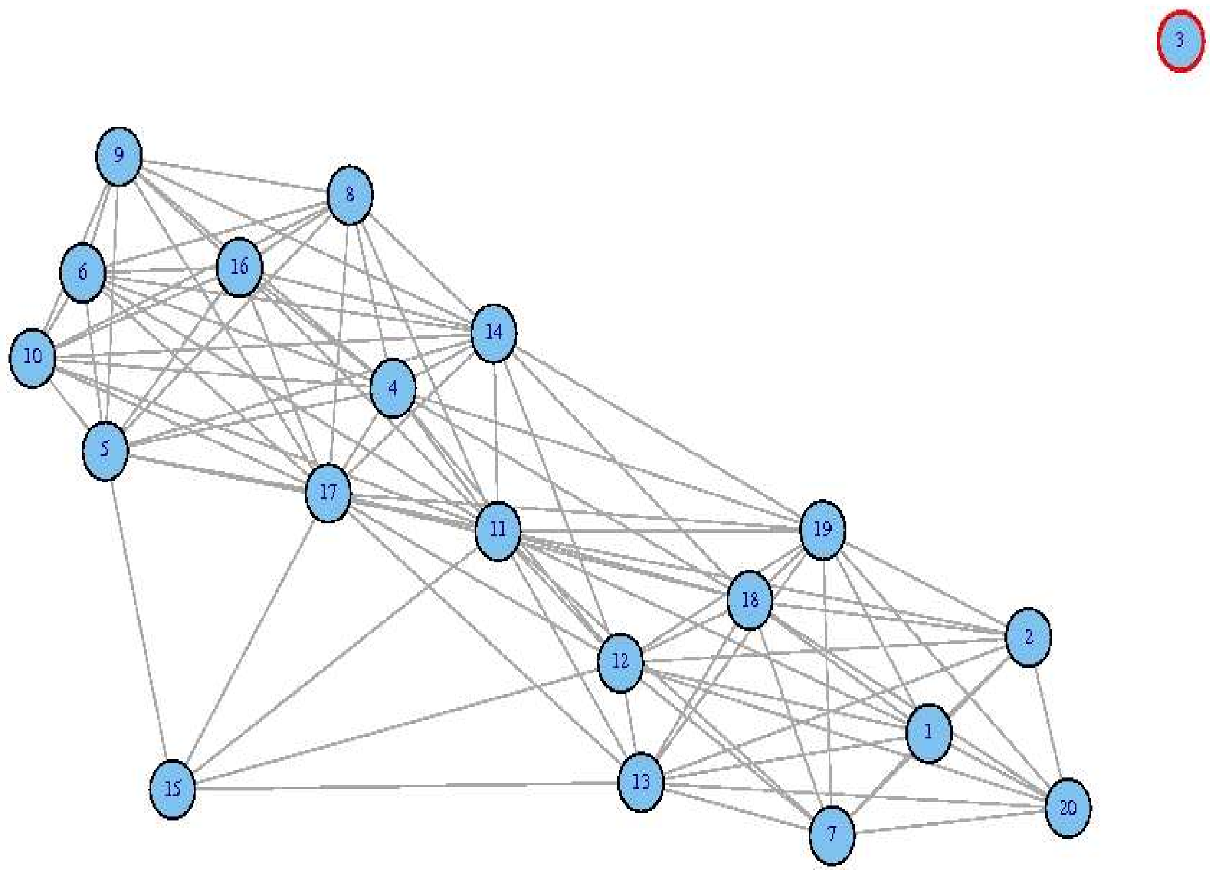
1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia
6. Malaysia 7. Mexico 8. South Korea 9. Taiwan
10. Australia 11. Austria 12. France 13. Germany
14. Hong Kong 15. Israel 16. Japan 17. Singapore
18. Switzerland 19. UK 20. US



Threshold (θ) = 0.3 Before crisis

Countries:

1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia
6. Malaysia 7. Mexico 8. South Korea 9. Taiwan
10. Australia 11. Austria 12. France 13. Germany
14. Hong Kong 15. Israel 16. Japan 17. Singapore
18. Switzerland 19. UK 20. US



Threshold (θ) = 0.4

Before crisis

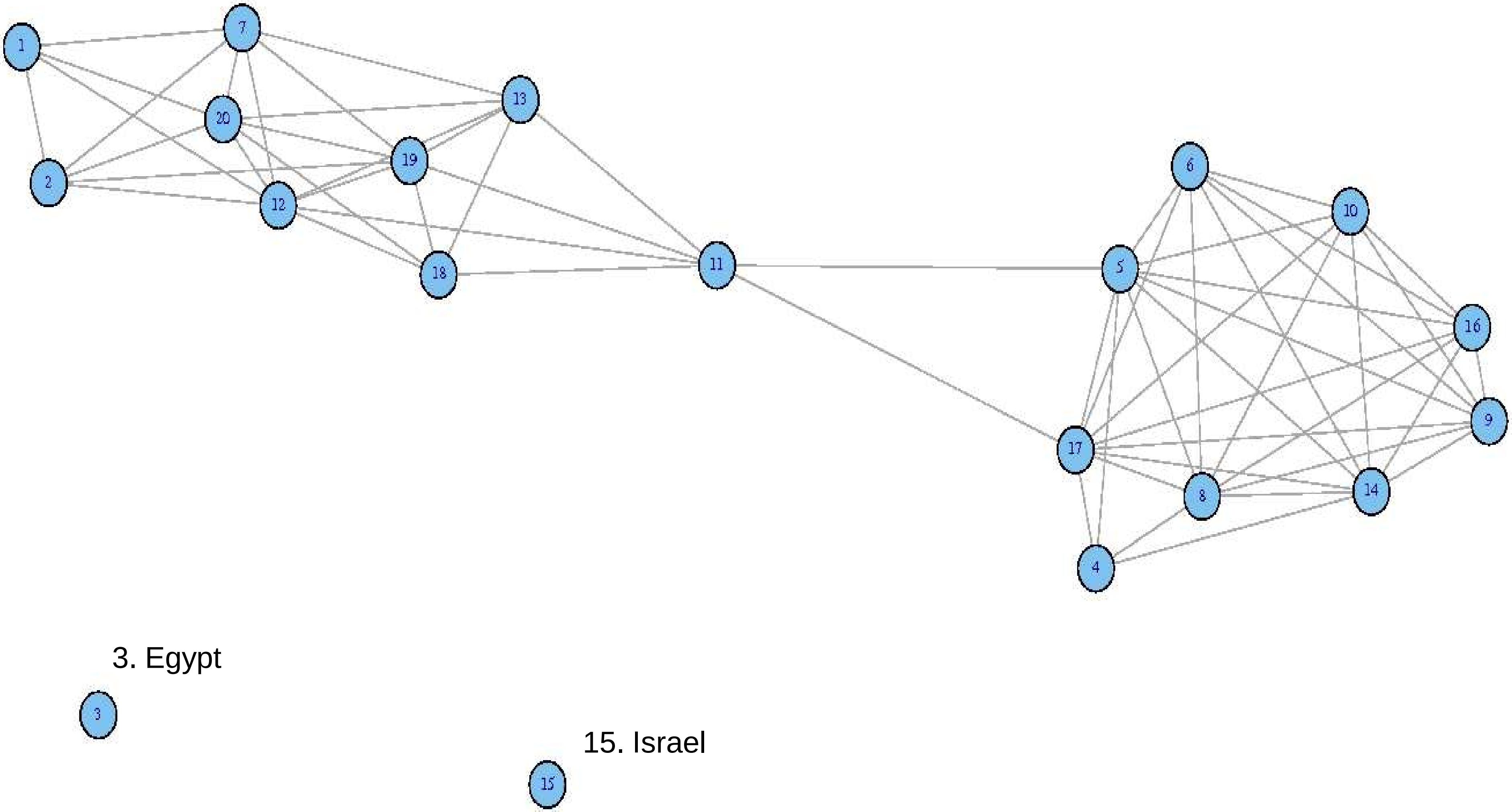
Countries:

1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia 6. Malaysia 7. Mexico
8. South Korea 9. Taiwan 10. Australia 11. Austria 12. France 13. Germany
14. Hong Kong 15. Israel 16. Japan 17. Singapore 18. Switzerland 19. UK 20. US

Threshold (θ) = 0.5 Before crisis

Countries:

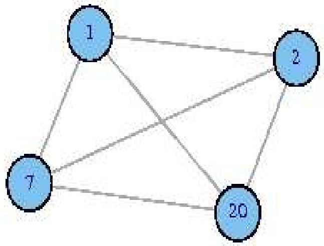
1. Argentina 2. Brazil 4. India 5. Indonesia 6. Malaysia 7. Mexico 8. South Korea 9. Taiwan 10. Australia 11. Austria 12. France 13. Germany 14. Hong Kong 16. Japan 17. Singapore 18. Switzerland 19. UK 20. US



3. Egypt

15. Israel

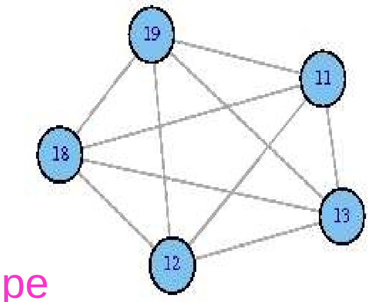
Threshold (θ) = 0.6 Before crisis



America

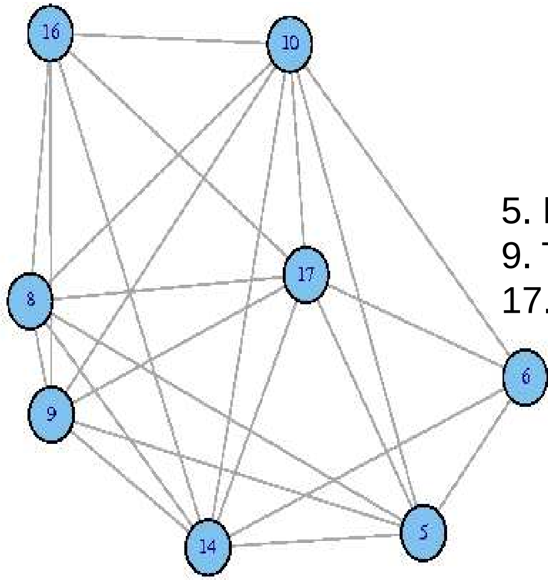
1. Argentina 2. Brazil
7. Mexico 20. US

3. Egypt



Europe

11. Austria 12. France 13. Germany
18. Switzerland 19. UK



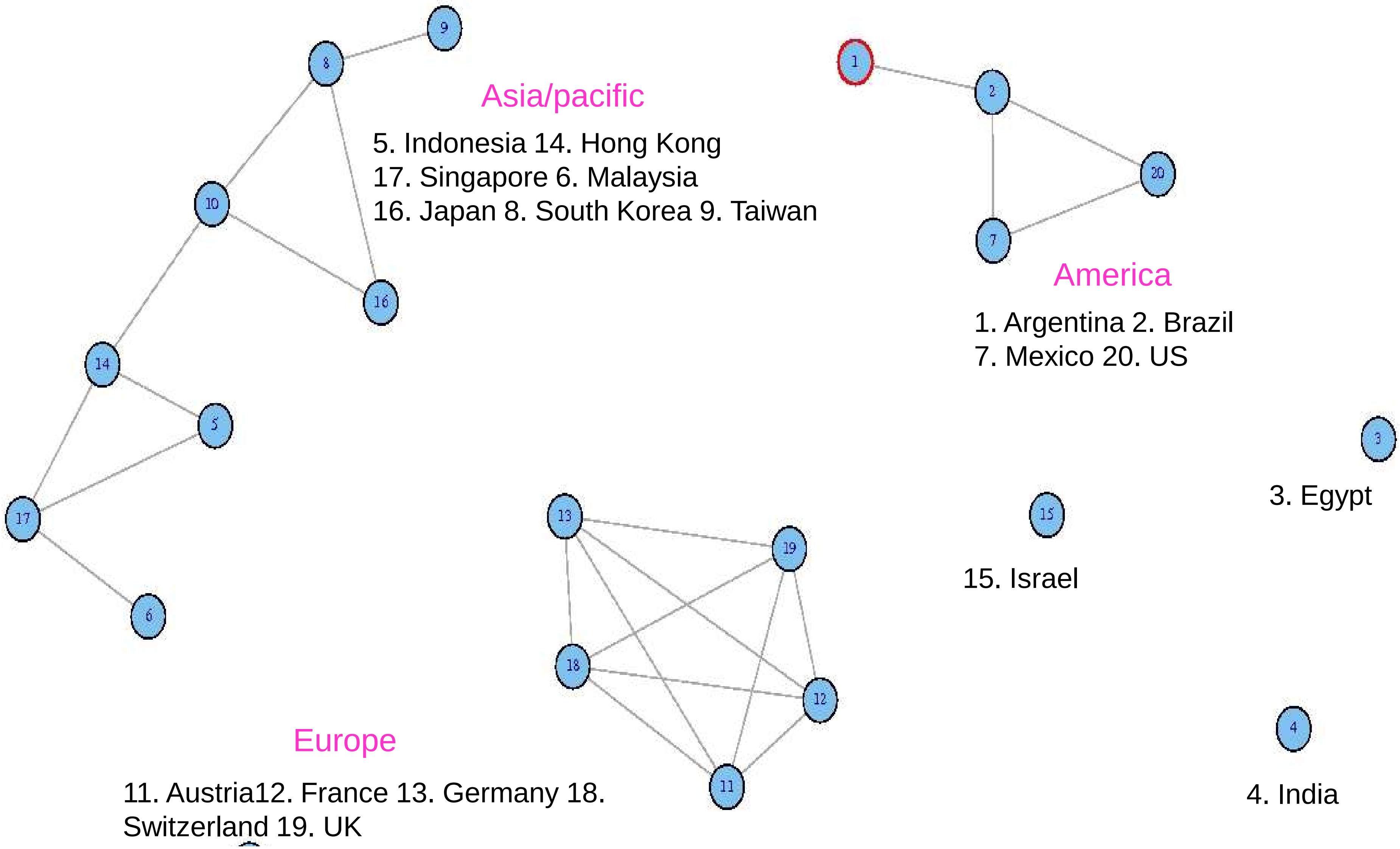
Asia/pacific

5. Indonesia 6. Malaysia 8. South Korea
9. Taiwan 14. Hong Kong 16. Japan
17. Singapore

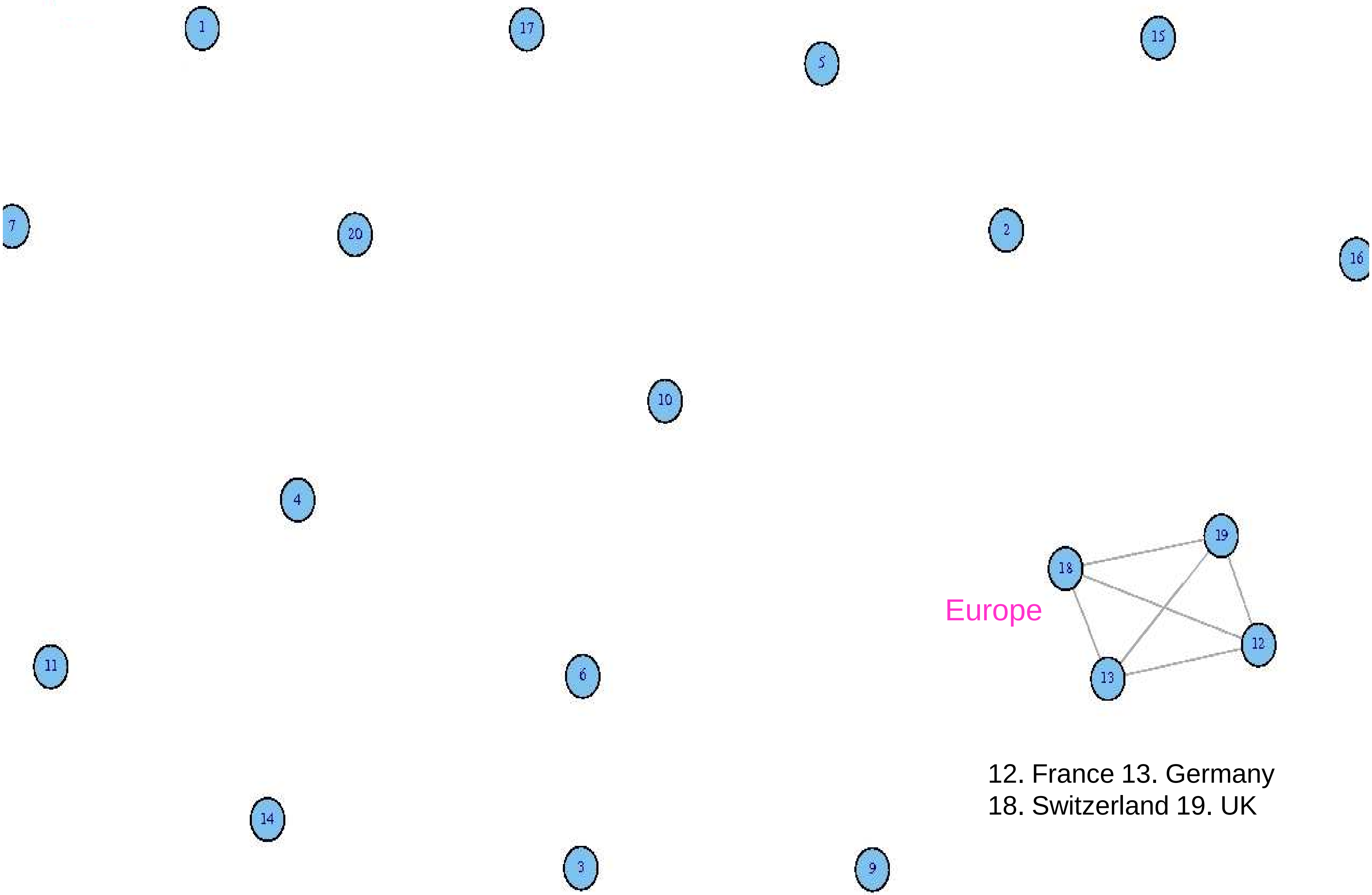
15. Israel

4. India

Threshold (θ) = 0.7 Before crisis

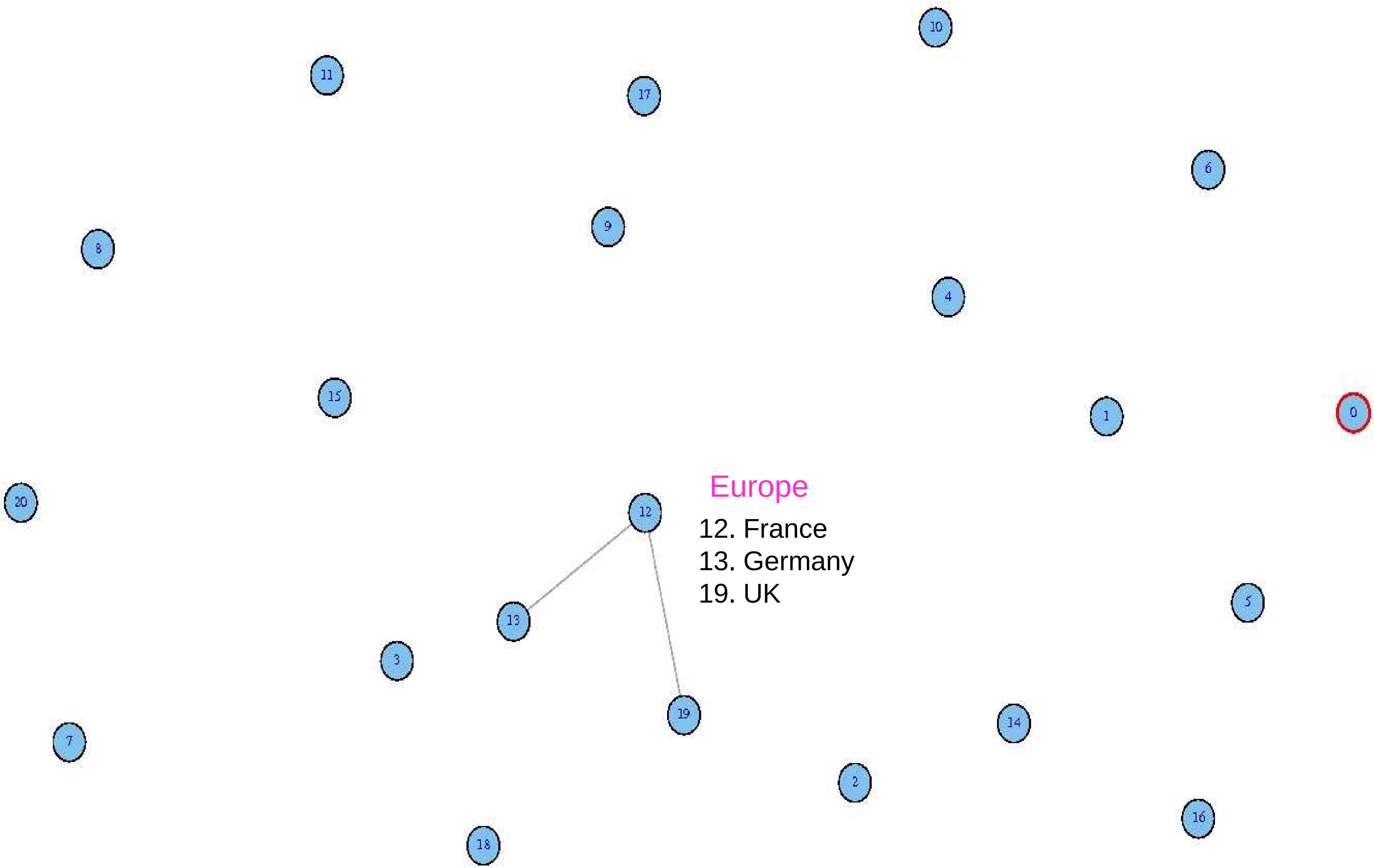


Threshold (θ) = 0.8 Before crisis

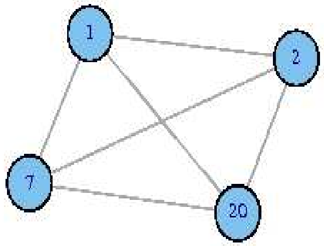


12. France 13. Germany
18. Switzerland 19. UK

Threshold (θ) = 0.9 Before crisis



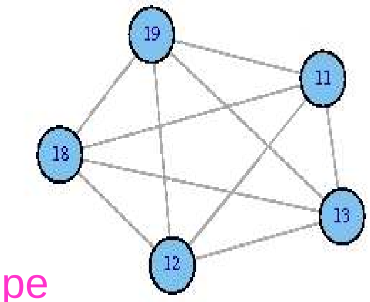
Threshold (θ) = 0.6 Before crisis



America

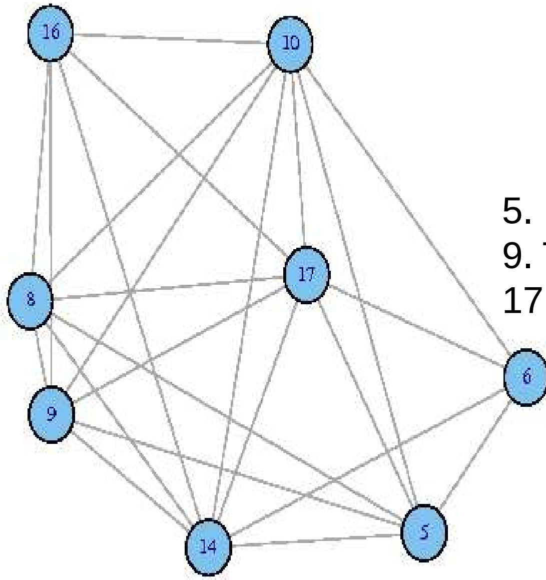
1. Argentina 2. Brazil
7. Mexico 20. US

3. Egypt



Europe

11. Austria 12. France 13. Germany
18. Switzerland 19. UK



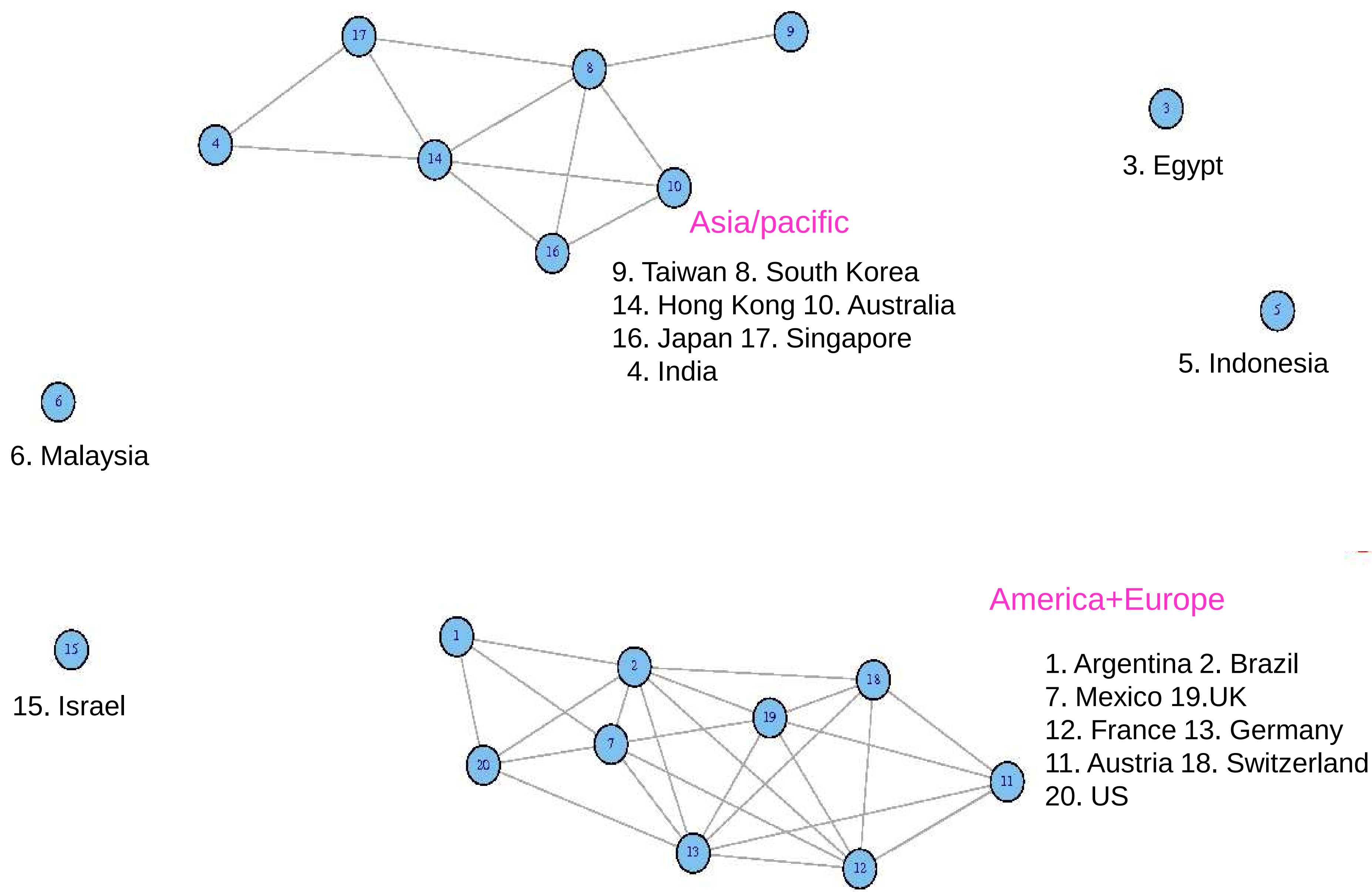
Asia/pacific

5. Indonesia 6. Malaysia 8. South Korea
9. Taiwan 14. Hong Kong 16. Japan
17. Singapore

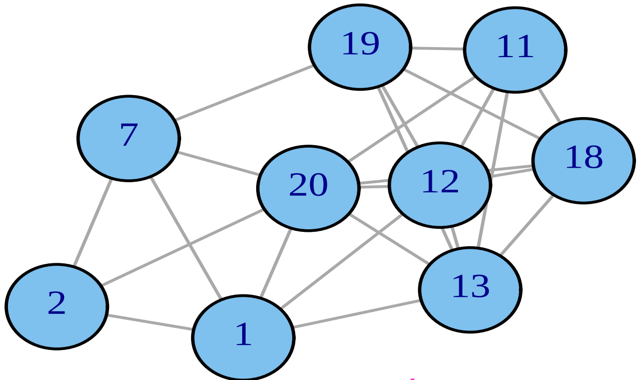
15. Israel

4. India

Threshold (θ) = 0.6 During crisis

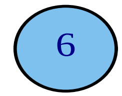


Threshold (θ) = 0.6 After crisis

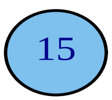


America+Europe

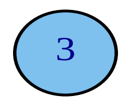
1. Argentina 2. Brazil
7. Mexico 20. US
12. France 13. Germany
11. Austria 18. Switzerland
19.UK



6. Malaysia



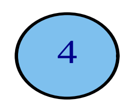
15. Israel



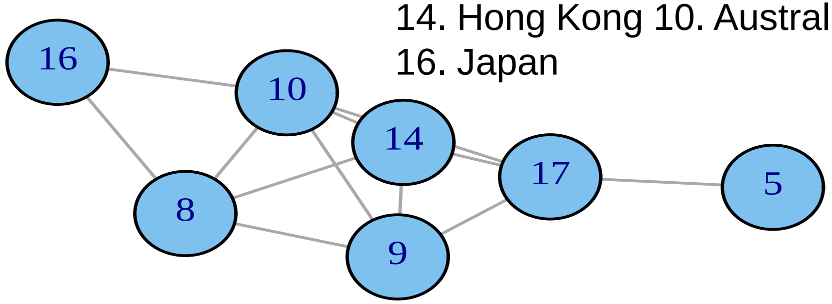
3. Egypt

Asia/pacific

5. Indonesia 17. Singapore
9. Taiwan 8. South Korea
14. Hong Kong 10. Australia
16. Japan



4. India



Hierarchical method: Minimum Spanning Tree (MST)

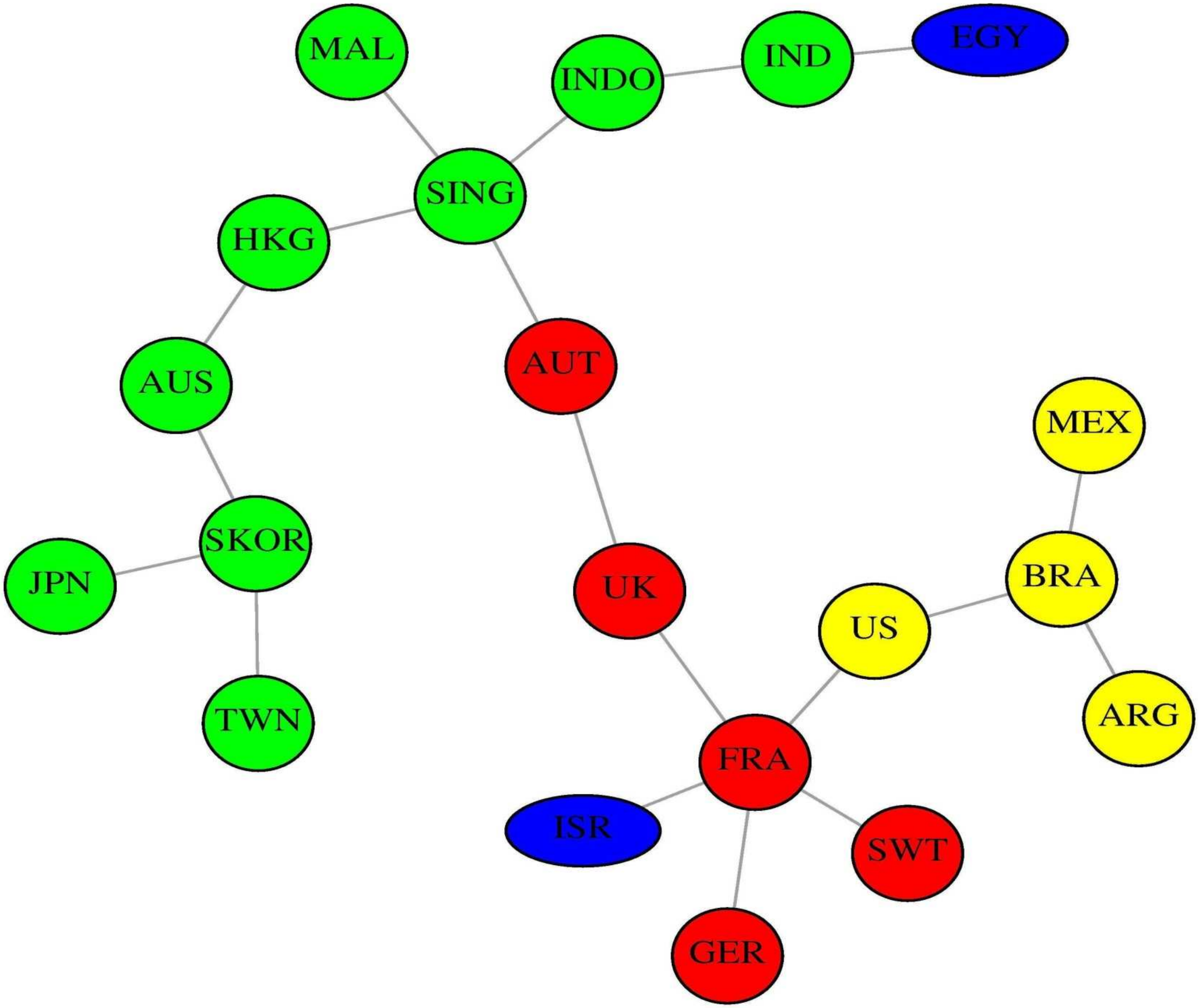
Spanning tree is a graph without loops connecting all the N nodes with N-1 links. The MST is the spanning tree of shortest length.

We construct the network of financial indices by using the metric distances $d_{ij} = \sqrt{2(1-C_{ij})}$ forming a N x N distance matrix whose elements lies between 0 and 2.

The number of possible nodal connections is very large i.e. N(N-1)/2. The MST reduces this complexity by showing only N-1 most important non redundant connections in a graphical manner.

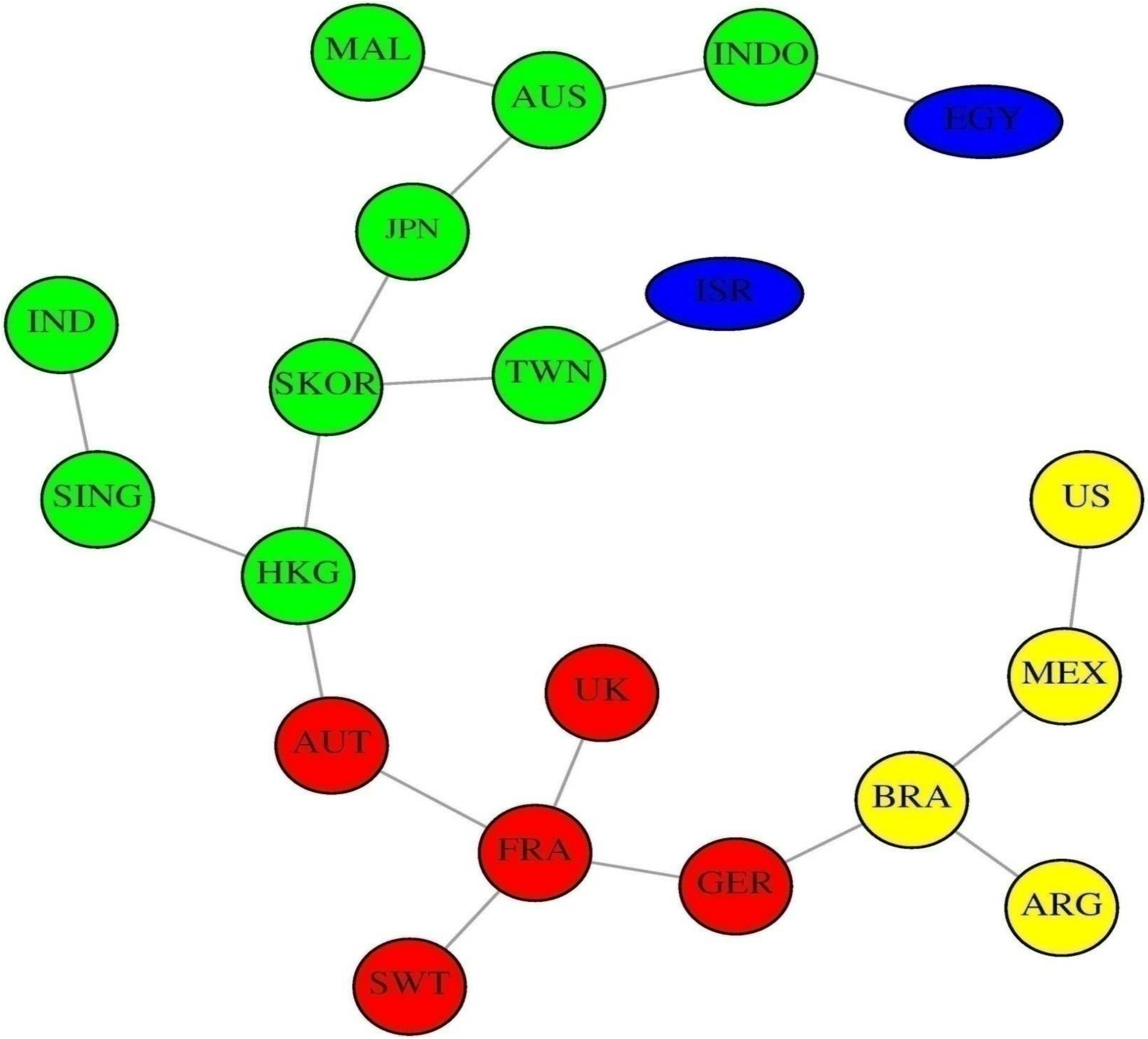
We have used the Prim's algorithm to draw the MST. It is a greedy algorithm that finds a MST for a connected weighted undirected graph. It finds a subset of edges that forms a tree that include every vertex , where total weight of all edges in the tree is minimized.

Minimum Spanning Tree (Before the crisis)



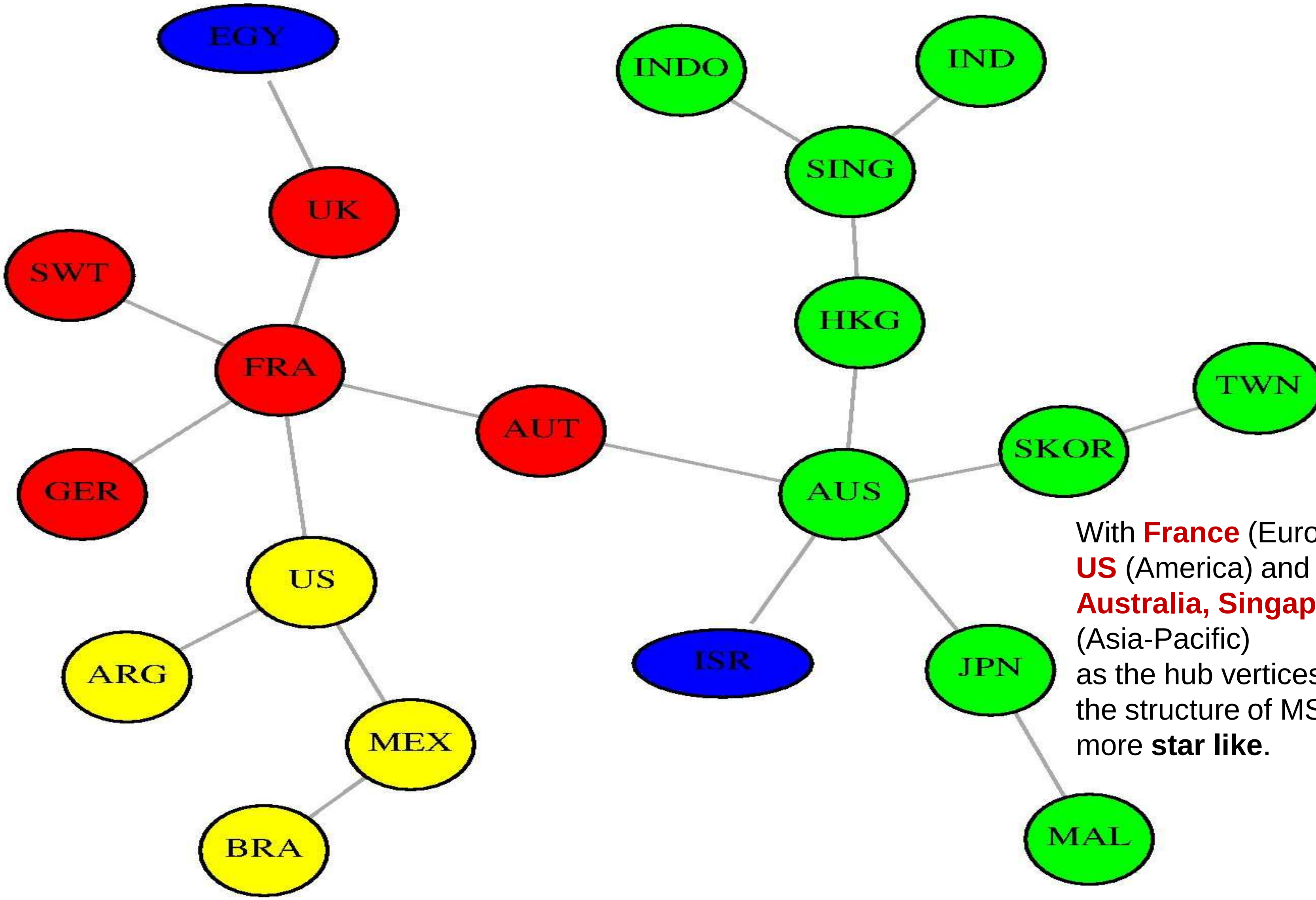
With **France** (Europe), **Brazil** (Americas) and **Singapore**, **South Korea** (Asia-Pacific) as the hub vertices the structure of MST is more **star like**.

Minimum Spanning Tree (During the crisis)



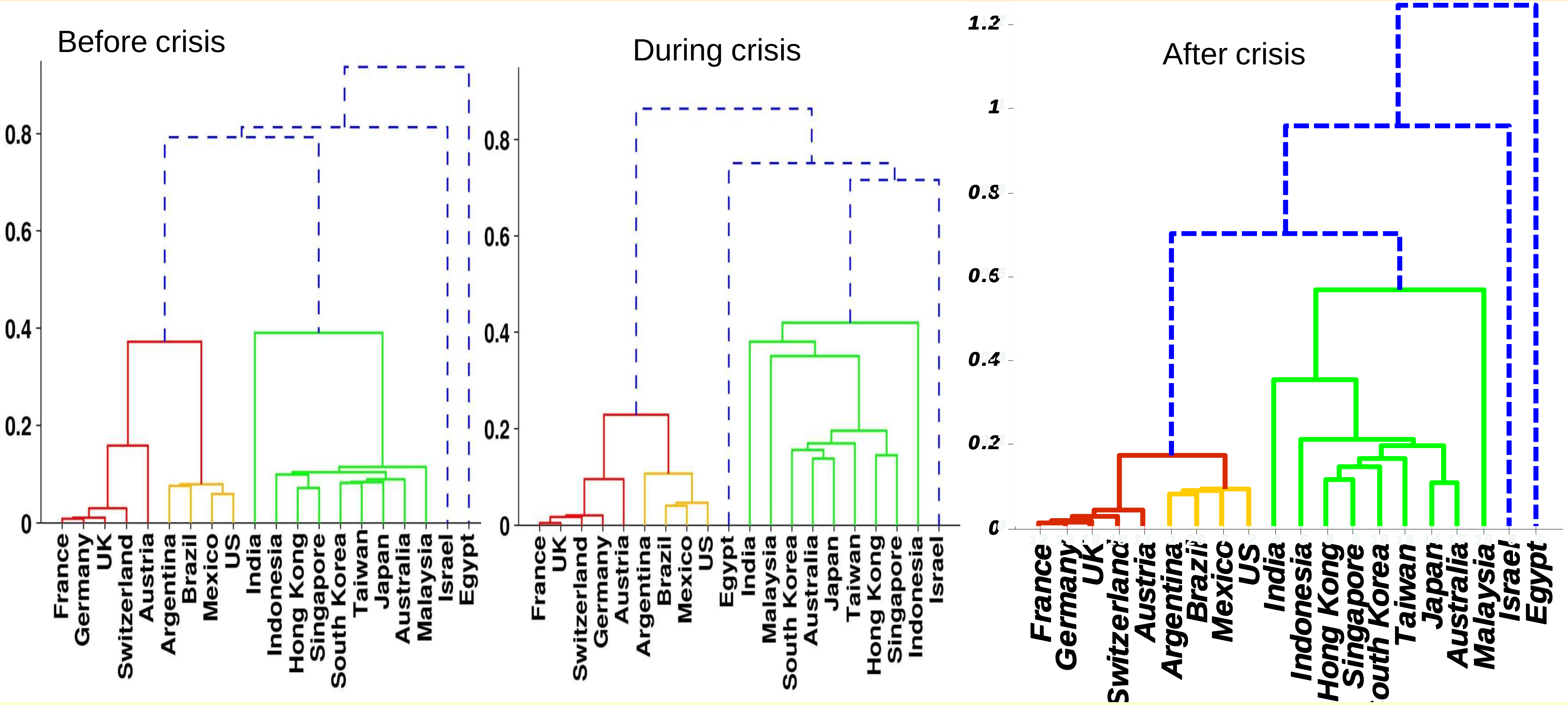
With **France** (Europe) **Brazil** (Americas), **Hong Kong**, **South Korea** and **Australia** (Asia-Pacific) as hub vertices the structure of MST is more **chainlike.**

Minimum Spanning Tree (After the crisis)



With **France** (Europe), **US** (America) and **Australia, Singapore**, (Asia-Pacific) as the hub vertices the structure of MST is more **star like**.

Hierarchical Clustering: **Average linkage hierarchical clustering algorithm** is applied to the distance matrix to produce the best treelike dendrogram.



During the crisis, the height of dendrogram of the European-American cluster decreases while the height of dendrogram of Asia-Pacific cluster increases.

This shows that the **European-American** indices interact (correlate) strongly while the **Asia-Pacific indices** (including Egypt and Israel) correlate weakly during the crisis. France is the tightly linked index in the European cluster.

This further **distinguishes the behavior of the European-American cluster from the Asia-Pacific cluster** and indicate that hierarchy of European and American indices increases while the hierarchy of Asia-Pacific indices decreases during the crisis.

Hierarchical Clustering

In hierarchical clustering objects are categorized into hierarchy similar to a tree like structure which is called a dendrogram. The dendrogram displays both the cluster-sub cluster relationship and the order in which the clusters were merged.

The nodes of dendrogram represent clusters and length of stems (heights) represent the distances at which the clusters are joined . By cutting the dendrogram at different heights we can easily determine the number of clusters.

Cophenetic matrix is generated from the dendrogram. Its elements are the branch distance where two objects become members of the same cluster in the dendrogram: for two nodes I, j we find the nearest common bifurcation point, the branch length for this point is the cophenetic element (c_{ij}) of these two nodes.
The Cophenetic Correlation Coefficient* (CCC) is defined as

$$CCC = \frac{\sum_{i<j}(d_{ij} - \bar{d})(c_{ij} - \bar{c})}{\sqrt{[\sum_{i<j}(d_{ij} - \bar{d})^2][\sum_{i<j}(c_{ij} - \bar{c})^2]}}$$

where d_{ij} and <d> are the element and average of **element of distance matrix** and
c_{ij} and <c> are the elements and average **of elements of cophenetic matrix respectively**.

The value of CCC is found to be 0.903 (before crisis), 0.933 (during crisis), 0.921 (after crisis).
We observe a significant change* in case of financial indices during the period of crisis.
This indicates that hierarchy in financial indices increases during the crisis of 2008.

***** J. He and M. W. Deem, Phy. Rev. Lett. **105**, 198701 (2010)

Conclusion

- Using RMT we find that there are major changes in the correlation before, during and after the global financial crisis of 2008.

- We apply techniques of complex network to study the structure and dynamics of global financial network before, during, and after the crisis.
- We construct networks at different correlation thresholds before, during and after the global financial crisis of 2008. Fruchterman-Reingold layout is used to find clusters in global financial markets.
- At threshold 0.6, we find that indices corresponding to the American, European and Asia-pacific forms separate clusters before the crisis but **during the crisis period American and European indices combined to form a strongly linked cluster** while the **Asia-Pacific form a separate weakly linked cluster**. When the value of threshold is further increased to 0.9 then the European indices (France, Germany and UK) are found to be the most tightly linked indices.

- Structure of MST is more star like before crisis and it changes to more chainlike during the crisis. After the crisis, the structure is found to be more star like.
- Our findings show that there are major changes in the structure of organization of financial indices during the financial crisis of 2008.

- Studying the crisis and finding the organizational changes of clusters during crisis period is useful and interesting as similar changes may occur during other crisis, leading to innovative ways for prevention and control.

Thank You

Publications

(1) "*Multifractal Properties of Indian Financial market*"
Sunil Kumar and Nivedita Deo
Physica A 388, 1593 (2009)

(2) "*Correlation and Network Analysis of Global Financial Indices*"
Sunil Kumar and Nivedita Deo
Physical Review E 86, 026101 (2012)

(3) "*Analyzing Crisis in Global Financial Indices*"
Sunil Kumar and Nivedita Deo
Econophysics of Systemic Risk and Network Dynamics
pp. 261-275 (Springer-Verlag, Italia, 2013)
Editors: F. Abergel, B.K. Chakrabarti, A. Chakrabarti, and A. Ghosh

(4) "*Studying Extreme Events in Financial Time series*"
Sunil Kumar and Nivedita Deo
Under Preparation

Conclusion

- We investigate and compare the structure and dynamics of a random system and financial system by using three methods:

  Random matrix theory and Network analysis.

- Using RMT we find that there are major changes in the structure of organization of global financial indices during the financial crisis.

- We apply techniques of complex network to study the structure and dynamics of global financial network before and during crisis. There is a change in the structure of organization of financial indices during the crisis

- Studying the crisis and finding the organizational changes of clusters during crisis period is useful and interesting as similar changes may occur during other crisis, leading to innovative ways for prevention and control.

Conclusion (Network Analysis)

- We constructed networks at different threshold (in the range 0 to 0.9) before, during and after the global financial crisis of 2008. Fruchterman-Reingold layout is used to find clusters in global financial markets.

- At threshold 0.6, we find that indices corresponding to the American, European and Asia-pacific forms separate clusters before the crisis but **during the crisis period American and European indices combined to form a strongly linked cluster** while the **Asia-Pacific form a separate weakly linked cluster**.

- When the value of threshold is further increased to 0.9 then the European indices (France, Germany and UK) are found to be the most tightly linked indices.

- Structure of MST is more star like before crisis and it changes to more chainlike during the crisis. After the crisis, the structure is found to be more star like.

- In MST, the financial indices are found to be organized by their geographical region.

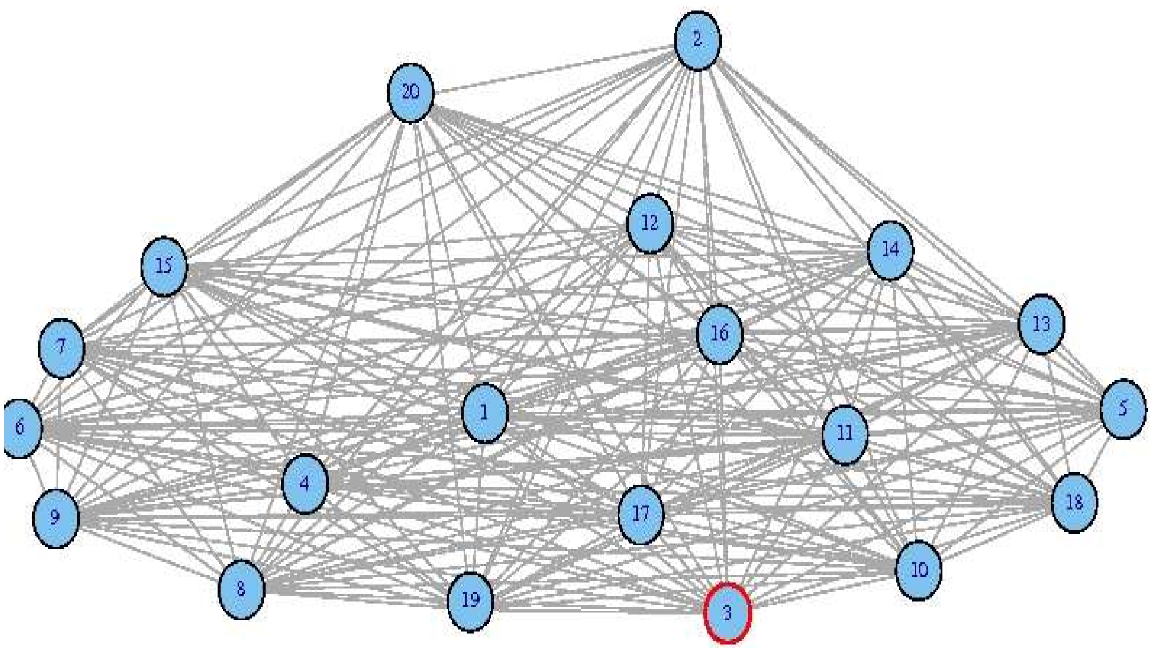
- Our findings show that there are major changes in the structure of organization of financial indices during the financial crisis.

Correlation Network of global financial indices

Threshold (θ) = 0.1 [During the crisis]

Countries:

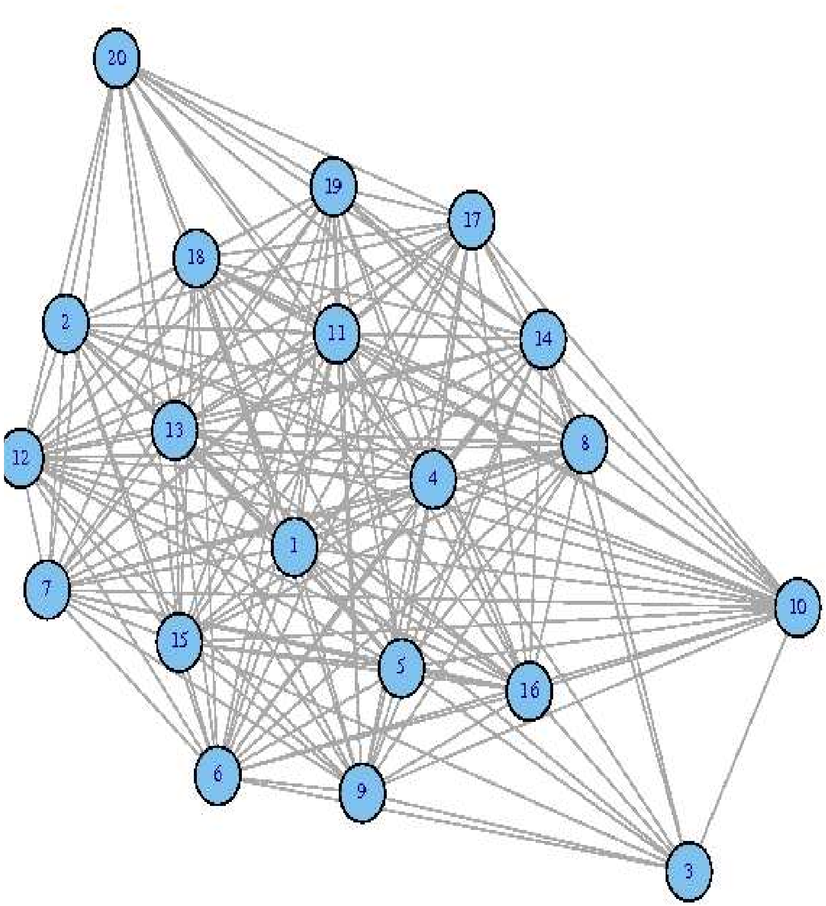
1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia 6. Malaysia 7. Mexico 8. South Korea 9. Taiwan 10. Australia 11. Austria 12. France 13. Germany 14. Hong Kong 15. Israel 16. Japan 17. Singapore 18. Switzerland 19. UK 20. US



Threshold (θ) = 0.2 During the crisis

Countries:

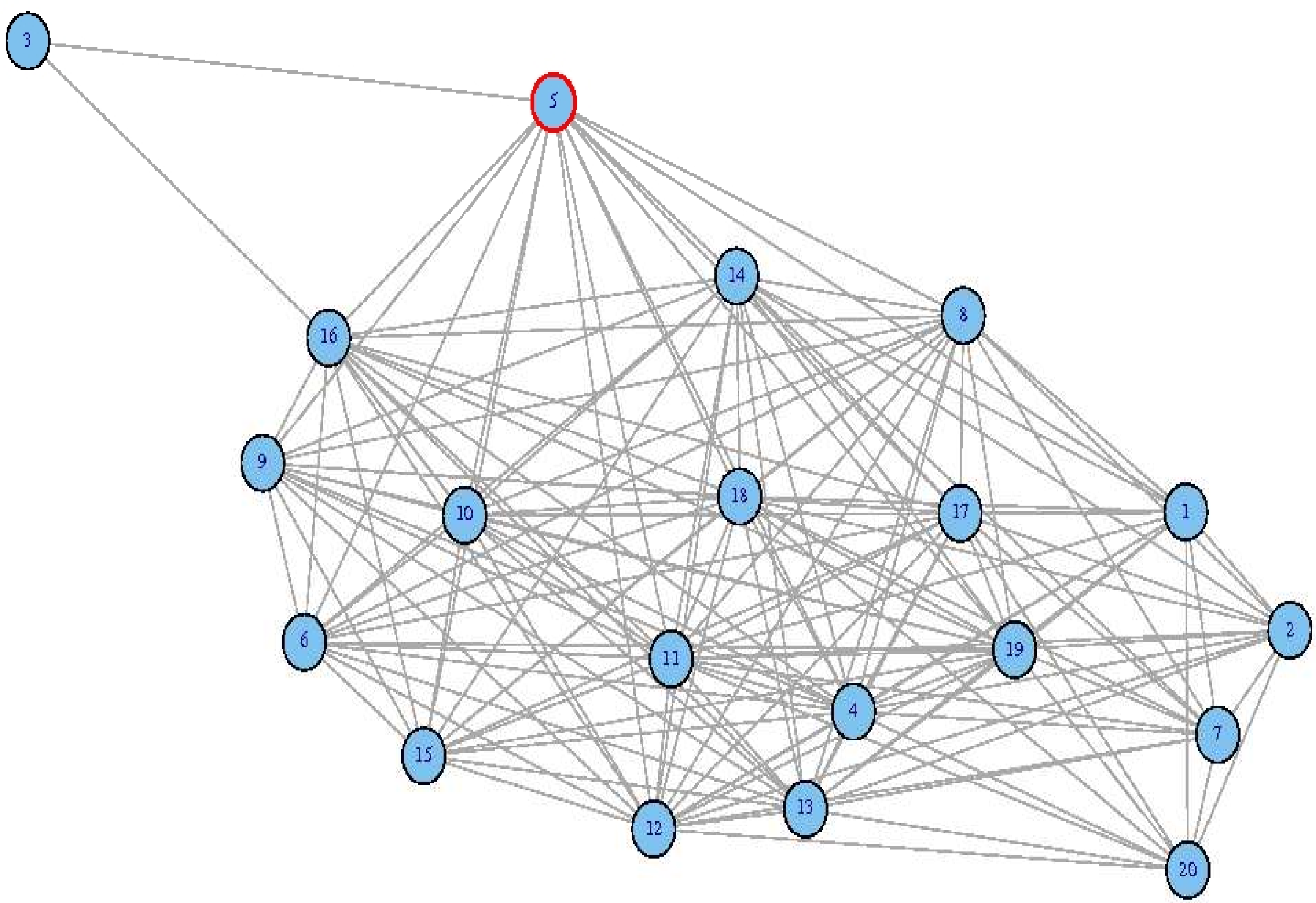
1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia 6. Malaysia 7. Mexico 8. South Korea 9. Taiwan 10. Australia 11. Austria 12. France 13. Germany 14. Hong Kong 15. Israel 16. Japan 17. Singapore 18. Switzerland 19. UK 20. US



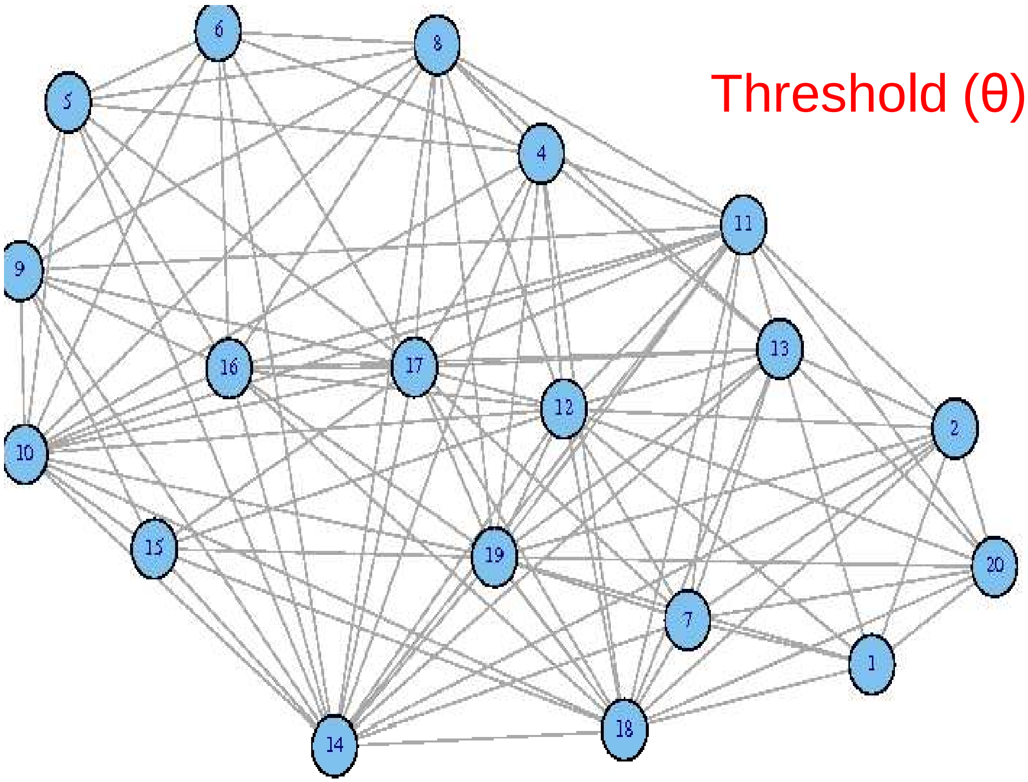
Threshold (θ) = 0.3 During crisis

Countries:

1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia
6. Malaysia 7. Mexico 8. South Korea 9. Taiwan
10. Australia 11. Austria 12. France 13. Germany
14. Hong Kong 15. Israel 16. Japan 17. Singapore
18. Switzerland 19. UK 20. US



Threshold (θ) = 0.4 During crisis



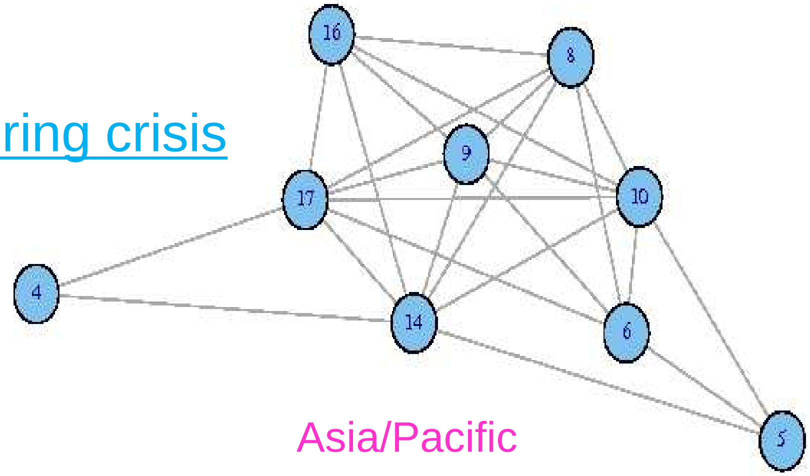
3. Egypt

America+Europe+
Asia/pacific

Countries:

1. Argentina 2. Brazil 3. Egypt 4. India 5. Indonesia 6. Malaysia 7. Mexico 8. South Korea 9. Taiwan 10. Australia 11. Austria 12. France 13. Germany 14. Hong Kong 15. Israel 16. Japan 17. Singapore 18. Switzerland 19. UK 20. US

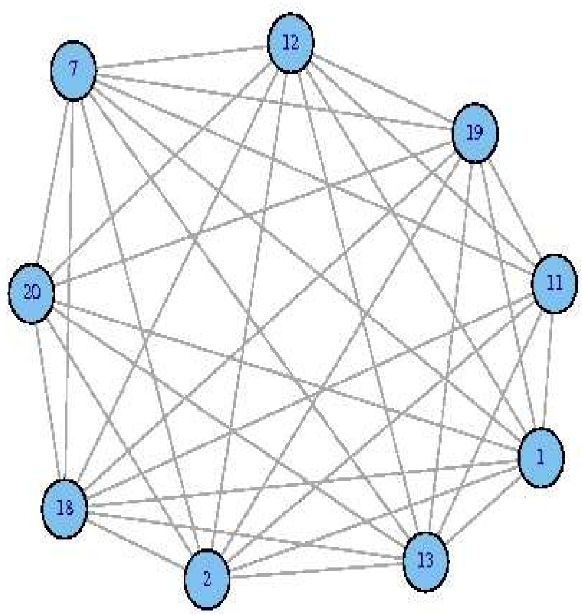
Threshold (θ) = 0.5 During crisis



Asia/Pacific

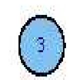
4. India 14. Hong Kong 17. Singapore 16. Japan
8. South Korea 9. Taiwan 10. Australia 6. Malaysia
5. Indonesia

15. Israel



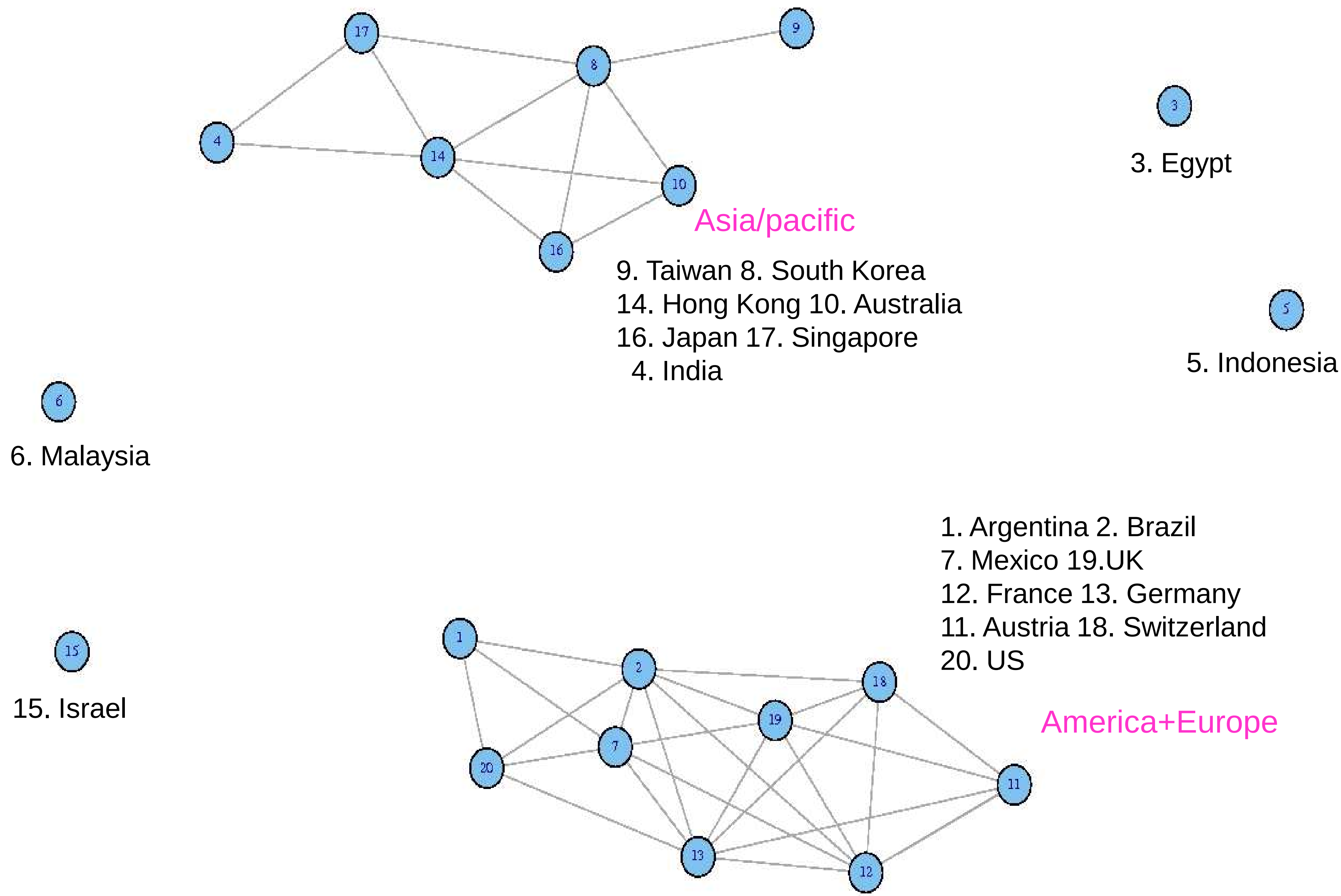
America+Europe

1. Argentina 11. Austria
13. Germany 2. Brazil
7. Mexico 11. Austria
12. France 13. Germany
18. Switzerland 19.UK
20. US

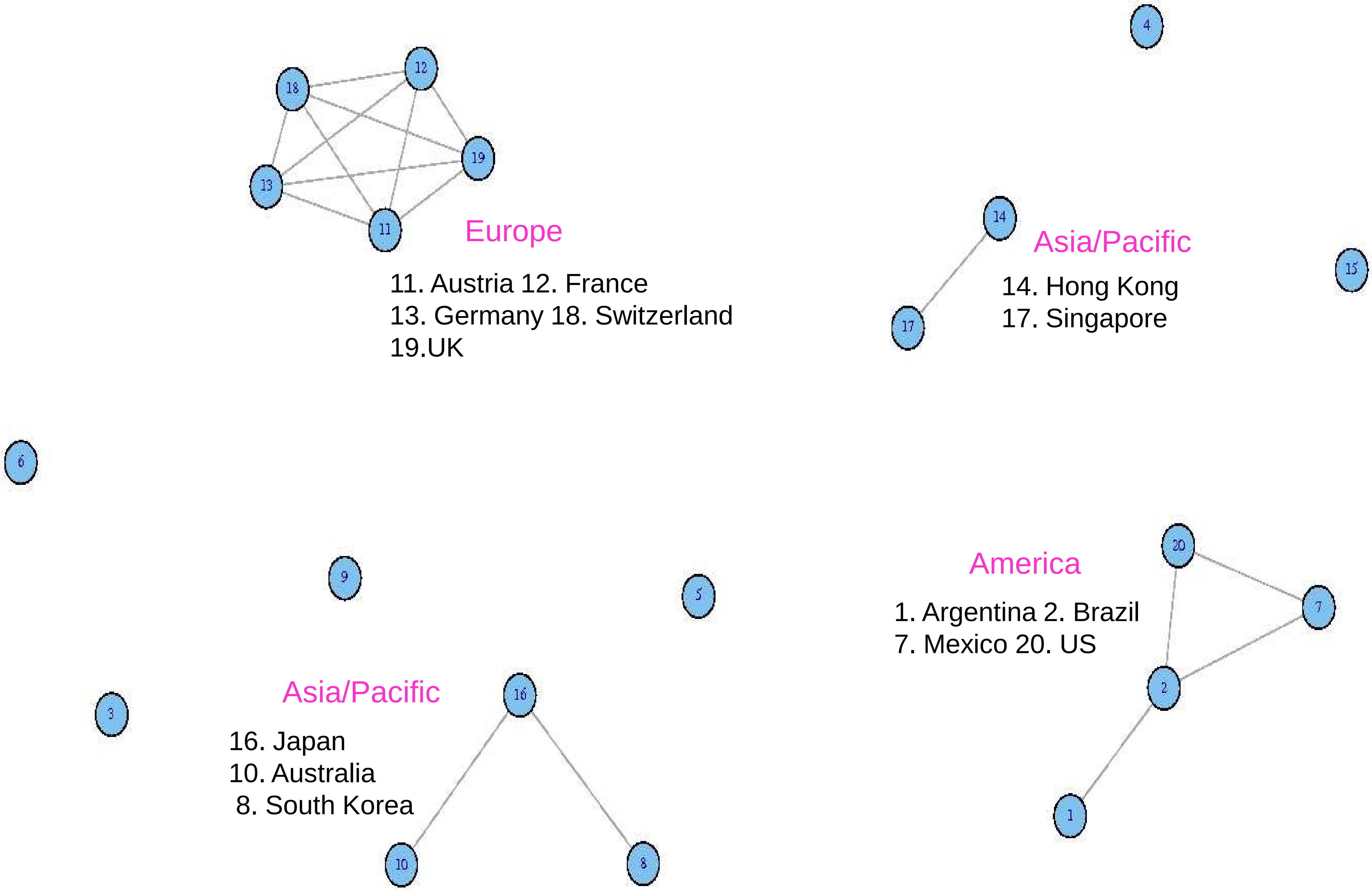


3. Egypt

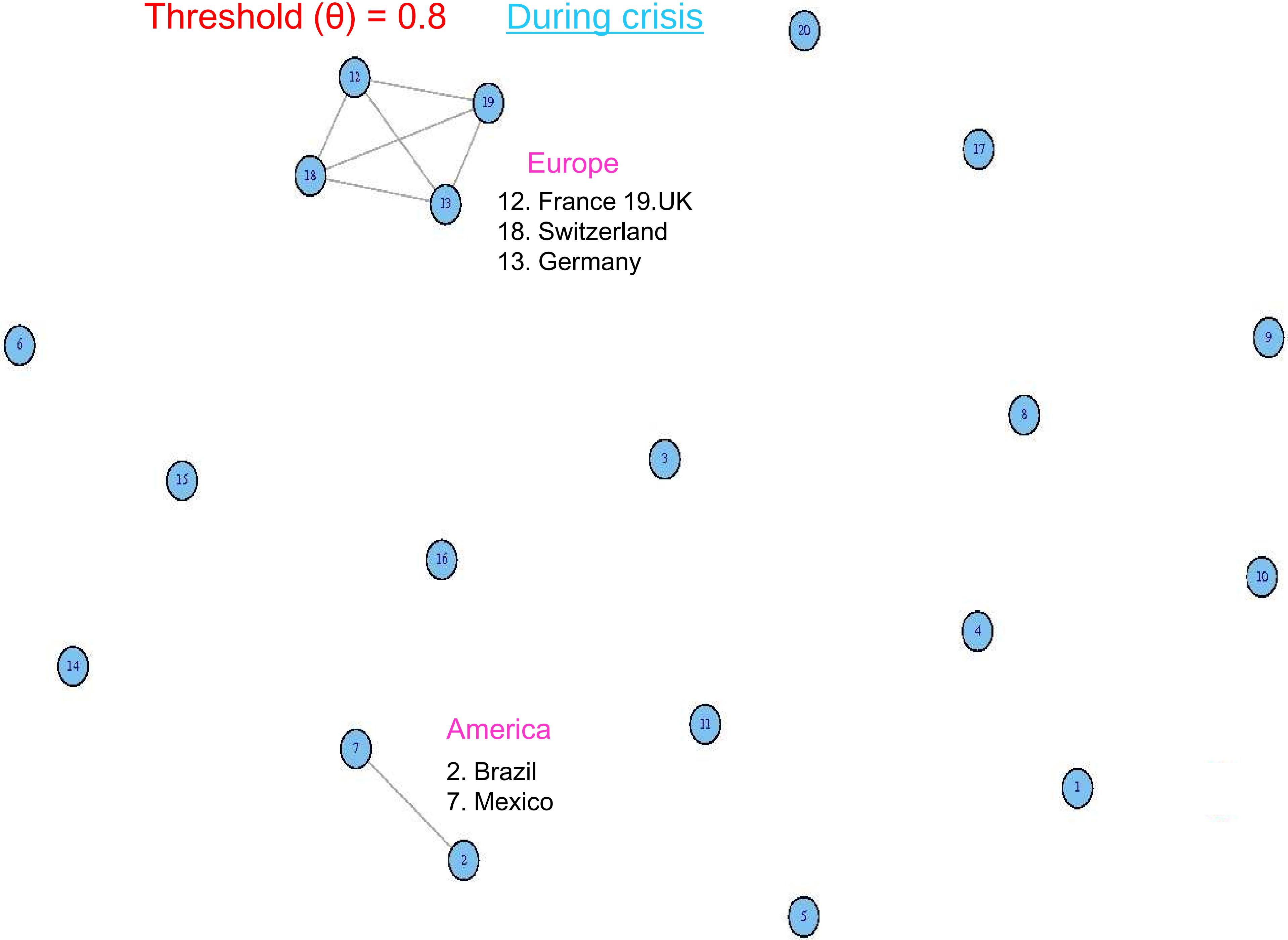
Threshold (θ) = 0.6 During crisis



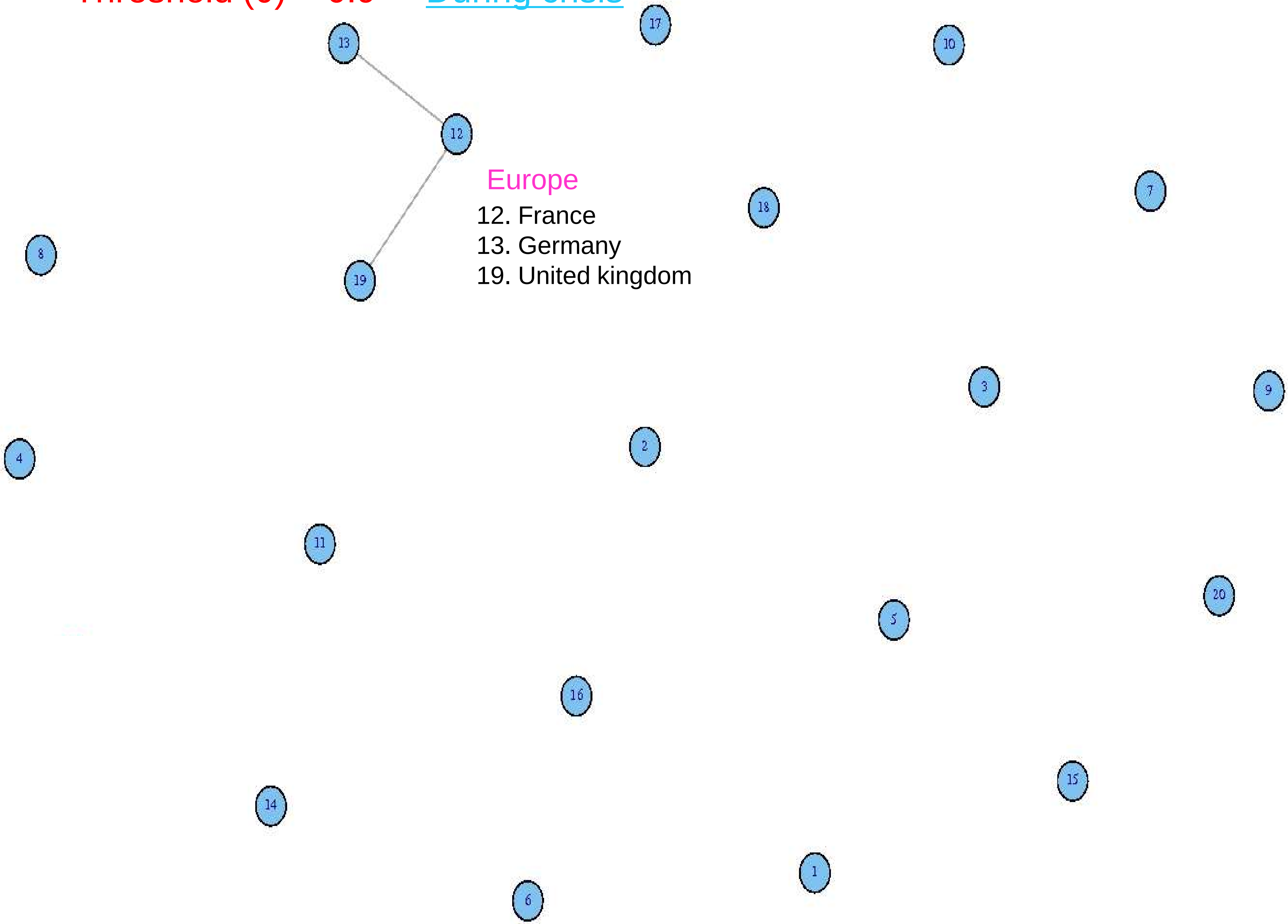
Threshold (θ) = 0.7 During crisis



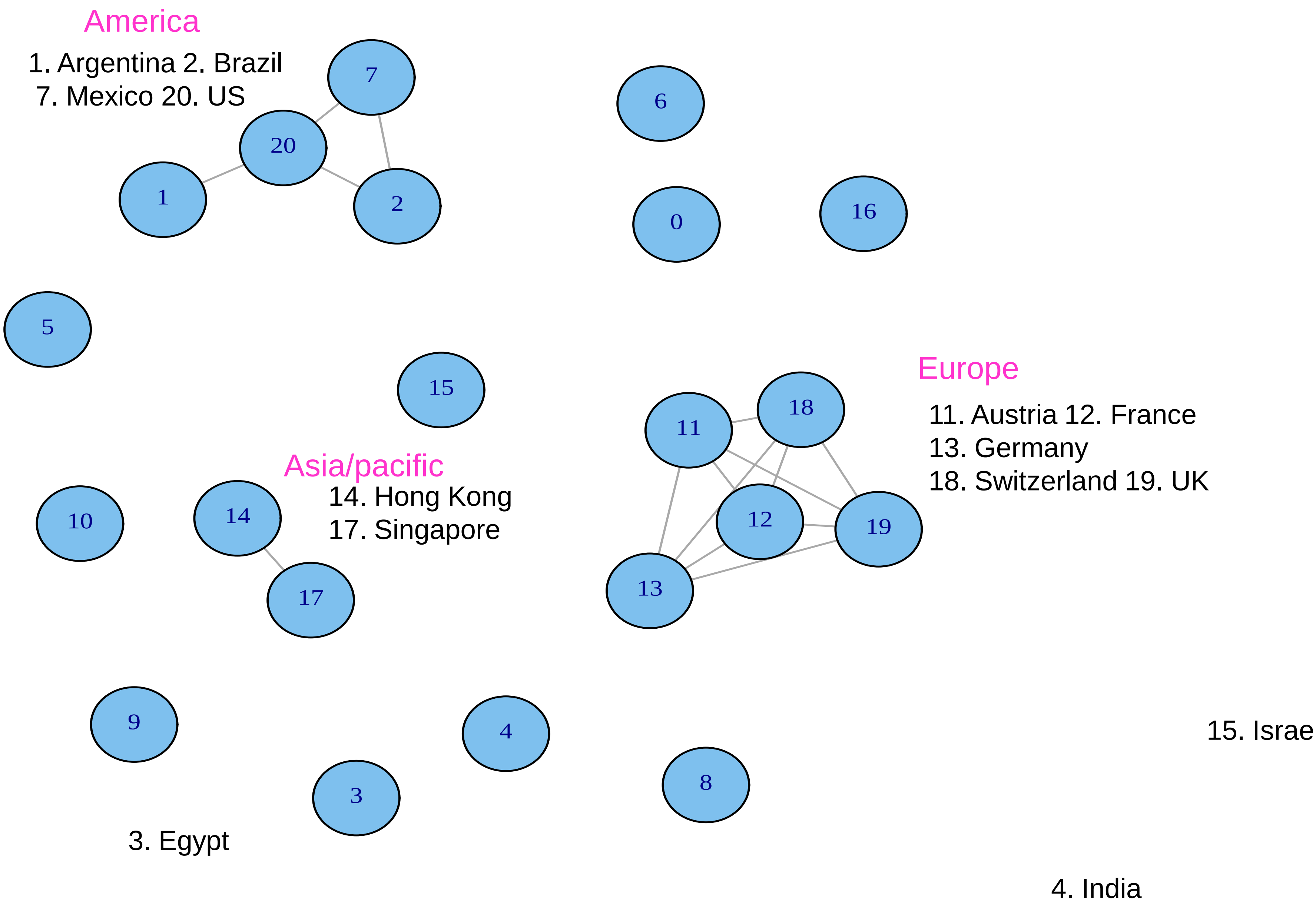
Threshold (θ) = 0.8 During crisis



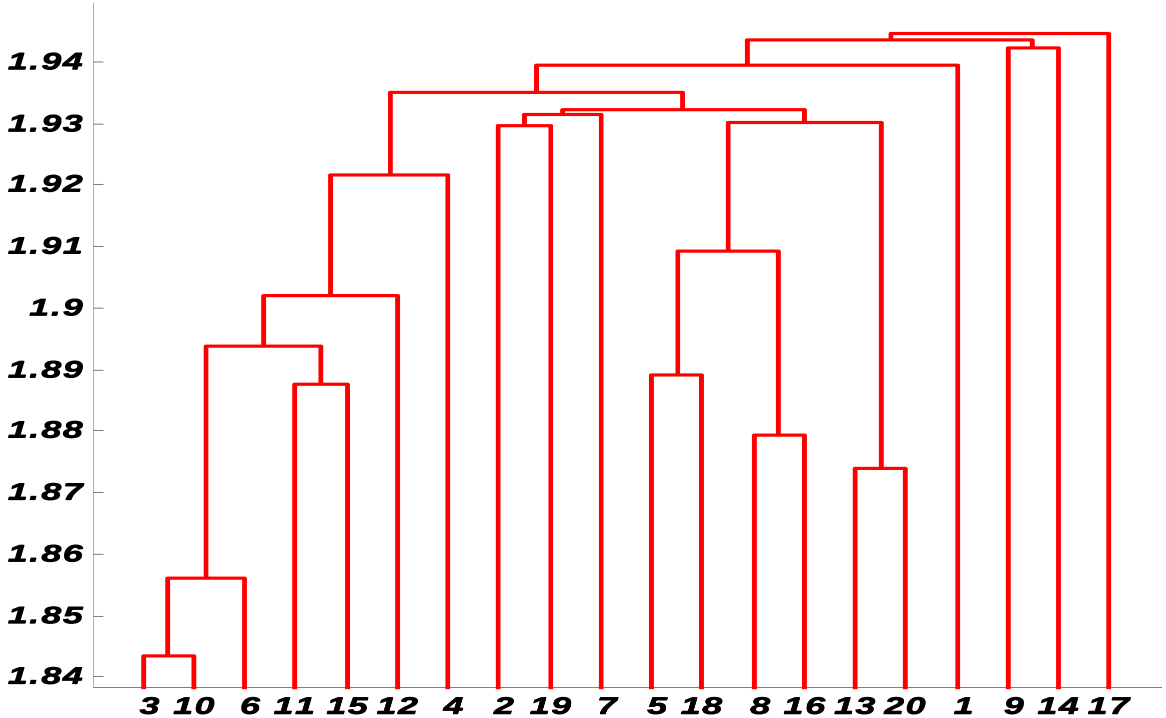
Threshold (θ) = 0.9 During crisis



Threshold (θ) = 0.71 After crisis



Hierarchical Clustering: **Average linkage hierarchical clustering algorithm** is applied to the distance matrix to produce the best treelike dendrogram.



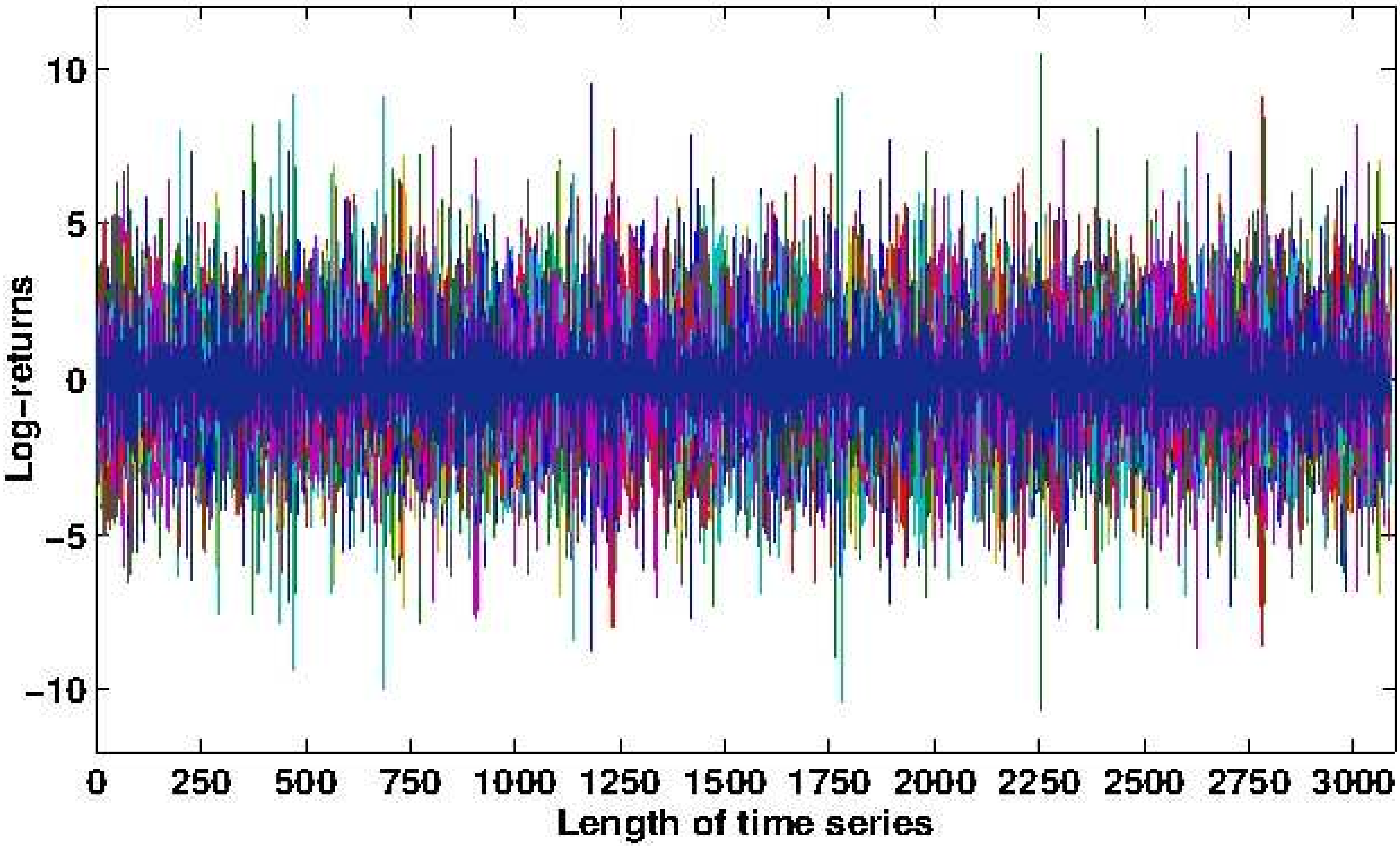
Dendrogram representation of the random correlation matrix

From the random correlation matrix, the value of CCC is found to be 0.3414

In case of global financial indices, the value of CCC increases from 0.903 (before crisis) to 0.933 (during crisis) which is a significant change* in case of financial indices 0.921 (after crisis.

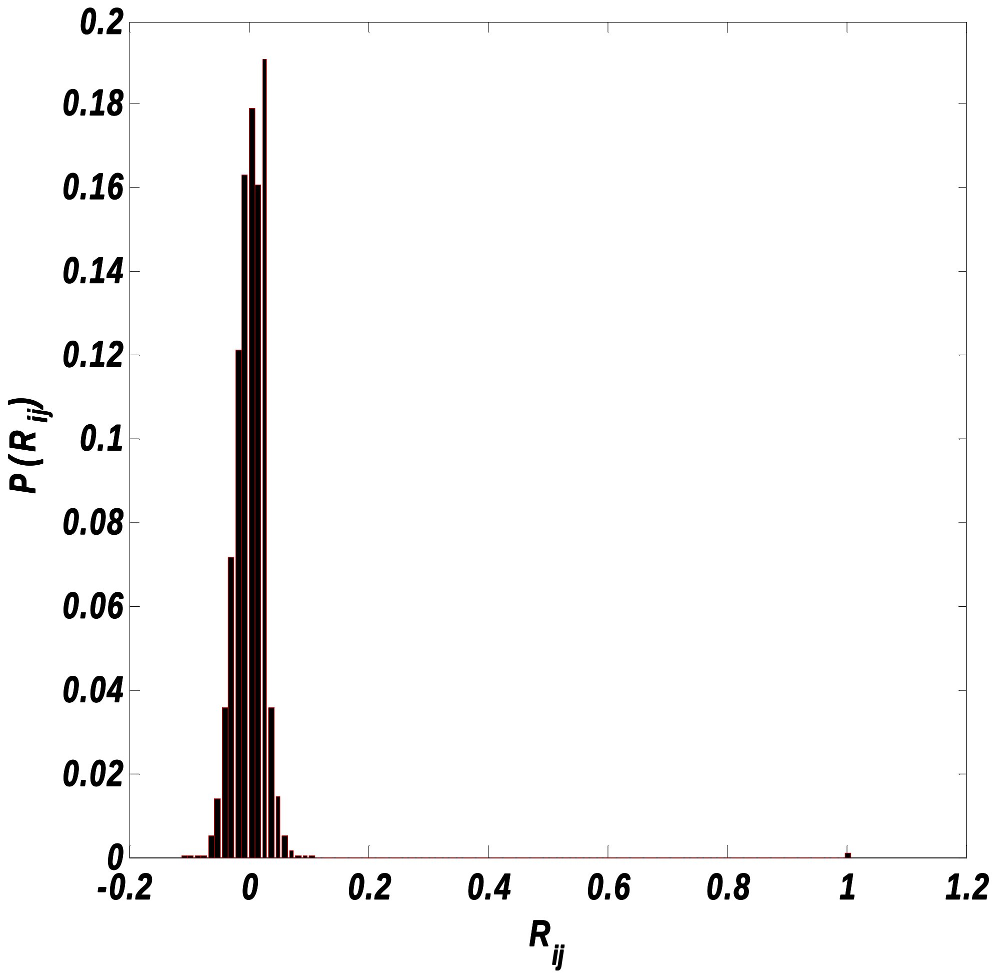
This indicates that hierarchy in financial indices increases during the crisis of 2008.

RMT approach to a random system



Log-returns computed from the random numbers having zero mean and unit variance

RMT approach to a random system



R_{ij} is computed using N=1000 random time series of length T=3088.

$\langle R_{ij} \rangle = 0.001$

$S_i(t) \rightarrow$ Random numbers with zero mean and unit variance

Log-returns:

$$\boxed{G_i(t) \equiv \ln S_i(t+1) - \ln S_i(t)}$$

Normalized returns:

$$\boxed{r_i(t) \equiv \frac{G_i(t) - \langle G_i \rangle}{\sigma_i}}$$

here $\sigma_i \equiv \sqrt{\langle G_i^2 \rangle - \langle G_i \rangle^2}$

Random correlation matrix (R) is computed with elements,

$$\boxed{R_{ij} \equiv \langle r_i(t) r_j(t) \rangle}$$

Which are limited to the domain [-1,1]

$R_{ij} = 1 \Longrightarrow$ Perfect correlation

$R_{ij} = -1 \Longrightarrow$ Perfect anti-correlation

$R_{ij} = 0 \Longrightarrow$ No correlation

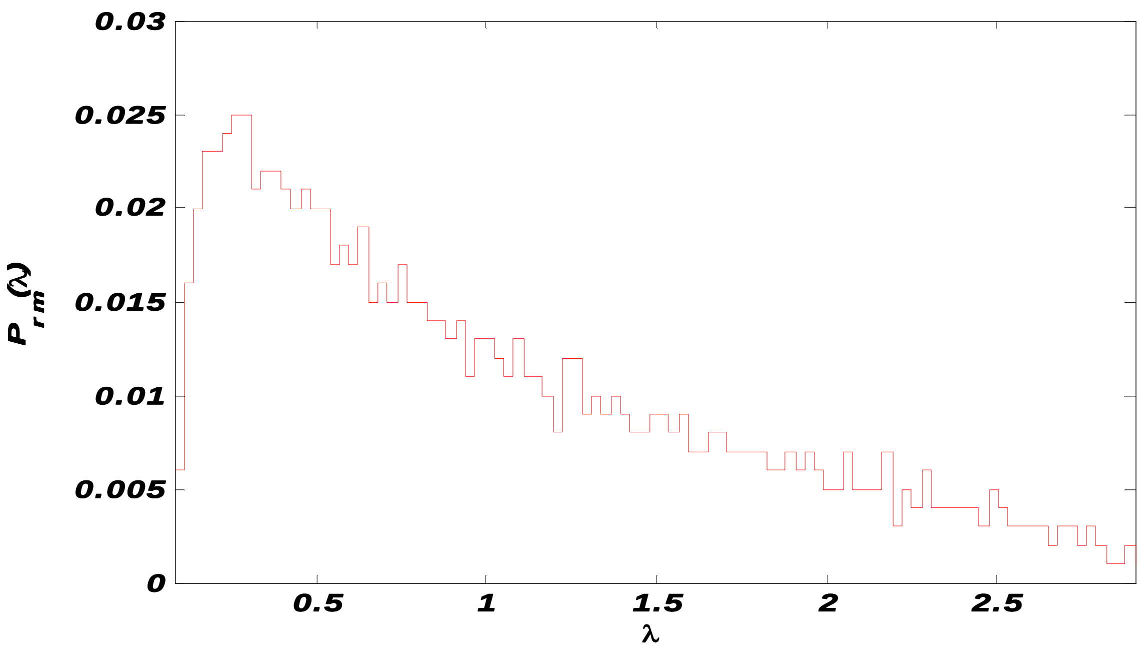
Statistics of eigenvalues of random correlation matrix

For the Wishart matrix (R), with Q = T/N(>1), probability distribution of eigenvalues,

$$P_{rm}(\lambda^{rm}) = \frac{Q}{2\pi}\frac{\sqrt{(\lambda^{rm}_{max} - \lambda^{rm})(\lambda^{rm} - \lambda^{rm}_{min})}}{\lambda^{rm}}$$

within the bounds $\lambda^{rm}_{min} \leq \lambda^{rm} \leq \lambda^{rm}_{max}$ and is 0 otherwise.

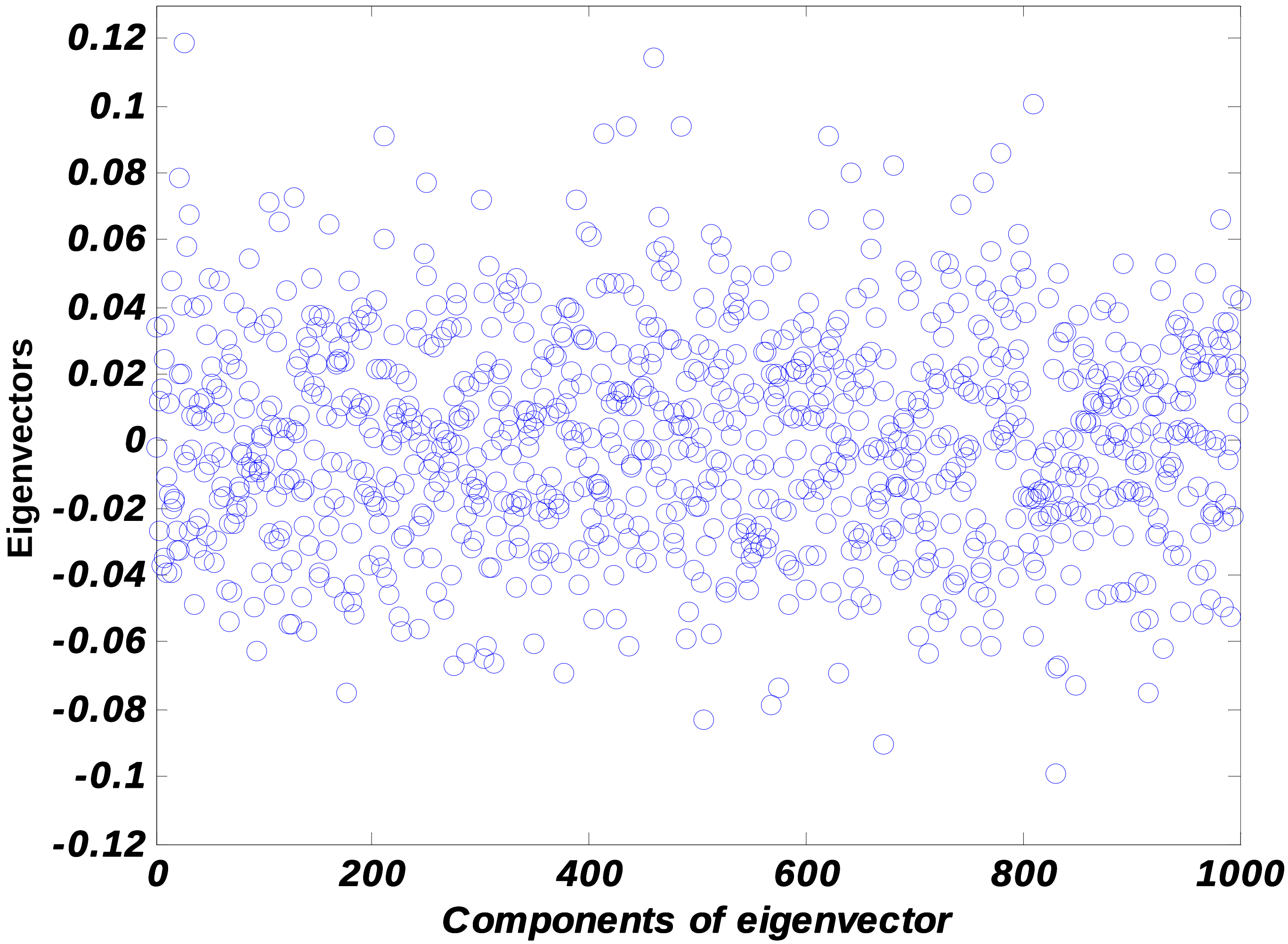
For Q=3.088, smallest(largest) eigenvalue given by $\lambda^{rm}_{min(max)} = [1 \mp (1/\sqrt{Q})]^2$ is 0.1857(2.462)



Numerically, for random correlation matrix, we find

$$\lambda^{rm}_{min(max)} = 0.0824(2.905)$$

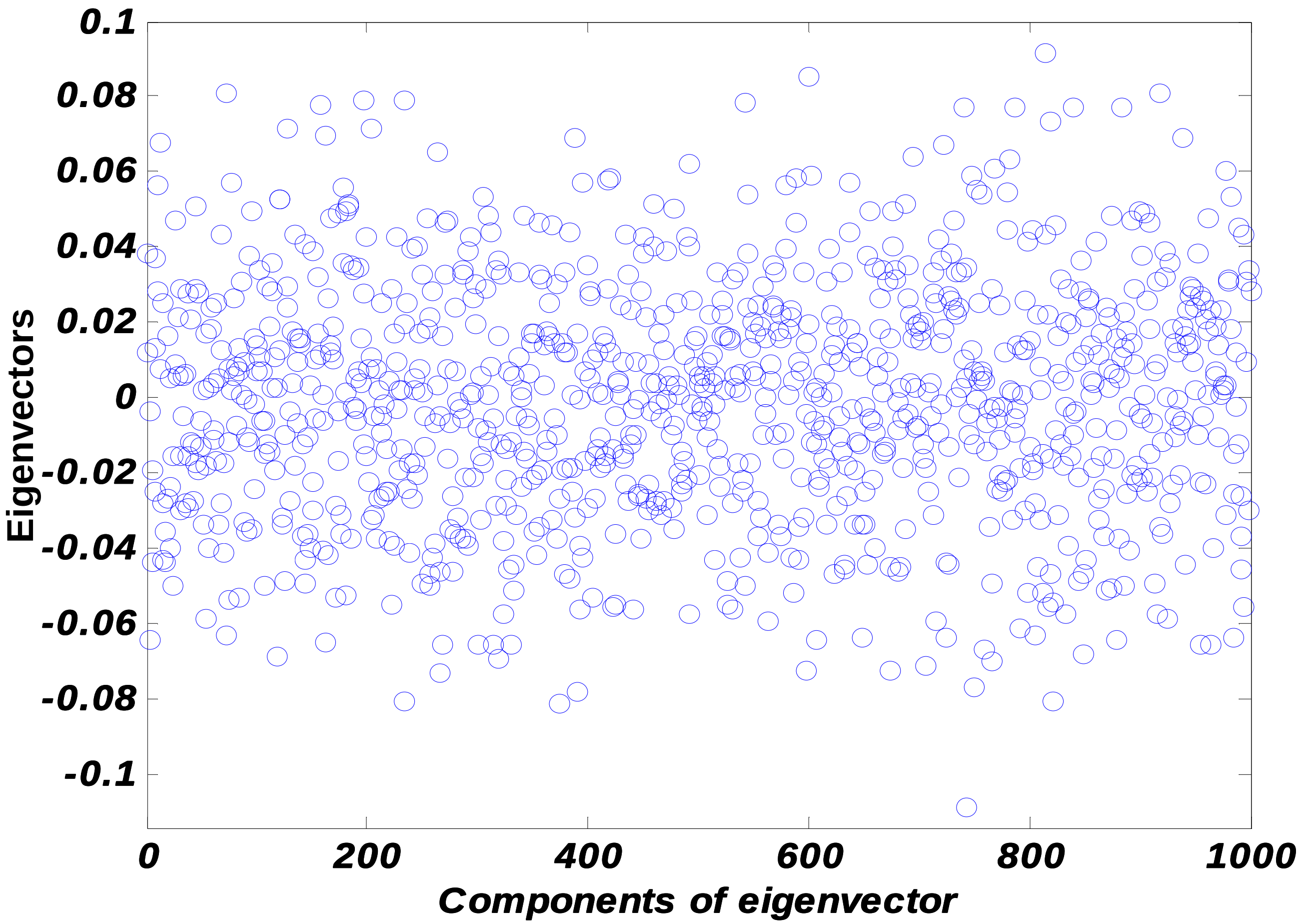
Components of eigenvectors corresponding to first largest eigenvalue



Random Correlation Matrix

Eigenvector components are distributed in negative and positive directions

Components of eigenvectors corresponding to <u>second largest eigenvalue</u>



Random Correlation Matrix

Eigenvector components are distributed in negative and positive directions

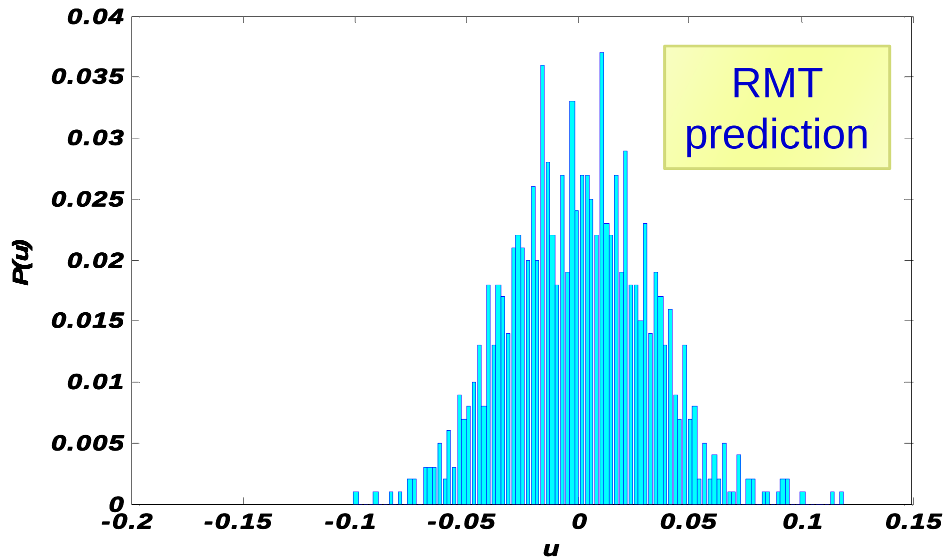
Distribution of components of eigenvector U^{20} corresponding to largest eigenvalue

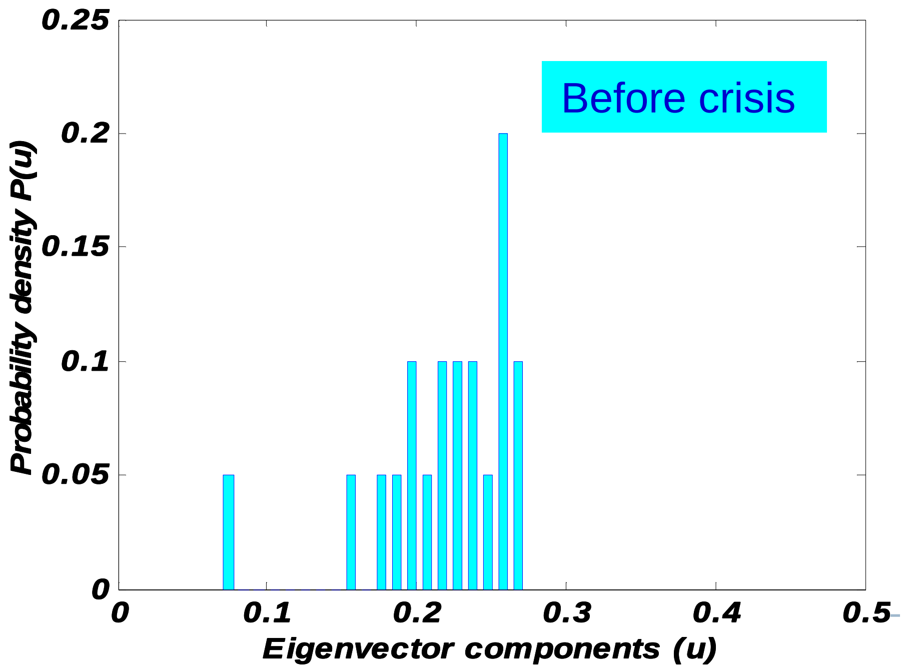
Eigenvector distribution follow the Gaussian distribution having zero mean and unit variance

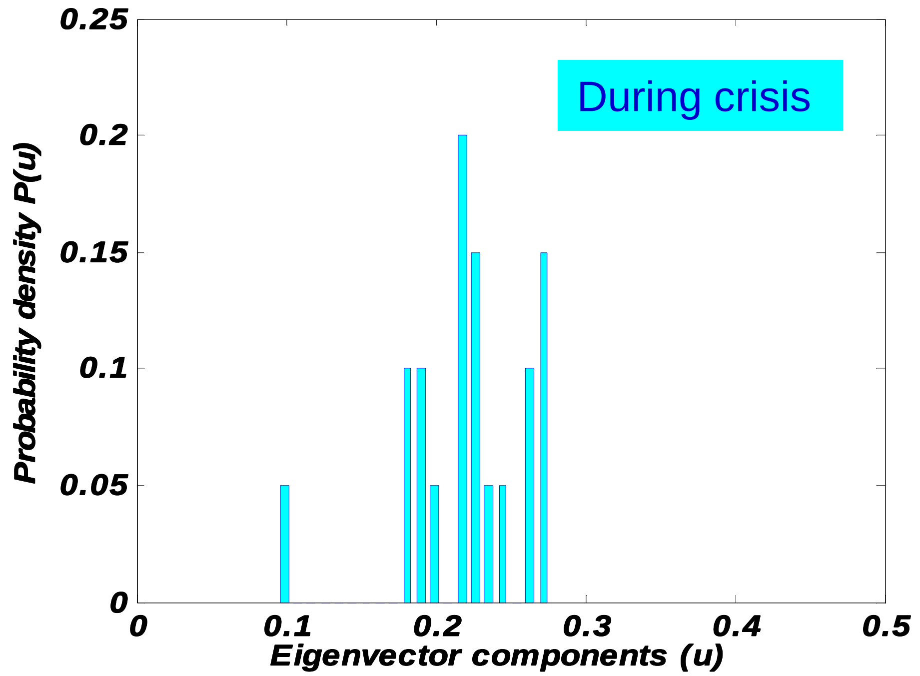
$$\boxed{P_{rm}(u) = \frac{1}{2\pi}\exp(-\frac{u^2}{2})}$$

Eigenvector components are distributed in positive side only for financial indices.

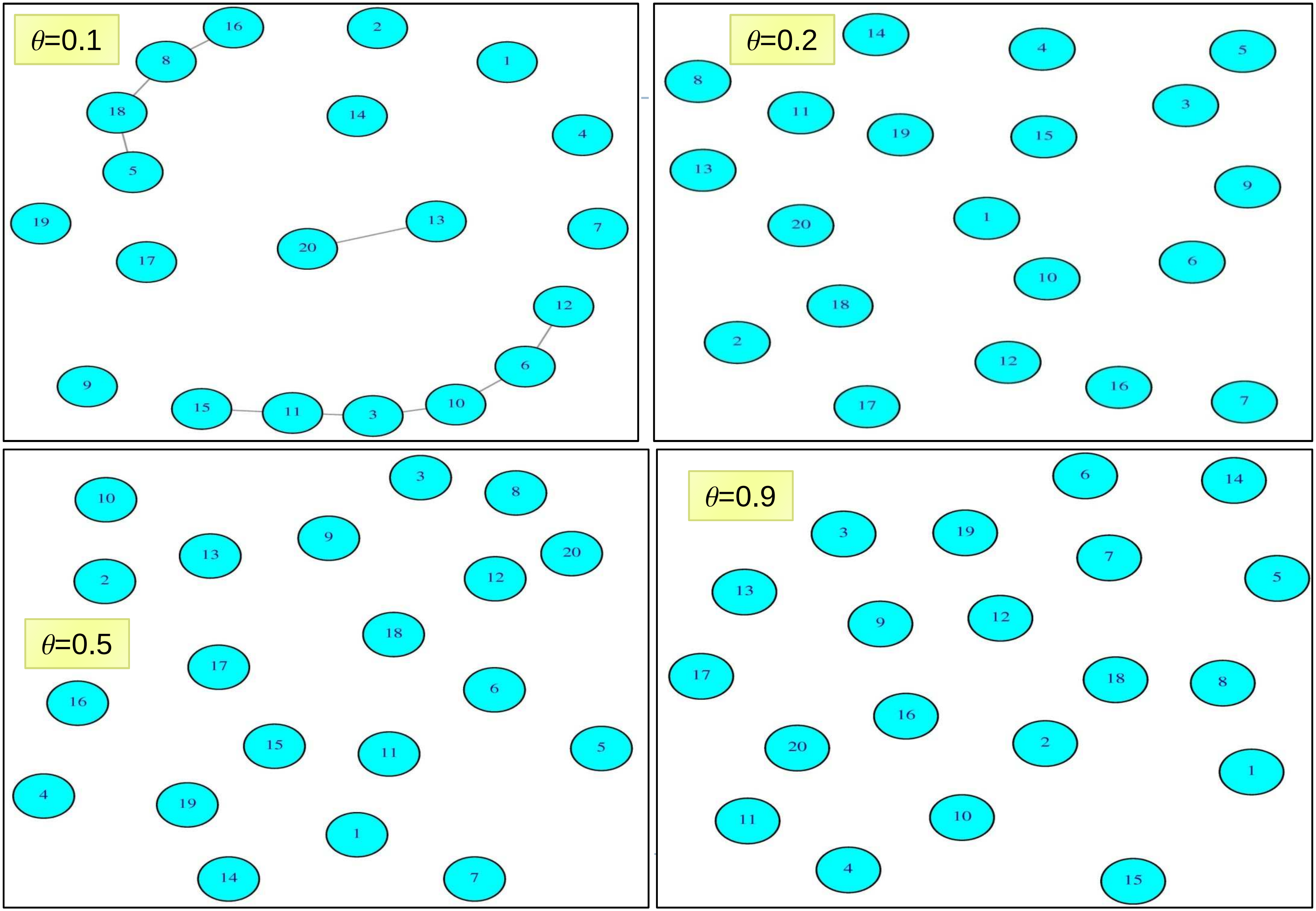
Suggest that all components participate in the eigenvector corresponding to largest eigenvalue.



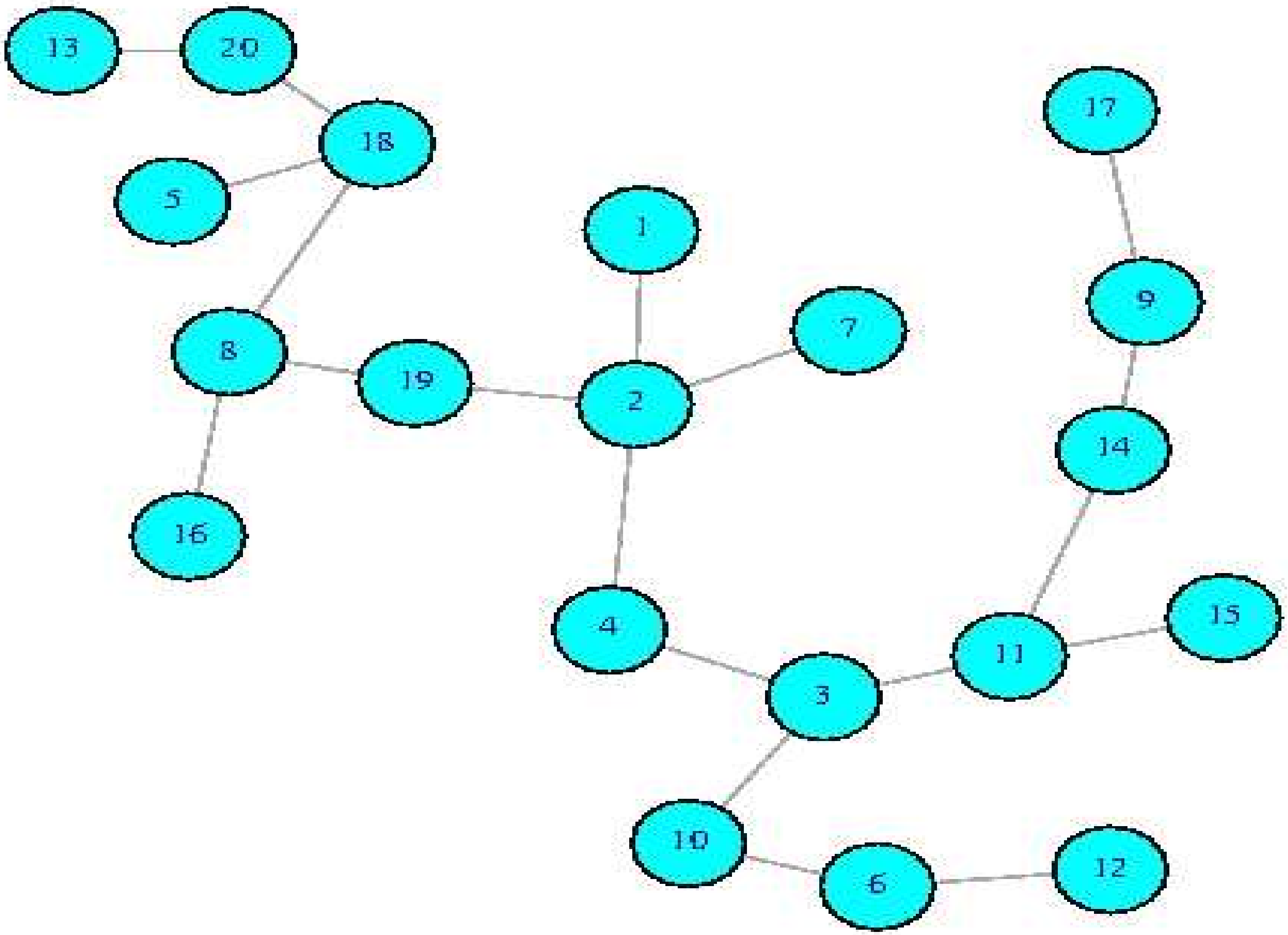




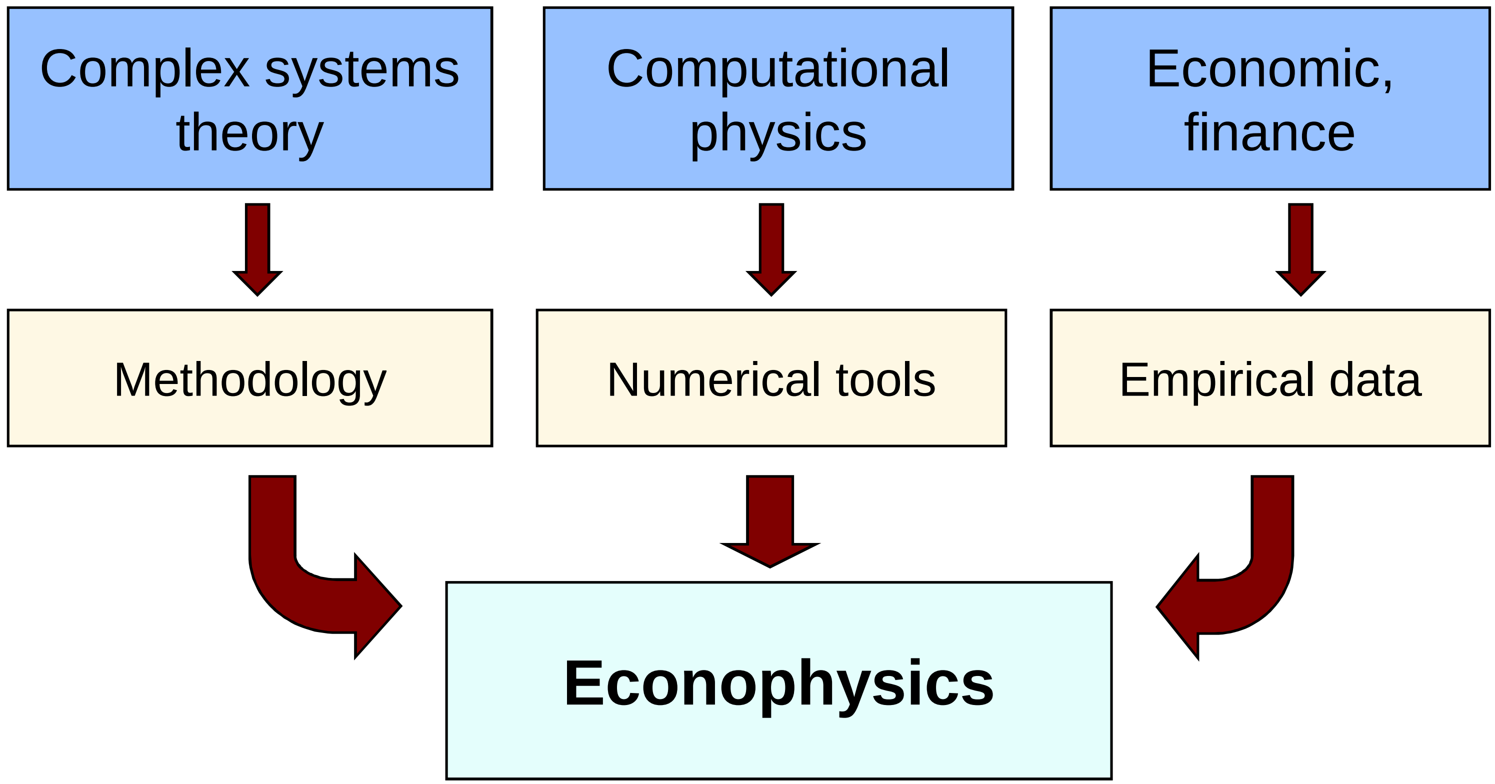
Random correlation networks at different threshold



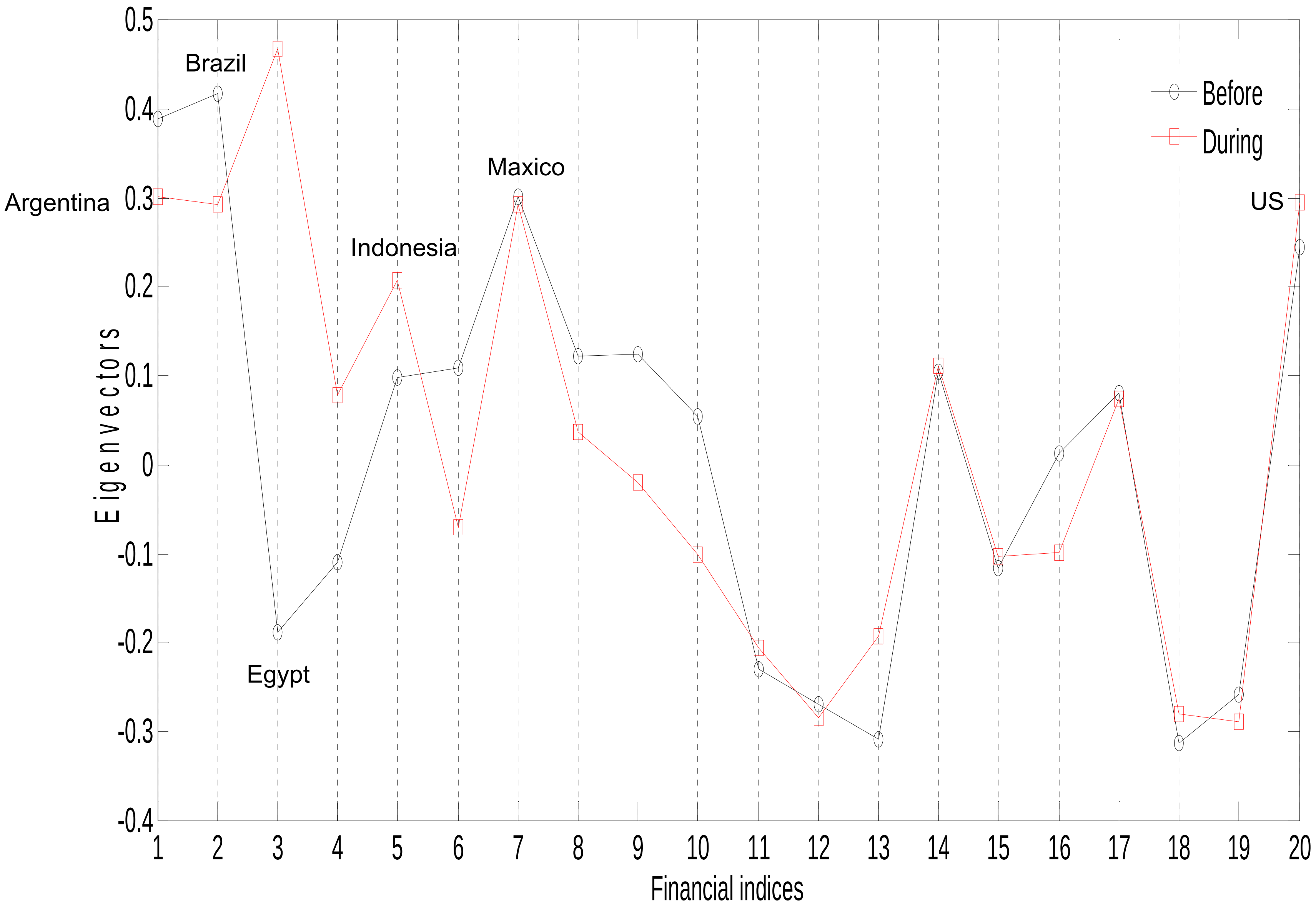
Minimum Spanning Tree (Random correlation matrix)



Introduction: What is econophysics?



Components of eigenvectors corresponding to third largest eigenvalue



Fruchterman-Reingold Algorithm:

A force based (or directed) algorithm which assign forces among the set of edges and set of nodes.

1. Assign forces as if edges were springs (Hooke's law) and nodes were electrically charged particles (coulomb's law).

2. Entire graph is then simulated as if it were a physical system. Forces are applied to nodes, pulling them closer or pushing them further apart.

3. This is repeated iteratively until the system comes to equilibrium state (their relative positions do not change anymore). At that moment the graph is drawn.

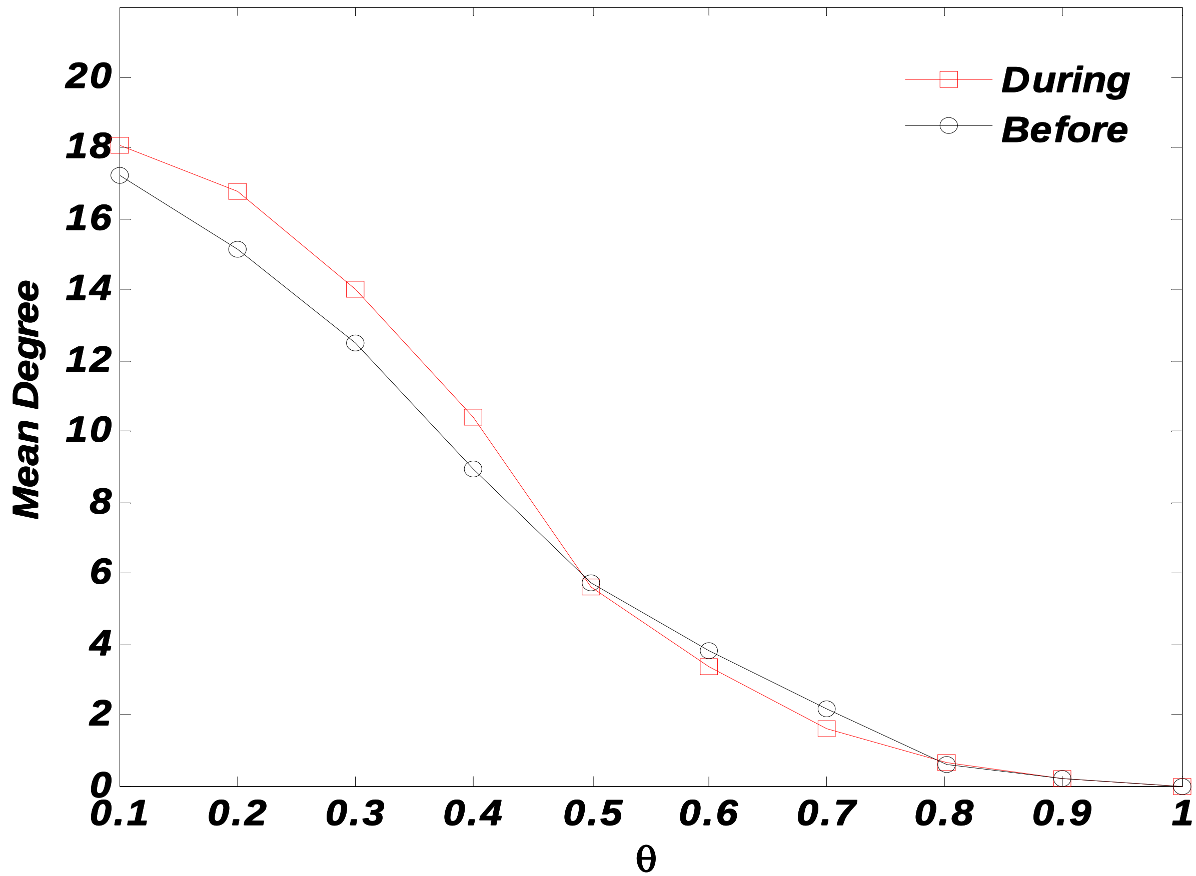
Physical interpretation of this equilibrium state is that all forces are in mechanical equilibrium.

Advantage: Good quality results, flexibility, intuitive, simplicity and strong foundation.

Topological structure of financial networks

(A) <u>DEGREE DISTRIBUTION</u>: The degree of vertex i can be defined as $K_i = \sum_{j \neq I} (e_{ij})$.

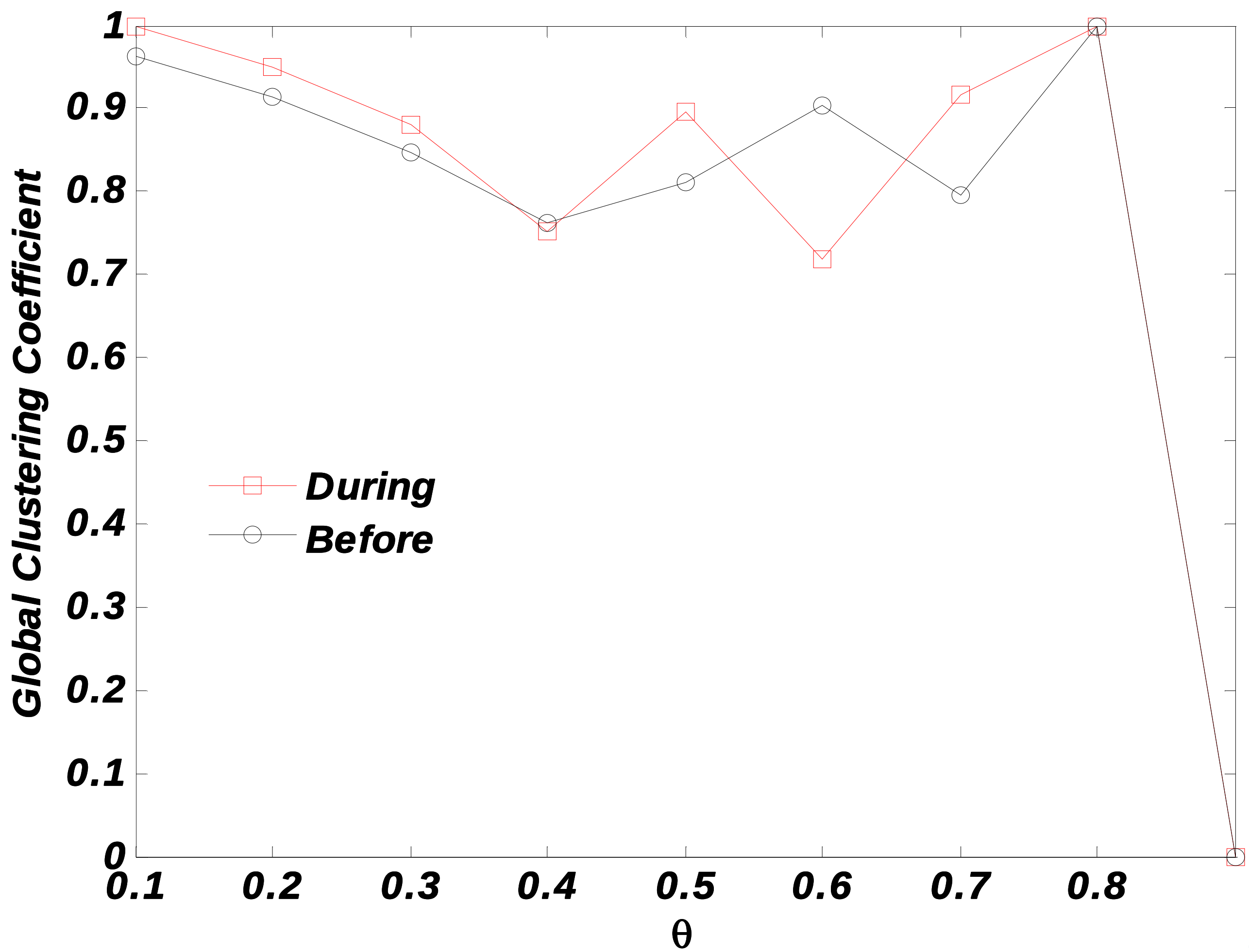
The mean degree is based upon the degree and shows how many neighbors a node in the network has on average.



The mean degree decreases with increase in the threshold as the number of connected vertices decreases with increase in threshold.

Topological structure of financial networks

(B) CLUSTERING COEFFICIENTS: If k_i nearest neighbors of vertex i have m_i edges among them, the ratio of m_i to $k_i(k_i-1)/2$ is the clustering coefficient of vertex i.

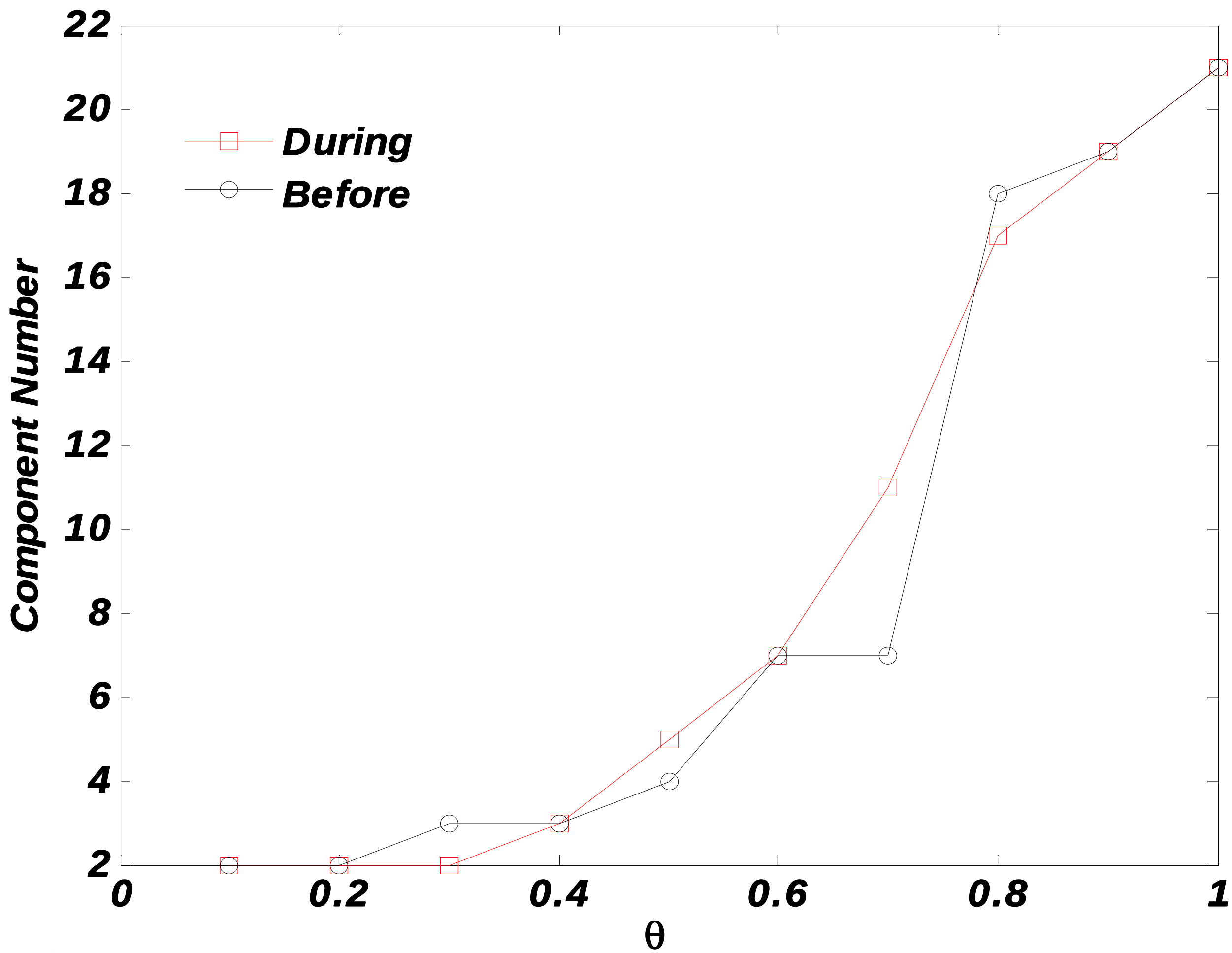


The global clustering coefficient is simply the ratio of triangles and connected triples in the correlation network of financial indices.

At θ = 0.9 there is no formation of triangles in the global financial network therefore its clustering coefficient is zero.

Topological structure of financial networks

(C) <u>CONNECTED COMPONENTS</u>: If the graph G=(V,E) is disconnected, it can be decomposed into several sub graphs which are known as connected components of G.

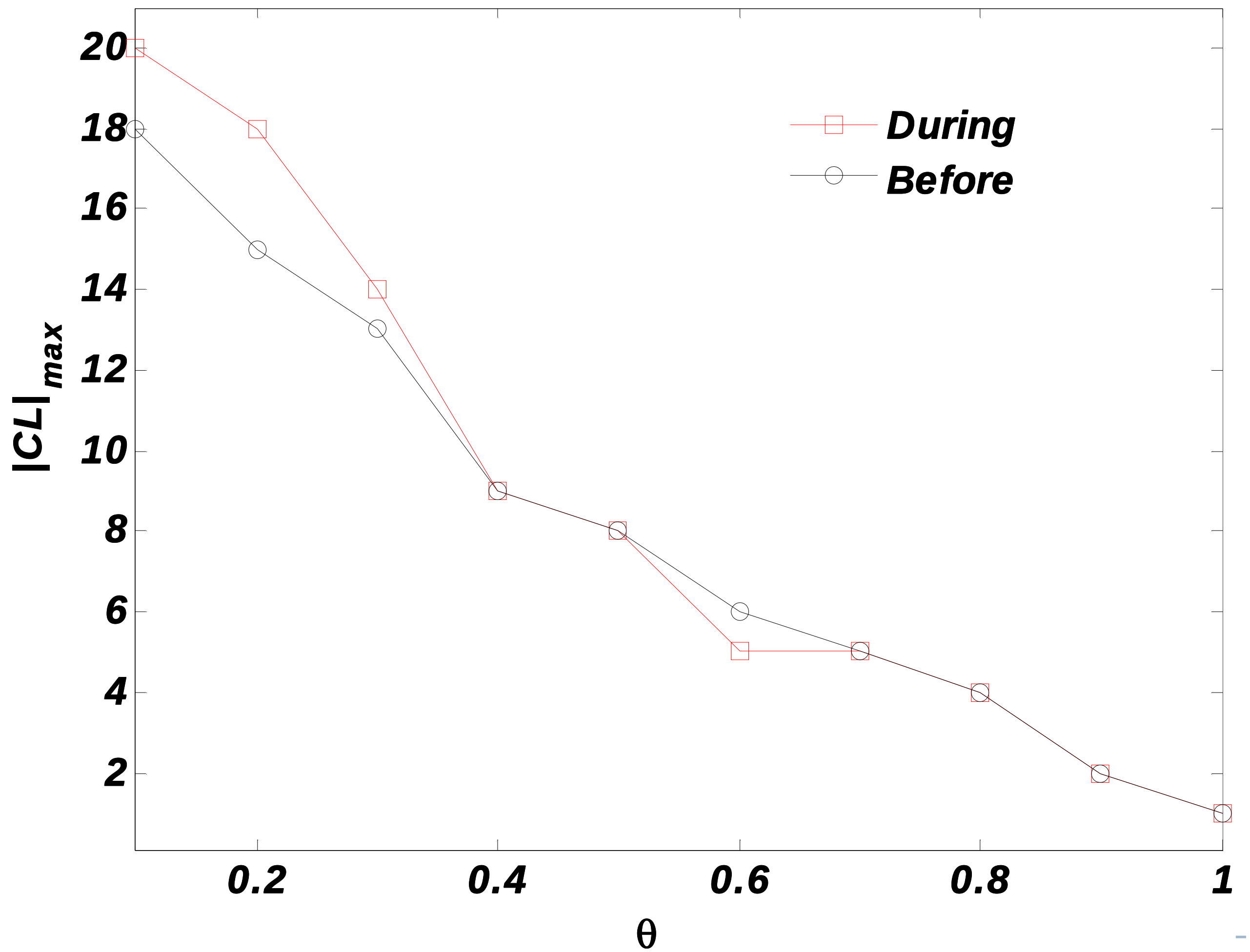


Component number in financial correlation network represents the financial indices that are correlated with each other.

At $\theta > 0.9$ vertices are nearly all isolated so the component number is approximately the vertex number.

Topological structure of financial networks

(D) <u>CLIQUE</u>: A clique in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge.

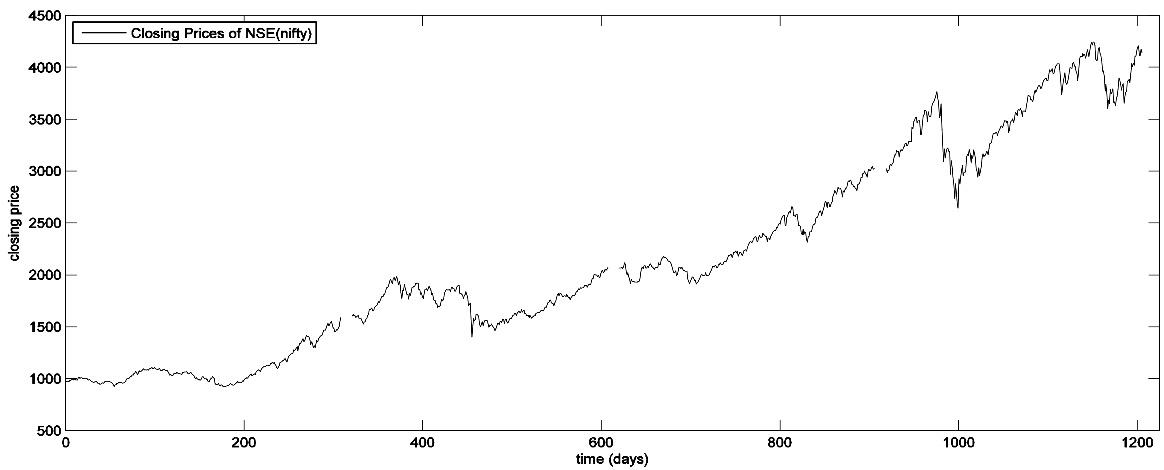


A maximum clique is a clique of the largest possible size in a graph.

In the network of financial indices, clique finds the cluster of indices that interact closely with each other.

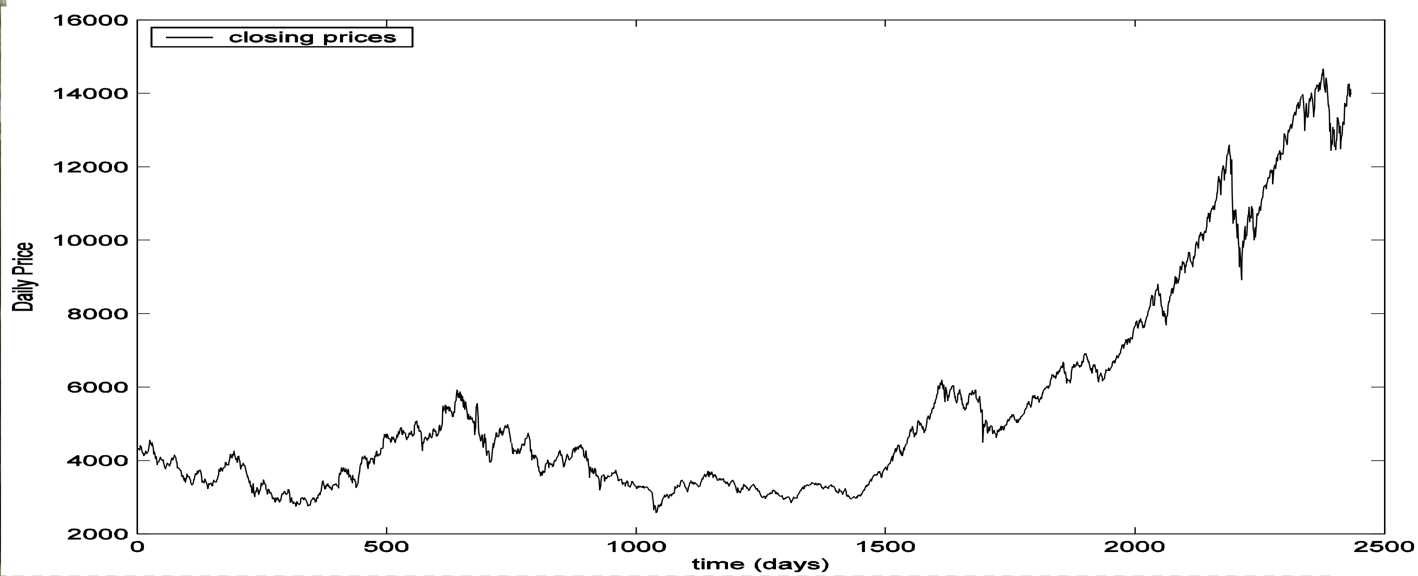
National Stock Exchange (NSE)

- Largest Stock exchange in India
- Third largest in the World in terms of volume transactions
- S&P CNX Nifty(nifty50 or simply nifty):
  - The leading index for large companies on National Stock Exchange of India
  - Well diversified 50 stock index accounting for 22 sectors for the economy
  - Used for variety of purposes such as benchmarking fund portfolios, index based derivatives and mutual



Bombay Stock Exchange

- Largest Stock exchange of Asia
- Established in 1875
- One of the Oldest Stock exchange in the world
  - Around 4,800 companies are listed.
  - BSE Sensex is widely used as market index for the BSE



Acknowledgement

We would like to thank:

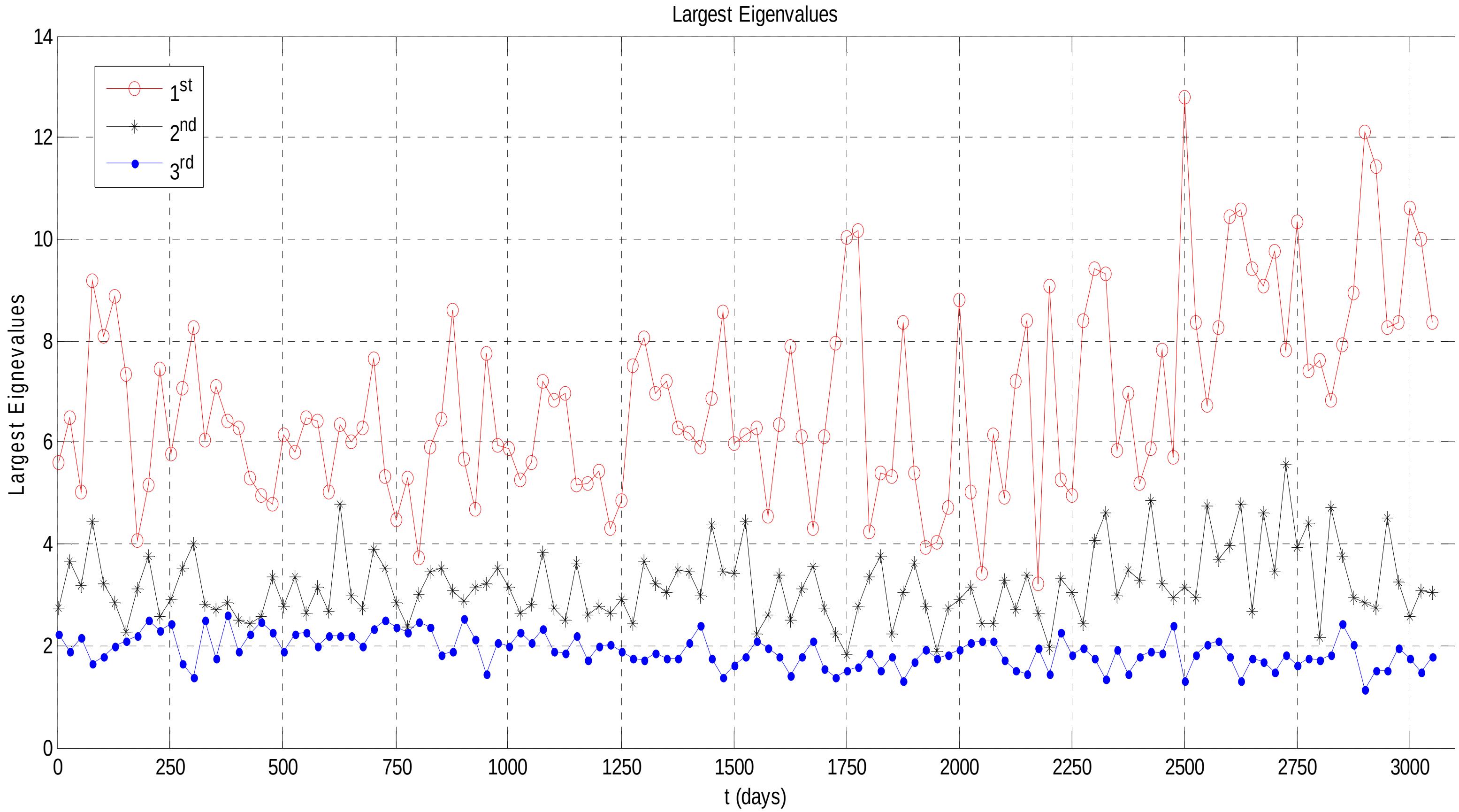
Prof. Bikas K. Chakrabarti (Saha Institute of Nuclear Physics, Kolkata) and Dr. Anirban Chakrabarti (Ecole centrale Paris, France) for discussions and hospitality during ECONOPHYS: KOLKATA international conferences (2003 and 2011).

Prof. Matteo Marsili (ICTP) for encouragement.

Prof. Sanjay Jain for encouragement, discussions, and suggestions in network analysis.

Dynamics of largest eigenvalues over time windows of 25 days

(July,1999 to June, 2009)



Largest eigenvalues represents the collective information about the correlation between different indices and its trend depends on the global financial market conditions.

There is an increase in first and second largest eigenvalue during financial crisis while third largest eigenvalue do not show significant variation as it is near RMT bound.

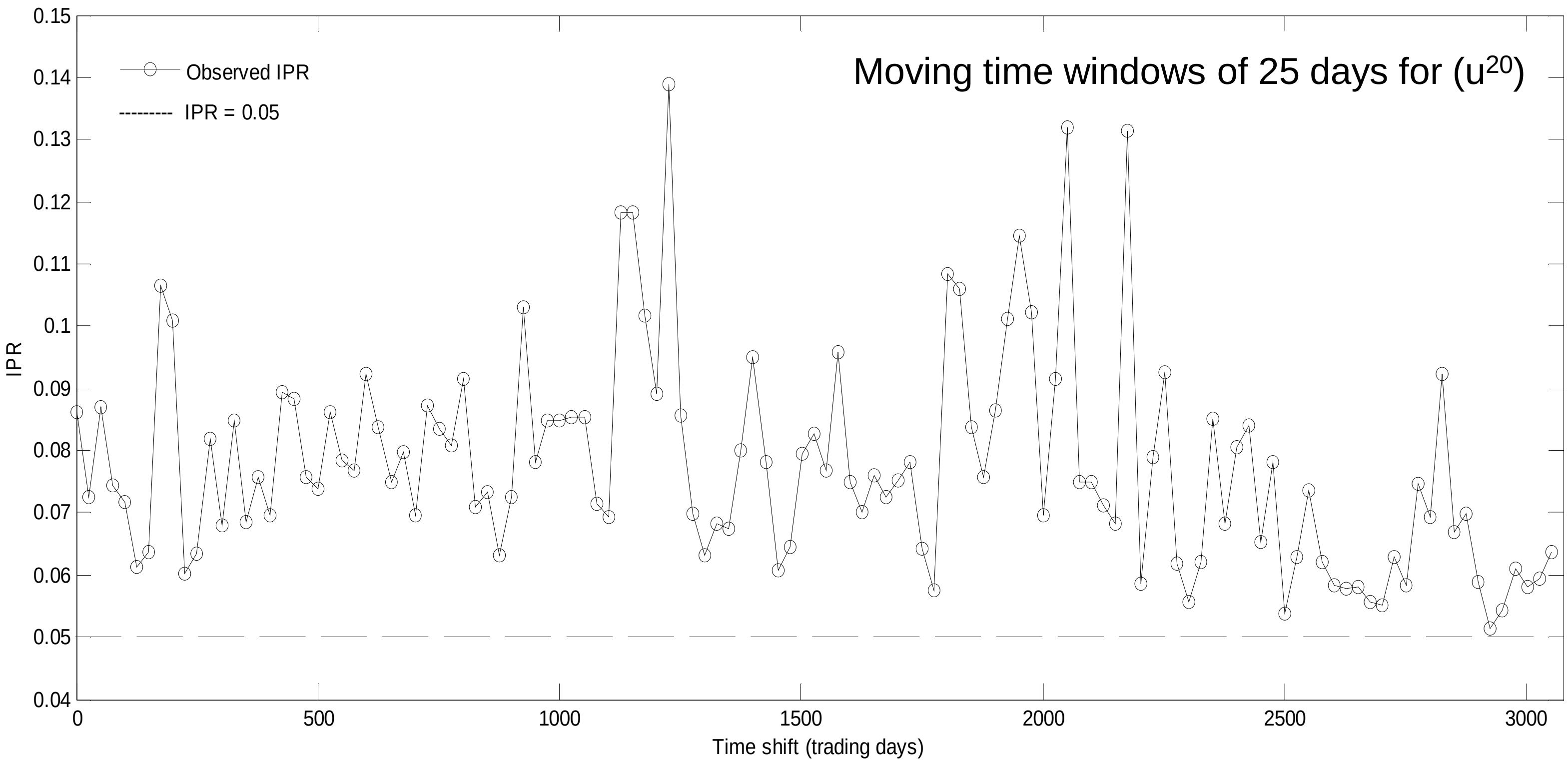
Inverse Participation Ratio

The inverse of the number of eigenvector components that contribute significantly to each eigenvector. IPR of eigenvectors u^k is defined by

$$I^k \equiv \sum_{l=1}^{N} [u_l^k]^4$$

where u_l^k , l = 1,….,N are components of eigenvector u^k.

Dashed line marks the IPR value 0.05 (= 1/20) when all components contribute equally.



Clusters:

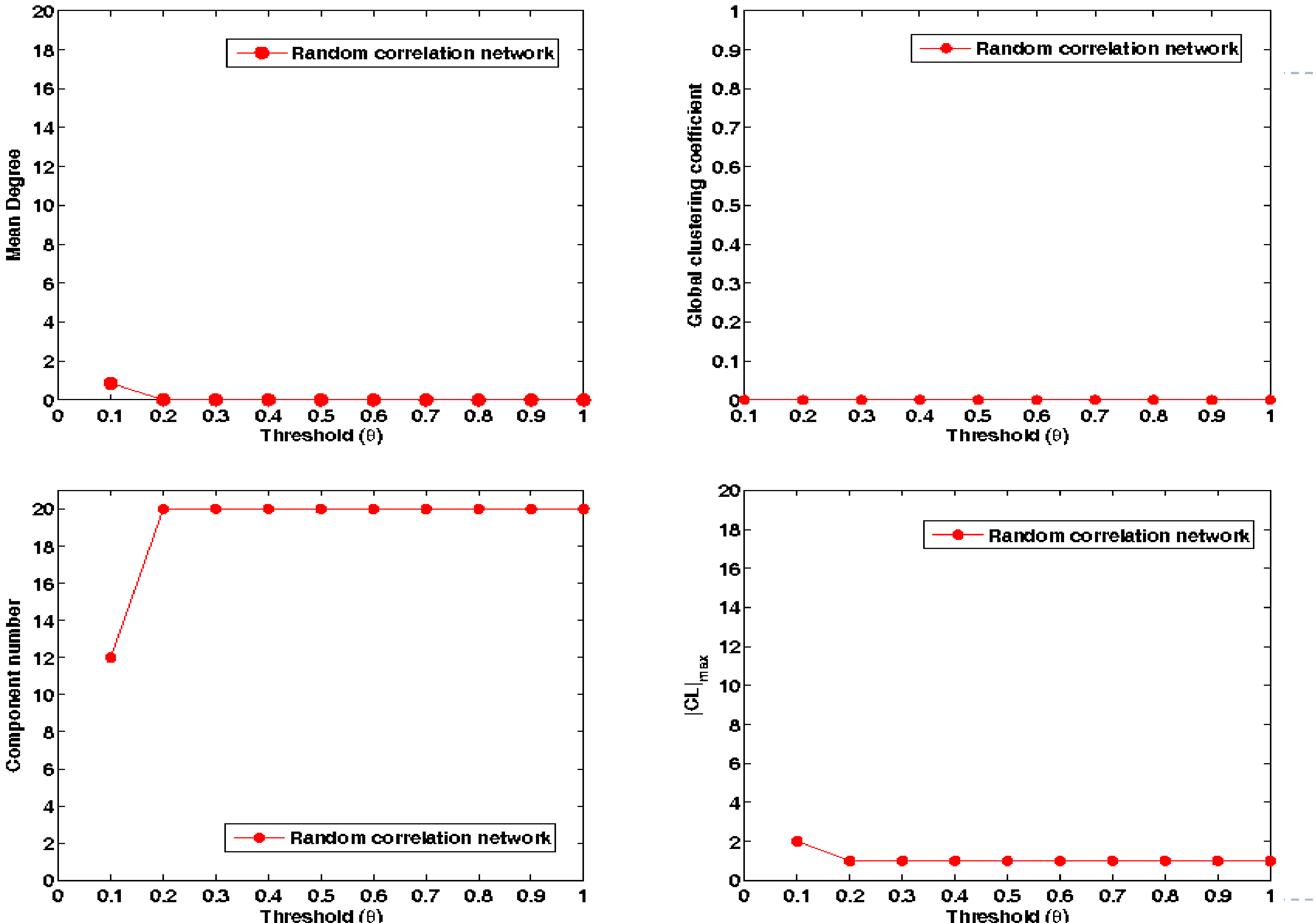
America : 1. Argentina 2. Brazil 7. Mexico 20. US

Europe : 11. Austria 12. France 13. Germany 18. Switzerland 19. UK

Asia-Pacific : 10. Australia 14. Hong Kong 4. India 5. Indonesia 6. Malaysia
16. Japan 17. Singapore 8. South Korea 9. Taiwan

Africa-Middle East: 3. Egypt 15. Israel

Topological properties of random correlation networks



Topological properties of global financial networks