Abstracts

Long presentations

A kinetic model of trading market and its analytic solution

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We analyze an ideal gas like model of a trading market with quenched random saving factors for its agents and show that the steady state income (m) distribution $P(m)$ in the model has a power law tail with Pareto index ν exactly equal to unity, confirming the earlier numerical studies on this model. The analysis starts with the development of a master equation for the time development of $P(m)$. Precise solutions are then obtained in some special cases.

[1] Arnab Chatterjee, Bikas K. Chakrabarti and S. S. Manna, Physica A 335 (2004) 155-163.
[2] Arnab Chatterjee, Bikas K. Chakrabarti and Robin B. Stinchcombe, cond-mat/0501413.

A Physicist's attempt to model wealth distributions in Economic systems

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We study a statistical model consisting of N economic agents in a closed economy which interact with each other by exchanging money, according to a given microscopic random law, depending on a parameter λ which controls the saving propensity of the agents. We focus on the equilibrium or stationary distribution of the money exchanged and verify through numerical fitting of the simulation data that the final form of the equilibrium distribution is that of a standard Gamma distribution. We also study variations of this model by introducing effects of commodity trading, formation of trade partners, etc.

A kinetic model of market with random saving propensity

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We numerically simulated the ideal-gas models of trading markets, where each agent is identified with a gas molecule and each trading as an elastic or money-conserving two-body collision. Unlike in the ideal gas, we introduce (quenched) saving propensity of the agents, distributed widely between the agents ($0 \leq \lambda < 1$). The system remarkably self-organizes to a critical Pareto distribution of money $P(m) \sim m^{-(\nu+1)}$ with $\nu \simeq 1$. We analyse the robustness (universality) of the distribution in the model. We also argue that although the saving ingredient is a bit unnatural one in the context of gas models, our model is the simplest so far, showing self-organized criticality, and combines two century-old distributions: Gibbs (1901) and Pareto (1897) distributions.

[1] A. Chakraborti and B. K. Chakrabarti, Eur. Phys. J. B 17 167 (2000).
[2]Arnab Chatterjee, Bikas K. Chakrabarti and S. S. Manna, Physica A 335 (2004) 155-163.

Income Inequality Dynamics: Evidence from a Pool of Major Industrialized Countries

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We analyze four sets of income data: the US Panel Study of Income Dynamics (PSID), the British Household Panel Survey (BHPS), the German Socio-Economic Panel (GSOEP), and the Italian Survey on Household Income and Wealth (SHIW). It is firstly demonstrated that a two parameter lognormal distribution can give very accurate fits to the low-medium income range (more than 90% of the population), whereas the high income range (less than 10% of the population) is well fitted by a Pareto or power-law function. This mixture of two qualitatively different distributions seems stable over the years covered by our data sets, although the indexes specifying them fluctuate over time. We quantify these fluctuations by establishing some links with the country specific business cycle phases, and show how the separation between the two regimes of the income distributions may be due to different income dynamics. In particular, we find that for the top percentiles of the distributions capital income rather than labour earnings accounts for a vaste share of the total income, so that its contribution to the latter may be responsible for the observed power-law behaviour in the tail. Secondly, to identify the contribution of the individual factors and to assess their relative importance to the overall inequality, we investigate income inequality using a decomposition analysis by income sources. Our results suggest that capital income makes by far the most significant contribution to overall inequality, confirming in this way its role in determining the Pareto power-law tail.

The monomodal and bimodal statistical distribution of money

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The individual distribution of money for various countries follows a Boltzmann distribution except at high values, where the experimental probability densities are systematically larger than the fitted values. This discrepancy can be settled by the use of nonextensive statistics, as developed by Tsallis, which conserves the Boltzmann shape at low values of money but follows a Pareto's power law in the tail, indicating fractal behaviour. In some cases, the distribution can be described by a single equilibrium function but in others two components appear. The evolution of an arbitrary initial distribution to the equilibrium, two components distribution, is explained as a consequence of the different state degeneracy associated with each of them. The social consequences, within a country and at global level are analyzed.

Equilibrium and nonequilibrium distribution of money

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Bimodal (or in general, polymodal) Boltzmann distributions are indicative of the presence of different ensembles of economic agents within a given population, which are at equilibrium. Examples of bimodal distributions far for equilibrium are not easy to find, because a sudden change in the economical situation is required to produce it. One clear example of this behaviour is the individual income distribution of Argentina. The crisis at the end of 2001 resulted in a strong and fast perturbation, which in the period of 4 months produced a devaluation of the local currency by a factor of 3. The resulting bimodal distributions are near Gaussian, indicating a nonequilibrium situation. The evolution through years 2002-2004 is analyzed. A generalization, using rate equations for a simple three level system provides further insight on the temporal evolution of the distribution of money to equilibrium.

Pareto-Zipf, Gibrat's laws, detailed-balance and their breakdown

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By employing exhaustive lists of personal income and firms in Japan, and also large firms in European countries, we show that the upper-tail of the distribution of income and firm size has power-law (Pareto-Zipf law), and that in this region their growth rate is independent of the personal income or firm size (Gibrat's law of proportionate effect). In addition, detailed balance holds in the power-law region; the empirical probability for an individual (or firm) to change its income (or size) from a value to another is statistically the same as that for its reverse process in the ensemble. We prove that Pareto-Zipf law follows from Gibrat's law under the condition of detailed balance. We also show that the distribution of growth rate possesses a non-trivial relation between the positive side of the distribution and the negative side, through the value of Pareto index, as is confirmed empirically. Furthermore, we also show that

these properties break down in the non power-law region of distribution, and can possibly do so temporally according to drastic change in financial or real economy.

Power Law Tails in the Italian Personal Income Distribution

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We investigate the shape of the Italian personal income distribution using microdata from the Survey on Household Income and Wealth, made publicly available by the Bank of Italy for the years 1977-2002. We find that the upper tail of the distribution is consistent with a Pareto-power law type distribution, while the rest follows a two-parameter lognormal distribution. The results of our analysis show a shift of the distribution and a change of the indexes specifying it over time. As regards the first issue, we test the hypothesis that the evolution of both gross domestic product and personal income is governed by similar mechanisms, pointing to the existence of correlation between these quantities. The fluctuations of the shape of income distribution are instead quantified by establishing some links with the business cycle phases experienced by the Italian economy over the years covered by our dataset.

Wealth distribution with correlations between links and success

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Different models of capital exchange among economic agents have been recently proposed trying to explain the emergence of Pareto's power law distribution of wealth. Most of these models consider the existence of risk aversion and also a probability that the poorer agent be somehow favored in each exchange. Here we add the hypothesis that the agent's connectivity is not only strongly related to its wealth but also to its success. So, starting from agents placed on a random lattice (i.e. with a gaussian distribution of links), an agent with success in its economic transactions will receive the expected monetary reward, but it will also increase its connectivity, at the expense of other agents (so that the total connectivity remains constant). When the system arrives to a stationary state, it is observed that the wealth distribution has been modified by the dynamics of the lattice, getting closer to a power law for some values of the parameters of the model. As expected, the lattice itself is different from the random initial one. The Gini coefficients are calculated and they show that the reconnection of the lattice induces a kind of "protective screening" of less favored agents, resulting in a distribution of wealth less unequal than on a static network for some values of the parameters.

Unequal Distribution of Wealth and Group Dynamics in an Artificial Financial Market

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This paper revisits the seminal Kareken-Wallace model of exchange rate formation in a two-country overlapping generations world. Following along the lines of Arifovic [Journal of Political Economy, 104 (1996) 510-541] and Lux and Schornstein [Journal of Mathematical Economies, in press] we investigate a dynamic version of the model in which agents decision rules are updated using genetic algorithms. Our main interest is in whether the equilibrium dynamics resulting from this learning process helps to explain the main stylized facts of free-floating exchange rates (unit roots in levels together with fat tails in returns and volatility clustering). Time series analyses of simulated data indicate that for particular parameterizations, the characteristics of the exchange rate dynamics are, in fact, very similar to those of empirical data. However, appearance or not of realistic time series characteristics depends crucially on the number of agents (not more than about 1000). With a larger population, this collective learning dynamics looses its realistic appearance and instead exhibits regular periodic oscillations of the agents choice variables. In the present paper, we investigate two extensions of the model with an emphasis on whether the fading away of realistic time series features for larger populations can be overcome in a setting with additional elements of heterogeneity among agents.

In particular we allow for (i) heterogeneity of wealth by imposing a realistic distribution of resources rather than assuming identical endowments of agents, (ii) subgroup dynamics of the genetic algorithm learning process (inspired by Darwin's continent cycle theory). While the stratification of wealth seems to have a surprisingly small effect on the dynamics, the introduction of a structured population of parallel genetic algorithms appears to amount to an effective decrease of the population (compared to the homogeneous case) and produces realistic time series properties for larger wealth numbers of agents.

From Bell Curve to J Curve

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Personal distributions of income and wealth are subject to Static and Dynamic theories.

These two papers will concentrate on Static theories but also look at Dynamic theories in relation to property distribution.

In general, we assume a market economy in which, at the age of ten, individuals have become endowed with varying attributes, created by both Nature and Nurture, which contribute directly and indirectly (via education, parental property or lack of these) to their future earning power.*

The Three Distributions

1. Bottom Distribution: the market rewards individuals according to single attributes (eg. physical strength or specific physical skill), which are normally distributed, subject to left truncation and right-side negative skewness.
2. Middle Distribution: individuals are rewarded according to multiple attributes which are normally distributed. The attributes, however, combine multiplicatively, thus generating log-normal distribution.
3. Upper Distribution: individuals are rewarded according to their 'responsibility', i.e, by the number of other individuals whose productivity is directly or indirectly influenced by the observed individual. It will be shown that this system directly generates a Pareto Distribution.

It follows that the Gini Coefficient of a society is,

(1) $G = G(C, V, A, e, f, P) \qquad C', V', A', e', f' > 0; P' < 0.$

- where C = Coefficient of Variation of low-level attributes, V is the Variance of the log-normal distribution generated as in (2), and A is the reciprocal of Pareto's Alpha, which last is in turn controlled by institutional factors determining the market forces influencing the intensity of the relation between responsibility and reward; f is the ratio of mean upper income to mean middle income and e is the inverse of the corresponding ratio for lower income. P is a measure of the redistributing effect, if any, of Public Policy.

Dynamic Theories: important processes, such the Gibrat process, in which

(2) $\log y(t) = \log y(t-1) + m$

- where $m(t)$ may be random or alternatively specifically correlated with $y(t-1)$. Such models can explain the development of economic concentration in both industrial and agricultural societies, without recourse to static theories.

If m is, in fact, random, the process generates a log-normal distribution whose Variance, V, (and hence G) increases linearly over time at the rate $Var(m)$. If $m(t)$ is negatively correlated with $y(t-1)$, V grows to a limit; in the opposite case, V grows explosively.

Distributional Philosophy

(3) $SW = W(y, 1-G)$

SW = Social Welfare, y = average income, pace Sen, derived from the law of Diminishing Marginal Utility.

(4) $y = k + bG; \;\; b > 0$

Production Function - k indicates national resources per capita; b = technical and behavioural coefficient reflecting, (i) productivity of effort (ii) effort-incentive associated with marginal rewards to effort.

(5) $SW = W(k + bG, 1-G)$

Sen shows that among societies with equal k, (3) is equivalent to $SW = y(1-G)$, in which case we have,

(6) $SW = W(k + bG)(1-G)$

This situation can be discussed from Philosophical, Religious or Game-theoretic (Rawls-Harsanyi) points of view.

Equation (1), in combination with (5) or (6) says that Social Welfare is influenced by Public Policy subject to constraint. If $SW^* = SW$ when P (pace Mirrlees) is optimised, the Good Lord has completely decided the Social Welfare of Nations, but, with respect to individuals, he does play dice.

* If parental wealth , which is usally skew-distributed, influences pre-school Nurture, or is genetically correlated with Nature, this assumption will be violated.

Turnover Activity in Wealth Portfolios

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We examine several named subsets of the wealthiest individuals in the US and the UK that are compiled by Forbes Magazine and the Sunday Times. Since we are dealing with named subsets it is possible to calculate the returns that wealth portfolios achieve over time. The data support conventional wisdom of a wealth distribution with power law-distributed right tail, and they allow us to calibrate a statistical equilibrium model of wealth distribution. The model accounts for the power law tail distribution and is also consistent with the observed asymmetric Laplace distribution of portfolio returns. Moreover, with information on the distribution of portfolio returns, the model provides an indicator for how often changes in the composition of the wealthiest portfolios occur—an indicator we call turnover activity. We also calculate a simple mobility measure from the subsets and look at trends in equality, mobility and turnover activity.

Concepts of thermodynamics in Economic Systems I: Lagrange principle and Boltzmann Distribution of Wealth

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In many particle systems of N elements (atoms, people, goods) and k categories (energy level, income class, price level) the most probable distribution is given by the Lagrange principle with constraints,

$$L = T \ln P + E \longrightarrow \text{maximum!}$$

L is the Lagrange function, P is the combinatorial probability of N elements in k categories, $\ln P$ is called entropy of the system. The constraints E are given by the energy or capital E_k required for each category (k),

$$E = \sum_k N_k E_k$$

T is the Lagrange parameter. In atomic systems it is the mean energy per atom or temperature, in social systems it is the mean income per person (GNP per capita), in markets it is the mean price level.

In natural systems the Lagrange function L corresponds to the negative free energy. The function E represents the bonding energy, $\ln P$ the entropy and T the temperature. At low temperatures the atomic system (solid) has strong bonds and is in a collective well ordered state. At higher temperatures the system (liquid, gas) has weaker bonds and is in an individual, disordered state. In binary systems the model of chemical interactions leads to solubility or insolubility of materials.

In economic systems the Lagrange function L corresponds to the utility of an economic system. The function E represents the capital of the system. The temperature T may be interpreted by the standard of living (GNP per capita). At low standard of living the economic structure will be in a collective and hierarchic state, at a higher standard of living the economy will become individual and capitalistic.

At equilibrium the derivative of L with respect to N_k is zero,

$$0 = \partial L / \partial N_k$$

and leads to the Boltzmann distribution

$$N_k = A \exp(-E_k / T)$$

This Boltzmann distribution may be observed in all systems, in the distribution of atoms in gases, in the distribution of people in classes of wealth, in the distribution of goods in price levels. This is demonstrated by wealth distributions of Germany 1993 and other countries, the distribution of income in Germany between 1950 and 1975 and other countries and the distribution of goods like automobiles in different price classes in Germany 1998.

However, a closer look on wealth shows two main levels of wealth in the distribution of world wealth as well as in national wealth. This is due to the Carnot process of economic growth and will be discussed in a second paper.

Concepts of thermodynamics in Economic Systems II: The Carnot Process of Economic Growth and Wealth Distribution

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Economic growth is a differential process of capital (K) and labor (L) and may be calculated by the non exact differential form $\delta g(K, L)$. The closed integral will be non zero and leads to economic growth. The integral may be split into two parts, production (Y) and consumption (C), both will depend on the production process. According to the laws of calculus the non exact differential δg may by transformed into an exact form by an integrating factor (T): $\delta g = T \delta S$. The new function S is called entropy. Depending on the economic system (market, country) T is a mean value of capital, a mean price level or a standard of living.

The closed integral of economic growth may be carried out at constant values of T or constant values of S and reveals that economic production corresponds to a Carnot process of motors and generators. Motors and economies both run on oil! The Carnot process explains the existence of rich and poor in economic processes. Like a motor that creates two different temperatures for inside and outside, or a freezer that is cold inside and warm outside, an economy creates e.g. rich people inside a company and poor people outside. A company is a Carnot machine that has to pay little to the workers and ask for much from the customers in order to make profit and survive. This leads to two level system of income within countries and within the world.

The production function of the Carnot process is determined by the entropy of mixing different production factors, $Y = T(x \ln x + y \ln y + z \ln z)$. In contrast to the Cobb Douglas function of standard economics the new function Y has may advantages: (1). It defines elasticity by the production factors x, y, z. (2). The new production function leads to a Boltzmann distribution, which may be found in distribution of economic goods like cars, in the distribution of wealth and other applications. It replaces the Pareto distribution, which leads to infinite demand at zero price and may not be integrated at the limits of zero or infinite prices. The Boltzmann distribution is more appropriate, it leads to finite demand at zero price, and finite values of the integral at $P = 0$ and $P =$ infinite.

Economic growth and stagnation can be modeled by the Carnot process for different countries, like Japan and USA after World War II, Japan and USA 1980 - 2000, West and East Germany 1990 - 2000, Argentina and USA 1997 - 2001.

Kinetic-gas like models of closed economy markets

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Some analogies between kinetic models of gases and statistical models of closed economy markets are reviewed. In such models, usually consisting of N agents interacting with each other by exchanging money according to a given microscopic random law, the average money plays the role of temperature. Generalized models with a saving propensity in the trades between agents are considered. In these models the analogy with the gas-kinetic model is confirmed by the equilibrium distribution being given by a gamma-distribution, which represents just the general Maxwell-Boltzmann kinetic energy distribution in a gas in D dimensions, where D, the effective dimension of the gas, is related to the effective dimension of the space. We also consider more general models with individual saving propensity and study the static and dynamic correlation between money and saving propensity. The results are analyzed in the light of recent studies of these models based on a Boltzmann equation or master equation approach.

Dynamics of Money and Income Distributions

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The distribution of income or wealth can be shown via numerous examples to have remained broadly unchanged over time for a range of different societies. We review various approaches to modelling these distributions of income in recent years. We then focus on the model of interacting agents proposed and studied numerically by Chatterjee and colleagues (2003). This model allows agents to both save and exchange wealth at random. Closed equations for the wealth distribution are developed using a mean field approximation. We show that when all agents have the same fixed savings propensity, subject to certain well defined approximations defined in the text, these equations yield the conjecture proposed by Patriarca, Chakraborti and Kaski (2003) for the form of the stationary income distribution.

If the savings propensity for the equations is chosen according to some random distribution we show further that the wealth distribution for large values of wealth displays a Pareto like power law tail, i.e, $P(m) \sim m^{1+\nu}$. However the value of ν for the model is exactly unity. Exact numerical simulations for the model illustrate how, as the savings distribution function narrows to zero, the wealth distribution changes from a Pareto form to an exponential function. Intermediate regions of wealth may be approximately described by a power law with $\nu > 1$. But, the tail exponent ν never reaches values of $1.6 - 1.7$ that characterize empirical wealth data. This conclusion is not changed if three body agent exchange processes are allowed. We discuss a number of modifications, such as the inclusion of agent memory and redistribution of income that can lead to more realistic values of the Pareto exponent.

Redistribution and Free Trade in Agriculture: Are they Complementary?

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The purpose of the paper is to look at the welfare effects of trade in agricultural goods in a less-developed country where the agricultural market is controlled by a handful of large farmers. It is shown that the success of trade reform depends upon the distribution of output between large and small farmers and the success of land reform leading to redistribution from the large to the poor depends on trade reform. In other words, if undertaken in isolation, each reform might lead to a fall in welfare, but if jointly undertaken, they will lead to an increase in welfare. Thus the two reforms are complementary.

The Rich Are Different!: Pareto-like power law distributions from wealth dependent asymmetric interactions in asset exchange models

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It is known that asset exchange models with symmetric interaction between agents show either a Gibbs distribution or a condensation of the entire wealth in the hands of a single agent, depending upon the rules of exchange. In this talk we will explore the effects of introducing asymmetry in the interaction between agents with different amounts of wealth. These could be implemented in several ways: e.g, (1) in the net amount of wealth that is transferred from one agent to another during an exchange interaction, or (2) the probability of gaining vs. losing a net amount of wealth from an exchange interaction. We show that, in general, the introduction of asymmetry leads to power laws. Distributing the asymmetry parameter randomly over the entire population of agents results in a wealth distribution very similar to that empirically observed in various societies, with the tail having a Pareto-like power law behavior, while the low-wealth region has an exponential nature.

Blockbusters, Bombs and Sleepers: The income distribution of movies

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The distribution of gross earnings of movies released each year show a distribution that is Gaussian for the most part but having a power-law tail with exponent -3. While this offers interesting parallels with income distributions of individuals, it is also clear that it cannot be explained by simple asset exchange models, as movies do not interact with each other directly. In fact, movies (because of the large quantity of data available on their earnings) provide the best entry-point for studying the dynamics of how "a hit is born" and the resulting distribution of popularity (of products or ideas). In this talk we will explore various interesting features of movie income distribution, and present a network-based model for explaining the same.

Empirical Study and Model of Personal Income

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This paper analyzes empirical personal income distributions and proposes a simple stochastic model to explain it. Using the personal tax returns data in the U.S. and Japan, we clarify the middle income range follows an exponential distribution and the top 1 percent follows an power law distribution. We then propose a minimal two-factor model that reproduce these characteristics. Our model of personal income consists of asset accumulation process and a wage process. The asset accumulation process is multiplicative due to the stationary random asset returns. The wage process is assumed additive, reflecting the productivity heterogeneity. We show that this simple process can successfully reproduce the empirical distribution of income. In particular, the model can reproduce the particular transition of the distribution shape from the middle part which decays exponentially to the tail part with decays in power. This model also allows us to derive the tail exponent of the distribution analytically.

The Economic Mobility in Money Transfer Models

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Mobility like distribution is also an important aspect of the study on income and wealth. To find out the mechanism of the income and wealth distribution, a series of models have been constructed, among which money transfer models attract much attention. In this presentation, we choose four transfer models as typical ones to demonstrate the mobilities by recording the agents' rank time series and by observing the volatility of them. Like the shape of distribution, the mobility is also determined by the trading rule in these transfer models. To compare the mobility quantitatively, an index raised by economists, "the per capita aggregate change in log-income", is employed to measure the value of reranking degree, and we find that the mobility decreases as the saving rate increases in one model. A stratification phenomenon is observed in other one model and the resulting mobility is quite small. It is worth noting that even though different models have the same distribution, their degrees of mobility may be quite different. These findings suggest that the characteristics of mobility should be checked when evaluating this kind of models.

Evidence for the Independence of Waged and Unwaged Income, Evidence for Boltzmann Distributions in Waged Income, and the Outlines of a Coherent Theory of Income Distribution

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Two sets of high quality income data are analysed in detail, one set from the UK, one from the USA. It is firstly demonstrated that both a log-normal distribution and a Boltzmann distribution can give very accurate fits to both these data sets. The absence of a power tail in the US data set is then discussed. Taken in conjunction with detailed evidence from the UK and Japanese income data, a strong case is made for the mathematically separate treatment of waged and unwaged income. The authors present a case for preferring the use of the Boltzmann distribution over the log-normal function, this leads to a brief review of the work of a number of researchers, which shows that a coherent theory for the distribution of all income can be postulated.

Laser Econodynamics: Relieving Poverty by Modifying Income and Wealth Distributions

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The paper starts with the assumption that income and wealth distributions are formed from exponential/ Boltzmann distributions with a power decaying tail.

The "fairness" of such distributions as a reflection of human abilities are discussed.

Parallels with energy distributions found in physical systems are discussed, and how they could be used to construct economic models that might allow alternative overall distributions of wealth and income in society, and so the relief of poverty.

Two-class structure of income distribution in the USA: exponential bulk and power-law tail

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Personal income distribution in the USA has a well-defined two-class structure. The majority of population (97-99%) belongs to the lower class characterized by the exponential Boltzmann-Gibbs ("thermal") distribution, whereas the upper class (1-3% of population) has a Pareto power-law ("superthermal") distribution. By analyzing income data from the US tax agency (Internal Revenue Service) for 1983-2001 [1], we show that the "thermal" part of the distribution is stationary in time, save for a gradual increase of the income temperature (the average income in nominal dollars). On the other hand, the "superthermal" tail is non-stationary, swelling and shrinking with the course of the stock market. We discuss the concept of equilibrium inequality in a society, based on the principle of maximal entropy, and quantitatively show that it applies to the majority of population.

[1] A. C. Silva and V. M. Yakovenko, "Temporal evolution of the 'thermal' and 'superthermal' income classes in the USA during 1983-2001", Europhysics Letters, v. 69, pp. 304-310 (2005).

Statistical mechanics of money, income, and wealth: foundations and applications

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We review foundations of statistical mechanics of money formulated in Ref. [1]. On the basis of analogy between conservation of energy in physics and money in economics, we argue that the equilibrium probability distribution of money in a closed economic system should follow the exponential Boltzmann-Gibbs law. We demonstrate how the Boltzmann-Gibbs distribution emerges in computer simulations of economic models. We also discuss the role of debt, and models with broken time-reversal symmetry, for which the Boltzmann-Gibbs law does not hold. We consider a thermal machine, in which the difference of money temperatures between different countries causes steady flow of money (trade deficit) and allows an intermediary to extract monetary profit from the non-equilibrium condition. We discuss relation with the kinetic Boltzmann and diffusion Fokker-Planck equations and illustrate how a combination of additive and multiplicative random processes can generate the "thermal" and "superthermal" classes empirically observed in income distribution in the USA and other countries.

[1] A. A. Dragulescu and V. M. Yakovenko, "Statistical mechanics of money", The European Physical Journal B, v. 17, pp. 723-729 (2000).

A stochastic trading model of wealth distribution

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We develop a stochastic model where the poorer end of the society engage in two-party trading while the richer end perform trade with gross entities. Using our model we are able to capture some of the essential features of wealth distribution in societies: the Boltzmann-Gibbs distribution at the lower end and the Pareto-like power law tails at the richer end. A reasonable scenario to connect the two ends of the wealth spectrum will be presented. Also, an analytic approach to obtain different power law exponents will be given. Furthermore, a link with the models in macroeconomics is also attempted.

Short presentations

Some Evidences of Power-Law Distribution in Indian Capital Market

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We examine whether power-law distribution emerges in Indian capital market similar to the wealth distribution of individuals in an economy and how the exponent of power-law behaves at various high wealth levels as well as how the relative growth and decline of firms over time affects the distribution. Using a database of 500 companies having highest market capitalization in Indian capital market, we observe that the power-law distribution emerges at the top 10% wealth level of the firms. As we go down on the ranks, the power-law becomes less conspicuous, which is consistent with other research findings. The value of the exponent is observed to compare well with respect to wealth distribution of the individuals in the economy. The behaviour of exponent in respect of high growth and declining firms is also investigated. We observe that the value of exponent in respect of high growth firms does not change over time, but the value reduces during the same period in the case of firms showing negative growth. The t-test shows that as the rank difference is high and statistically significant for the declining firms as against the growing firms, it has expectedly caused the exponent to change its value.

Funds Management by Banks in India: Solution to a persisting optimization problem

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Even after liberalization of financial markets and a consequential shift in the role of commercial banks, extending working capital facilities to industry and trade continues to be a major activity for the banking sector in India. Working capital loans are granted in India normally in the form of cash credit. Under this facility a borrower can avail of any amount of credit not exceeding the sanctioned limit and pays interest based on daily actual balances outstanding in the loan account. While some fluctuation in level of utilization based on actual requirement is only to be expected, the actual level of fluctuation often is much beyond what is projected by borrowing companies at the time of submission of loan proposals. This creates a serious problem of fund management for banks and the solution lies in ensuring that borrowers take the task of forecasting their requirements seriously. The Tandon Committee, set up by the central bank of the country under the chairmanship of Mr. P. L. Tandon for going into various aspects of follow-up of bank credit, examined, inter alia, this issue and recommended that working capital loans should be bifurcated into a fixed demand loan component and a fluctuating cash credit component carrying differential rates of interest. The bifurcation should be worked out based on projected fund requirement in such a way that any departure from target results in payment of penalty by borrower in the form of additional interest. In other words, the process of bifurcation should ensure a minimum value for the overall annual interest burden for a borrower in case it adheres strictly to its projections. The problem is thus basically one of optimization.

A subsequent review suggested that this particular recommendation, though accepted, has not been implemented. This is due to absence of any methodology for its implementation. But, the problem, being a real one, lingered and there have been attempts to mitigate it through ad-hoc thumb rules.

This article shows how the real life problem can be solved with the help of some simple tools used in Physics. Correctness of the result is also demonstrated by considering some illustrative cases. The method employed is quite simple and the result is exact. Moreover, the final prescription is easy enough to be implemented by any practical banker.

Stochastic model of income distribution

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We propose a stochastic model of evolution of income in a society of economic agents. In the model, an economic agent (may be an individual or a group), can be in two states: inactive and active. Transitions between the states occur at random time intervals with activation and deactivation rate constants k_a and k_d respectively. In the inactive state, addition to an agents income occurs at the rate b_w. In the active state, there is an additional rate, j_w, for the increase in income, i.e., the total rate at which income increases is $b_w + j_w$. In any state, active or inactive, income

diminishes at a rate proportional to the current income, the proportionality constant γ_ω is the rate constant for income decay. In the stochastic model, the only stochasticity is associated with the random transitions between the inactive and active states of an agent. Let $P(X,t)$ be the probability density function describing income distribution with X representing income. We write down an equation describing the time evolution of $P(X,t)$. In the steady state, the density function has the form of a beta distribution. Income distributions for the poor, middle and rich classes of society are further obtained separately. In economic literature, beta distribution and its generalizations, namely, the generalized beta distribution functions have been proposed to describe the income distributions of different economic societies. Our stochastic model provides a simple basis for the appearance of beta-type distributions.

Role of extreme events and nonlinearity in wealth distribution

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We attempt to investigate the problem of wealth distribution from the viewpoint of asset exchange. A simple asset exchange model called Yardsale model gives wealth condensation asymptotically. We show that if few members in population choose different rules for asset exchange, we are able to reproduce a power law tail. We also study a nonlinear variant of Yardsale model in this context. In another problem, we study a population which exchanges assets by Yardsale rule alone. However, if extreme events like disasters, revolution, bancruptcy or taxation (essentially anything that will not allow wealth condensation as a stable steady state) are introduced, the asymptotic distribution under such conditions also has a power law tail.

Continuously tunable Pareto exponent in a random shuffling money exchange model

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A group of N traders trade by the method of pairwise conservative money reshuffling. In a trade, two traders i and j $(1 \leq i,j \leq N, i \neq j)$ having moneys m_i and m_j are selected with probabilities depending on their individual wealths like, $\pi_i \propto m_i^\alpha$ where α is a pre-assigned m independent constant parameter. On the board both the traders put all their monies and just reshuffle their total money $m_i + m_j$ by randomly partitioning with uniform probability. In the stationary state the money distribution is observed to follow a Pareto distribution with a power law tail: $P(m) \sim m^{-(1+\nu)}$. Our numerical results show that the Pareto exponent is continuously tunable and $\nu = \alpha - 1$ for all values of $\alpha > 0$.

How the rich get richer

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Our model describes wealth aggregation amongst conglomerates. Interacting conglomerates compete for growth as agents in this 'winner-takes-all' model; for finite assemblies, the largest conglomerate always wins. In mean-field, our model exhibits glassy dynamics, with two well-separated time scales, corresponding to individual and collective behaviour; the survival probability of a conglomerate eventually falls off according to a universal law $\ln t^{-1/2}$. In finite dimensions, this glassiness is enhanced: the dynamics manifests both ageing and metastability. Pattern formation is manifested in each metastable state: some conglomerates survive forever, and can be vastly rich, provided each has a sphere of influence totally isolated from the others.

Wealth Distribution in the Boltzmann-Pareto framework

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In every economic system a set of agents, called economic agents, participate in the process of an operation just as a set of particles (e.g, a collection of gas molecules forming a box of gas) join together to form a physical system. The physical system is described by a set of parameters which limit its operation. For example, a gaseous system may be described by the size of the box in which the gas moves. The length, breadth and height (i.e, volume) fix the boundary and hence the parameters. So also the production system of an economy may be described by the types of activities (services, industry, agriculture etc.) in which economic agents (like gaseous particles) take part. An economic agent (a person, say) may have access to one or all the activities and acquire the capabilities (energy) to enter into the market. The income-capability hence differs from persons to person in the system. To put it otherwise the total income-capability of the economic system at a time point is distributed among the agents (persons) depending upon their actual participation in the diverse processes of production (different types of activities are the parameters).

But how is the distribution process in the economy? Does it follow some regularity? Is it a statistical regularity? Can the statistical regularity be approximated by some functional process? Does the process have some inherent tendency to concentrate? How does the process behave over time? Is there any close similarity between the economic process and the physical process? To be more precise is the Pareto process, for example, closely related to the Boltzmann process? Has the physical concept of entropy any use in the process of wealth/income/asset distribution? The present paper attempts to examine all these issues in the econophysics framework, using India's income data during 1990-2000.

Poster presentations

Age Dependence in Networks

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We consider a growing network in which an incoming node gets attached to the i^{th} existing node with the probability $\Pi_i \propto K(k_i)T(\tau)$, where $K(k_i) \sim {k_i}^\beta$ and $T(\tau) \sim \tau^\alpha$, k_i being the degree of the i^{th} node and τ its present age. We find the phase diagram in the $\alpha - \beta$ plane. While the network shows scale free property only along a curve, small world property exists over a large region in the phase diagram. The degree distribution also shows interesting features in different parts of the phase diagram. We also discuss a real world network, viz, the citation network, where the age of the nodes plays an important role in deciding the attachment probability of the incoming nodes. We observe here that very old papers are seldom cited, while recent papers are cited with greater frequency. We find out the distribution $T(t)$ of t, the time gap between the published and the cited paper. For different sets of data, we find a universal behaviour: $T(t) \sim t^{-0.9}$ for $t \leq t_c$ and $T(t) \sim t^{-2}$ for $t > t_c$ where $t_c \sim O(10)$. We then analyse the results of the model system in light of the results obtained for the real network.

The Dynamic Character of the Transfer Model with Saving

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To explore the mechanism behind the distribution, a series of transfer models are constructed basing on the analogies between money transfer in business and energy transfer in molecule collision among which the model with uniform saving rate draws much attention. Many efforts are put into to find out the exact mathematic presentation of the static distribution. However, investigating the static distribution only cannot provide the whole picture of the dynamic mechanism behind the distribution. For this aim, some researchers observed dynamics of the models: the money circulation and mobility. If the distribution can be take as the cross section of the system, the circulation and the mobility are the lengthways sections which are helpful for us to expose mechanism of distribution. And investigating the two dynamic characteristics is also meaningful to economics research. Monetary circulation is the dynamics about how the money moves in the economy. Based on the dynamics, the monetary velocity, an important macroeconomic variable could be discussed at the micro-level. While mobility is the dynamics about how the people move in the economy. It is an indispensable supplement to distribution in the study on inequality.

The purpose of this paper is to show how the saving behavior affects the dynamic characteristic of the transfer model with uniform saving rate. From the simulation results, it can be seen that the bigger the saving rate, the slow money circulates and the slow agents mobile. There is something interesting should be noted. We find that the mobility index increases when the sampling time interval increases and reaches the same maximum no matter what value the saving rate is.

Exact maximum likelihood estimation of stationary Arma models

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Since the Normal distribution is symmetric, stationary Arma Gaussian models are not adequate for data exhibiting strong asymmetry. Many time series encountered in practice are non-Gaussian. This paper illustrates for a first order auto regressive and first order moving average model with nonconsecutively observed or missing data, the closed from the exact likelihood function obtained, and the exact maximum likelihood estimation for a stationary Arma(1, 1) model with incomplete data asymmerty with zero-mean process:

Xt = j Xt -1 + Nt - q Nt -1

where j and q are real parameters, and Nt is the Gaussian white noise with mean 0 and variance $s2$.

Random trading market: Drawbacks and a realistic modification

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We point out some major drawbacks in random trading market models and propose a realistic modification which overcomes such drawbacks through ‘sensible trading’. We apply such trading policy in different situations: (a) Agents with zero saving factor (b) with constant saving factor and (c) with random saving factor. In all the cases the richer agents seem to follow power law in terms of their wealth (money) distribution and this is consistent with Pareto’s observation.

Is small-world network disordered?

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Recent renormalization group results predict non-self-averaging behaviour at criticality for relevant randomness. Contrary to this expectation, strong self-averaging behaviour is found in the critical region of a quenched Ising model on an ensemble of small-world networks, despite the relevance of the random bonds at the pure critical point.Single realisation finite-size data of various physical quantities show as good a data collapse (finite-size scaling) as the average.

Statistical Studies on the Price Index of Bombay Stock Market

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We have carried out extensive studies on the daily close price index of the Bombay Stock Exchange (BSE) for the period of 1997 to 2004. To reveal the exact scaling behavior of the BSE price index we made use of four newly developed robust methods of scaling analysis namely (i) Finite Variance Scaling Analysis (ii) Diffusion Entropy Analysis (iii) Detrended Fluctuation Analysis and (iv) Hurst Rescaled-Range Analysis. We have also performed recurrence analysis on the BSE price indices. Our analysis clearly revealed the statistical independence i.e, the absence of scaling in the BSE price index.
