Wealth Exchange Models with preference in interaction

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Economy and Agents

- Economy : involves a large number of people/agents.
- Study of an individual extremely difficult.
- According to P. Ball : "At face value, there might seem to be little room left for statistical physics to make a realistic contribution. But if there is one message that emerges clearly from this discipline, it is that sometimes the details do not matter."

Kinetic Exchange Model : Gas = Economy

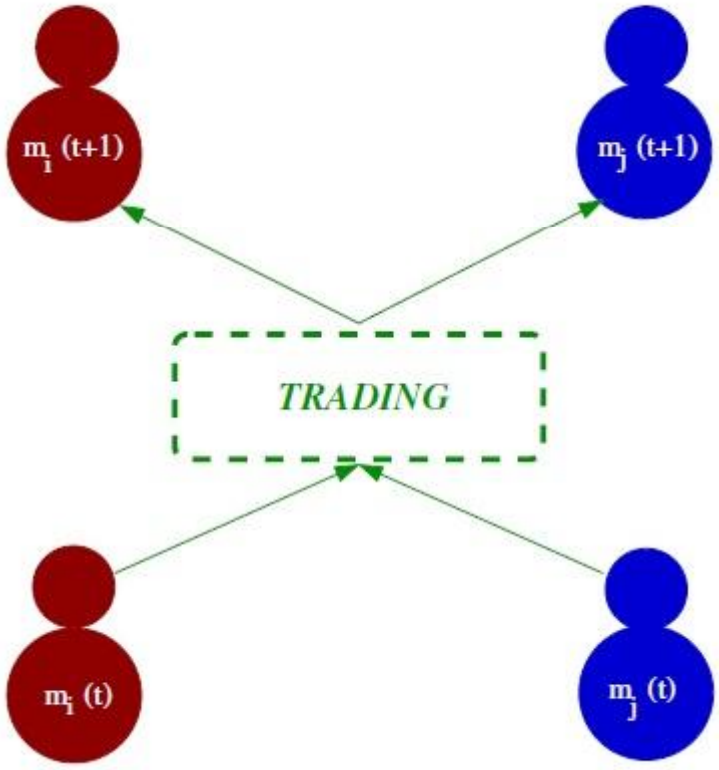
- *Kinetic exchange model is based on the exchange of energy among gas molecules due to elastic collisions occurring among them.*
- *Derives the average macroscopic behaviours from the microscopic properties of gas molecules.*
- *Energy is locally conserved.*

Gas Molecules → Agents
Energy → Wealth

- *In a trading, a pair of traders exchange wealth, respecting local conservation in any trading.*
- *These models have a microcanonical description and nobody ends up with negative wealth (i.e., debt is not allowed).*

Basic description of the models

Wealth distribution in a real economy decay as $P(m) \sim m^{-(1+\nu)}$ for large m (Pareto law; Cours d'economie Politique, F. Rouge, Lausanne (1897)). Two agents interact and exchange money; total money M is conserved.



- DY Model: *N agents exchange money randomly → no saving → Steady state follows Gibbs distribution. (Eur. Phys. J. B* ***20****, 585 (2001)).*
- CC Model: *Agents save a fixed fraction* λ *of their money.. Same saving trend for all..(Eur. Phys. J. B* ***17****, 167 (2000)). P(m) is similar to a Gamma distribution ; no power law tail.*
- CCM Model: *Different agent has different saving trend..* λ*s are different.. (Physica A* ***335****, 155 (2004)). P(m) not exactly known but power law tail exists.*

But there are preferences…

- No random selection of agents.
- Preferential selection of agents.

- Basic assumption : two agents will interact only when their wealth are "close".
- Selection of richer agents.
- Repeated interaction between selected pair of agents.

V. Hatzopoulos, G. Iori, R. N. Mantegna, S. Micciche, M. Tumminello, *Quantitative Finance (2013).*

M. Tumminello, F. Lillo, J. Piilo, R. N. Mantegna, *New Journal of Physics 14, 013041 (2012).*

Models and parameters

Number of agents N, Total money M (=N),
1 Monte Carlo (MC) step = N pairwise interaction

- Model A : Probability of interaction between agents i and j

$$P_{ij} \propto \left|m_i - m_j\right|^{-\sim} \; ; \quad \sim > 0$$

  - When $\sim = 0$ it gives back the DY model.
  - When $\sim$ is large, tail of the distribution has a power law form.

- Model B : Probability of interaction between i and j

$$P_{ij} \propto \left|m_i - m_j\right|^{-\sim} \left(C_{ij} + 1\right)^{¥}$$

- Model C : Probability of interaction between i and j

$$P_{ij} \propto \left|m_i - m_j\right|^{-\sim} m_i$$

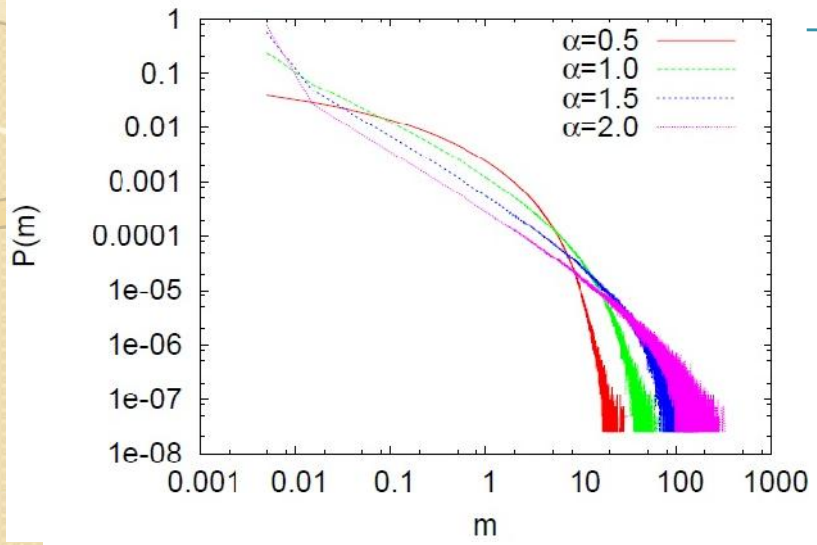
- Model D : All three parameters are present.

$$P_{ij} \propto \left|m_i - m_j\right|^{-\sim} m_i \left(C_{ij} + 1\right)^{¥}$$

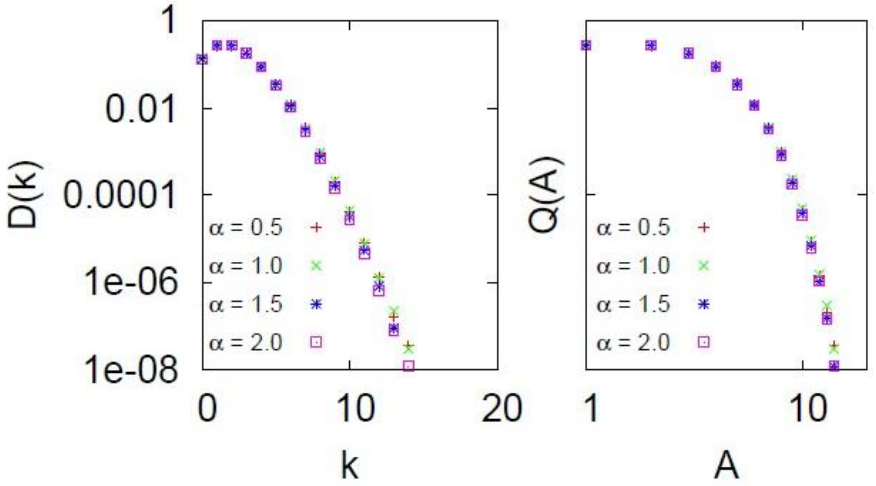
Features studied

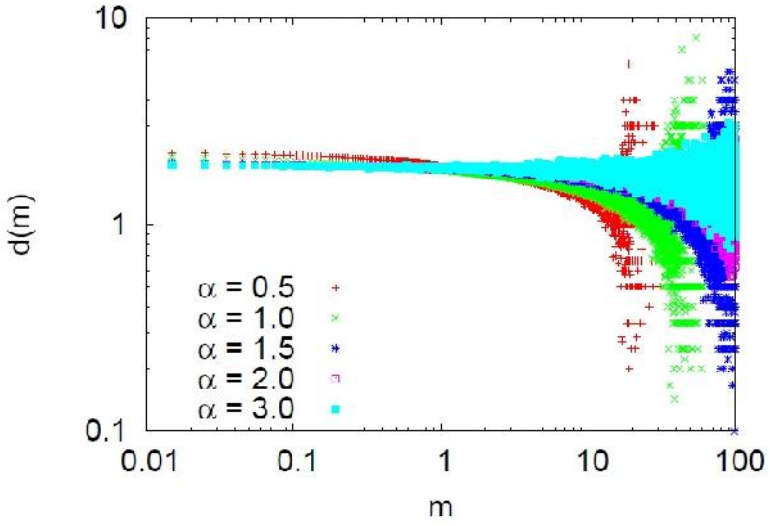
- Wealth Distribution (already introduced at the beginning).
- Degree Distribution: The number of people with whom one particular agent interacts within one MC time step, averaged over all time step is the degree of an agent. D(k) denotes the probability that an agent has degree k.
- Activity Distribution: Activity distribution is defined as the number of transactions made by one individual in one MC timestep, averaged over all timesteps. We use Q(A) to denote the activity distribution.
- Average degree with wealth m: d(m), the average degree of an agent with money m is also calculated to investigate whether the degree is correlated to wealth.

Features for Model A



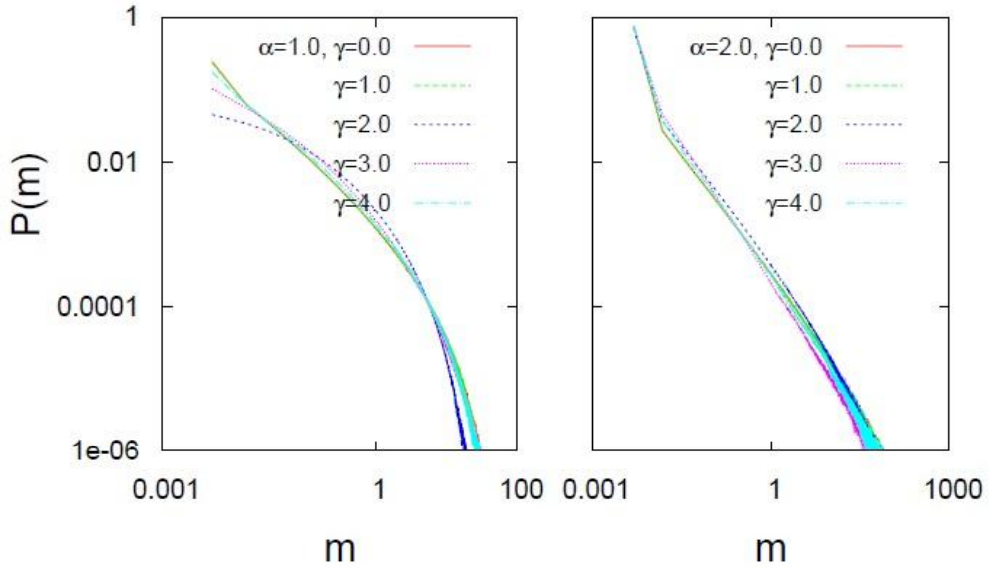
Power law distribution with an exponential cut-off decreasing with α.

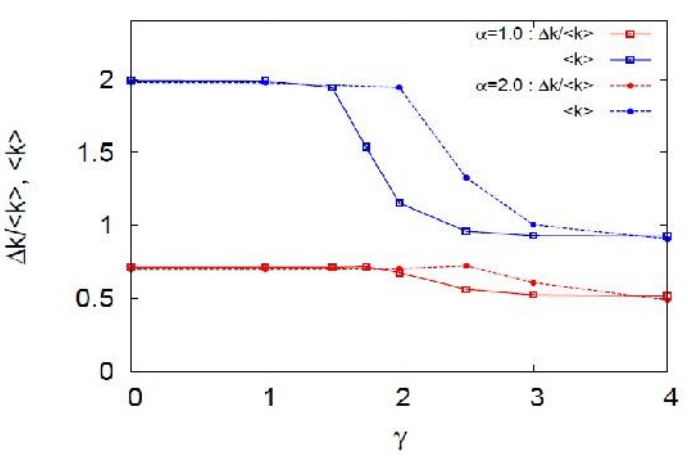


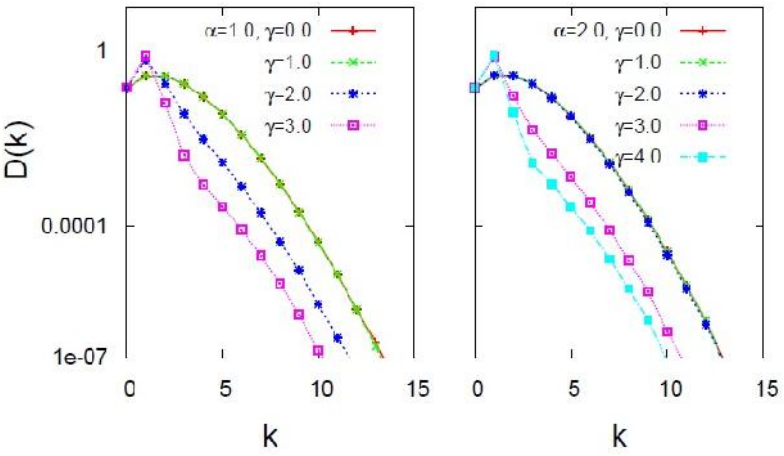


d(m) is independent of the wealth possessed by an individual; more so for larger values of α.

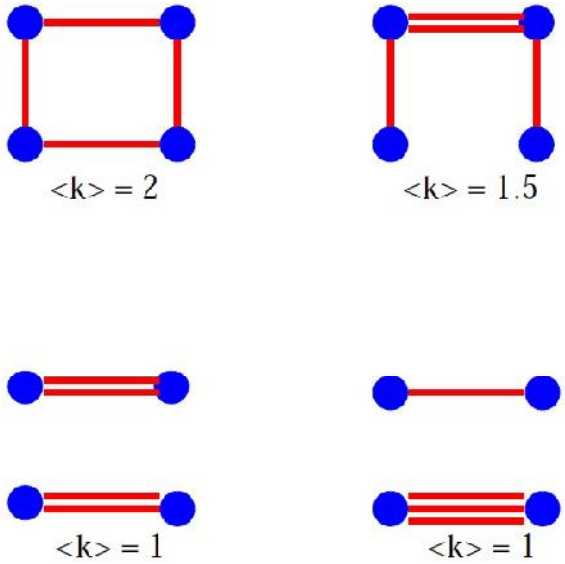
Features for Model B



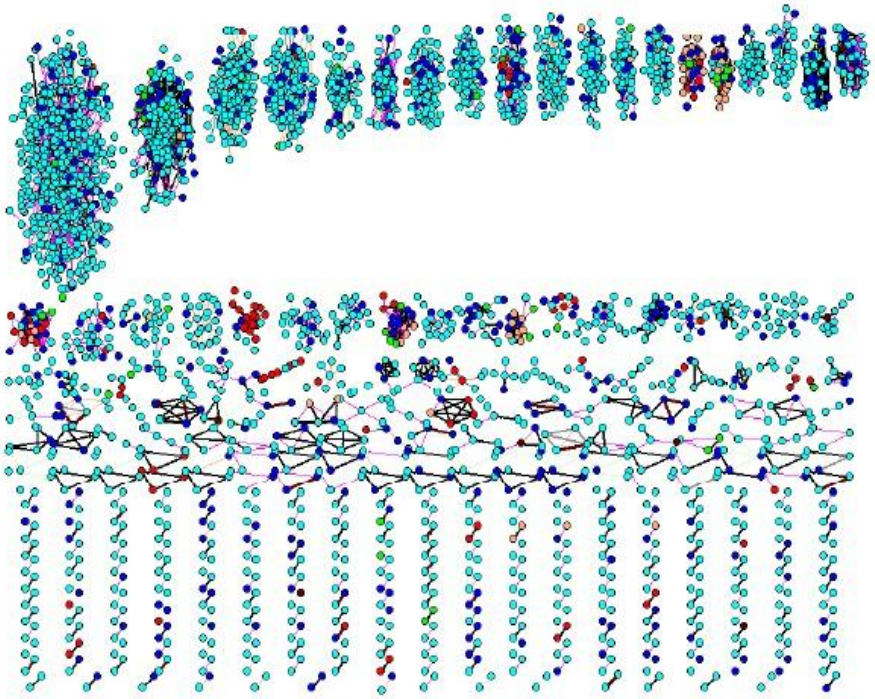




Dimerisation

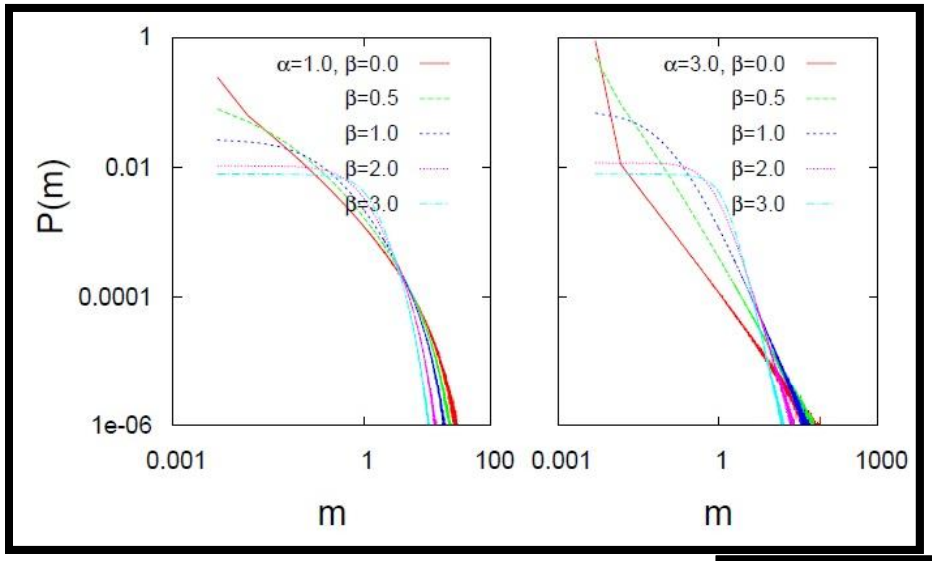


- Average degree decreases from 2 to 1.
- Dimers and small clusters are observed in real data.



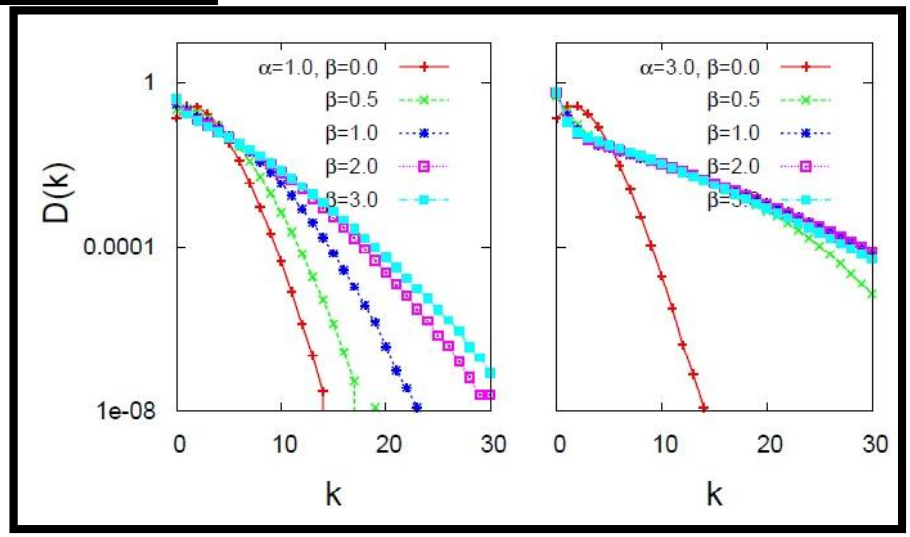
Courtsey : M. Tumminello, F. Lillo, J. Piilo, R. N. Mantegna, *New Journal of Physics 14, 013041 (2012).*

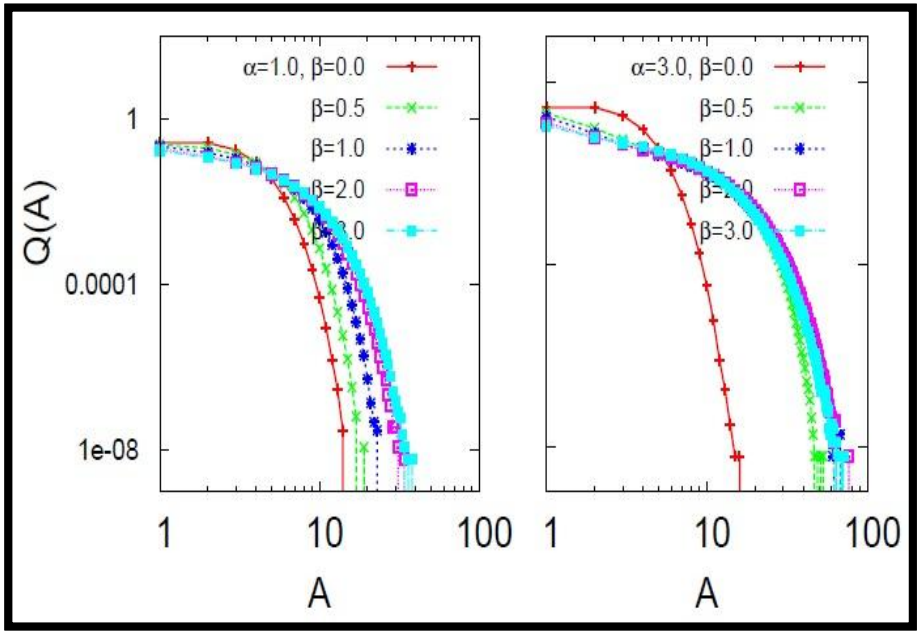
Features for Model C



A flat region is found for small m, and a power law region for a narrow range of m follows it.

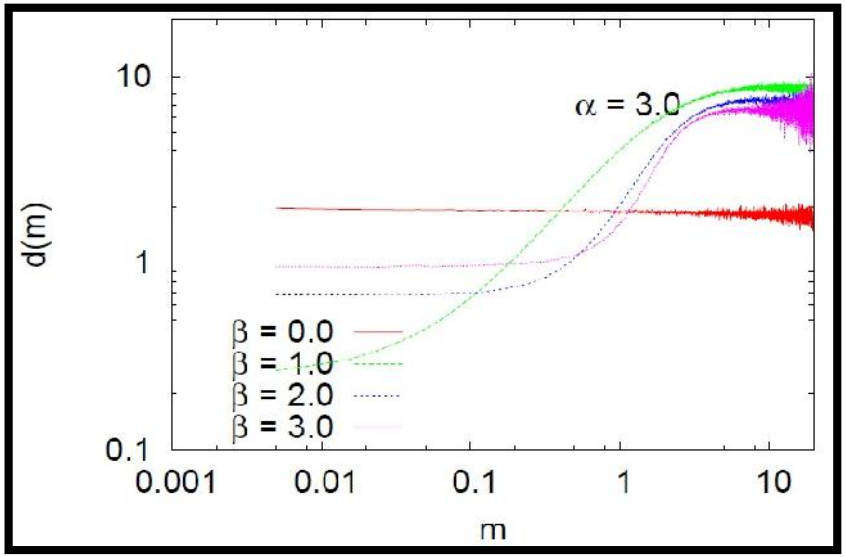
Degree distribution is more spread out.



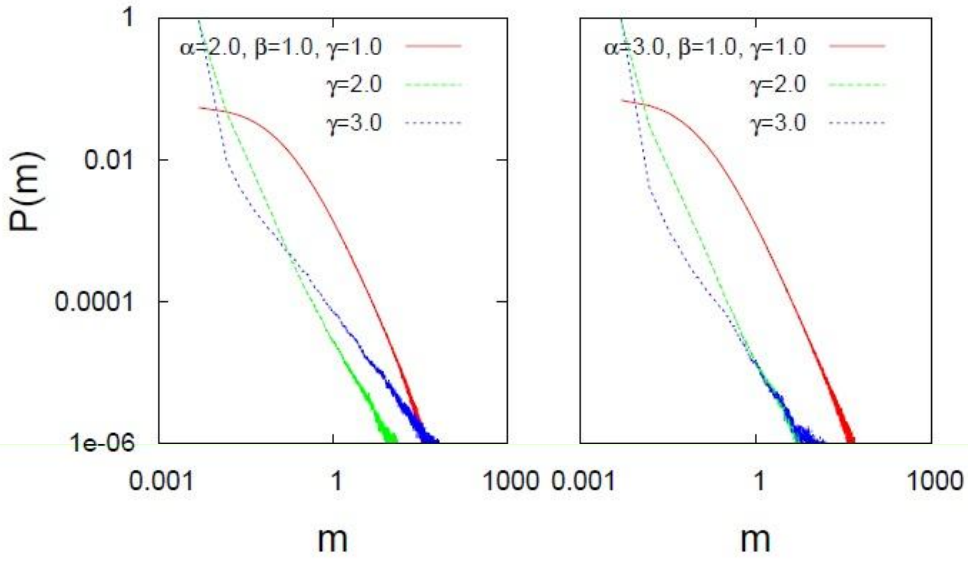


Activity distribution has a distinct parameter dependence.

It shows a different behaviour compared to models A and B.

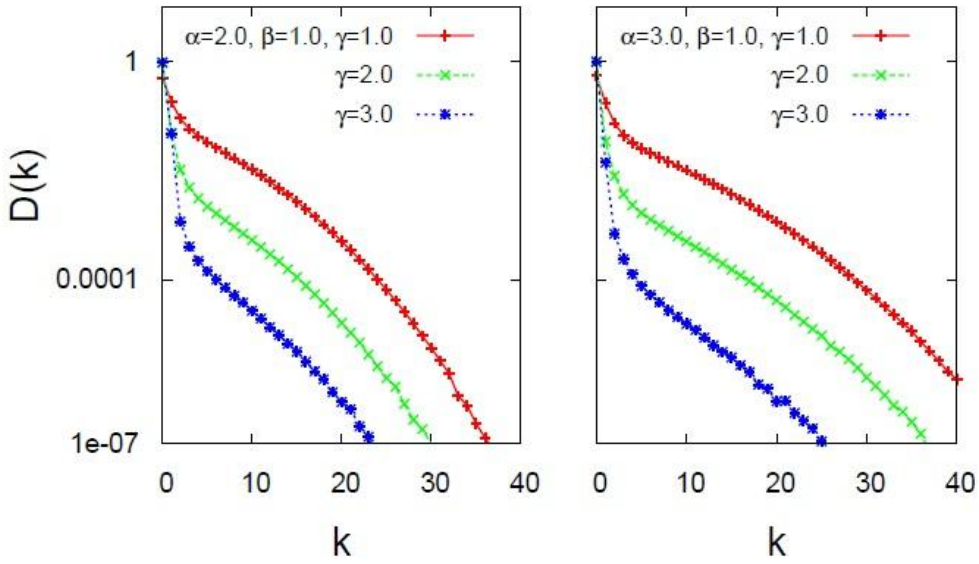


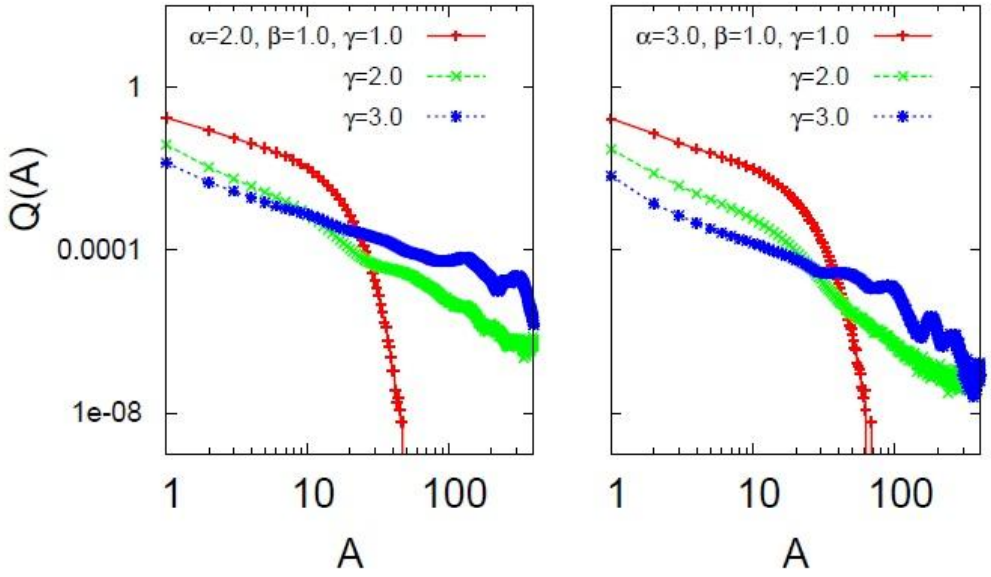
Model D



The value of the Pareto exponent is close to 1 as γ is small and decreases as γ increases.

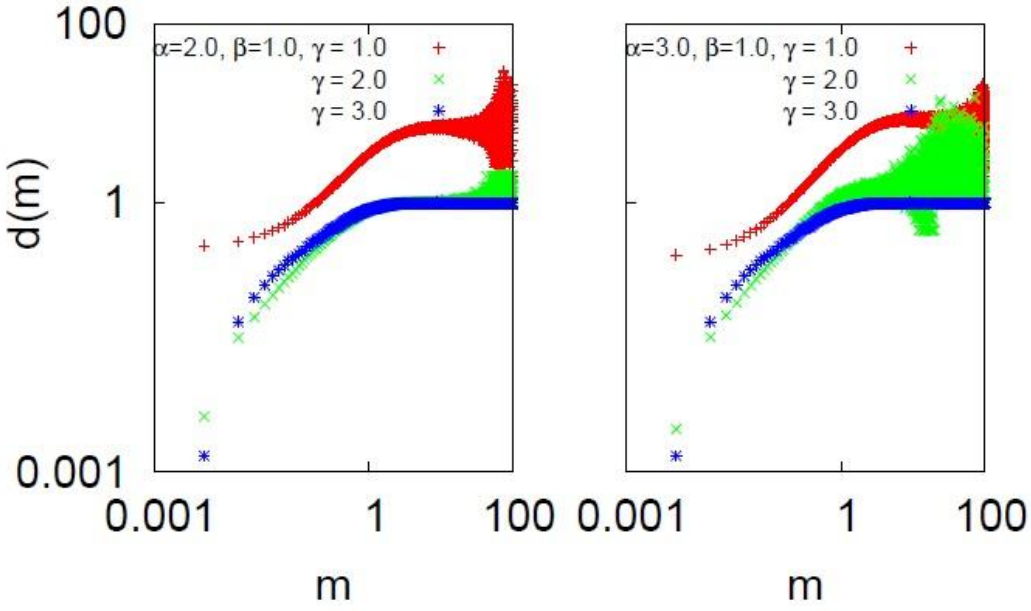
Degree distribution has highest value for k = 0 and then drops off suddenly to a low value.





- Power law in activity distribution.
- Observed in real data.[*X. Gabaix, P. Gopikrishnan, V. Plerou and H. E. Stanley,* MIT Working Paper Series **03-30**, 1-46(2004).]

Dimerisation observed as in model B.



Type of the Model	A	β	γ	ν
Model B	2.0	0.0	1.0	0.088
	2.0	0.0	2.0	0.096
	2.0	0.0	3.0	0.279
	2.0	0.0	4.0	0.174
Model C	3.0	1.0	0.0	0.798
	3.0	2.0	0.0	1.432
	3.0	3.0	0.0	2.134
Model D	2.0	1.0	1.0	0.671
	2.0	1.0	2.0	0.400
	2.0	1.0	3.0	0.091
	3.0	1.0	1.0	0.792
	3.0	1.0	2.0	0.519
	3.0	1.0	3.0	0.196
	3.0	3.0	2.0	2.341

Summary

- For all the models we assume that two agents will interact only when they are "closely" located in the wealth space. This is controlled by the parameter α.
- The parameter β helps to select the agents with a probability proportional to their wealth. This parameter has role at the initial region of the wealth distribution.
- γ takes care of the "memory" that a pair of agents have interacted already. Probability of interaction increases with the number of past interactions controlled by the parameter .
- All the parameters together are considered in model D. There α and γ controls the power law region. The flat region generated by the parameter β is reduced by the parameter γ.