Perturbative and Non Perturbative Aspects of Lattice Quantum Chromodynamics

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A thesis submitted to the
Board of Studies in Physical Sciences
In partial fulfillment of requirements
For the Degree of
DOCTOR OF PHILOSOPHY
of
HOMI BHABHA NATIONAL INSTITUTE

September, 2014
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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree/diploma at this or any other Institution/University.

Abhishek Chowdhury

Abhishek Chowdhury
DEDICATIONS

Dedicated to my mother.
ACKNOWLEDGEMENTS

Looking back through my journey at Saha Institute, I am indebted to a great many people for teaching, guidance, help and encouragement throughout in carrying out the work presented in this dissertation and as well as in the preparation of this document. This work could not have been completed without their support.

First and foremost among them is my advisor Prof. A. Harindranath with his vast wisdom and knowledge, his cheerful demeanor, his willingness to be accessible at all times, and his penchant for being tough when required all make him as close to a perfect advisor as there can be. I have learned something from each and every conversation with him, be it about mathematics, physics, politics, literature or even most interesting spirituality - I wish to thank Harindranath for the treasure trove of memorable moments.

During my thesis, I had the chance to collaborate with a number of people. First of all, I am also thankful to Prof. Asit Kumar De and Prof. Pushan Majumder for their active collaboration with me and for their valuable suggestions in different time of my research works.

I have to mention a name Santanuda (Dr. Santanu Mondal) who not only collaborated with me but also spent a valuable moment in my life. I still remember all those days, where I had several valuable discussions with him that often forced me to think more carefully about physics part of the problems. I have to admit that those discussions helped me to understand the subject of LQCD in a better way.

I would like thank to one of my active collaborators, Jyotirmoyda (Prof. Jyotirmoy Maiti) for his constant support and guidance. The thing I admire most in Jyotirmoyda is his attention to detail and I learned from him that the style of writing numerical codes, is as important as understanding the theory part of the codes itself. If I ever get any compliments about my technical code writing skills, then it should go to Jyotirmoyda. Jyotirmoyda is one of the nicest people I ever met and he has helped me with my personal problems as
my elder brother. Again, I have to admit that without his mentoring, I would not have finished this thesis.

In the thesis, all the numerical calculations are carried out on the Cray XT5 and Cray XE6 systems supported by the 11th-12th Five Year Plan Projects of the Theory Division, SINP under the DAE, Govt. of India. For the prompt maintenance of the systems and the help in data management, I want to thank Mr. Richard Chang.

On a personal level, I would like to thank my family and my parents for their love and for their encouragement. They had a strong belief and confidence in me, for that, I shall forever be grateful to them. Most importantly, I was so fortunate to enjoy the social environment, support and love from my family in all these years. I am deeply indebted to them.

During my stay at Saha Institute I had the opportunity of sharing my way with many a people whose company made the life most enjoyable. Special thanks to Prof. Munshi Golam Mustafa (he has helped me quite a lot in many situations), Susnatadi (Susnata Seth) and Susmitadi (Susmita Kundu), and Santanuda (Santanu Mondal). I would like to thank all my contemporary boarders of Meghnad Saha Hostel for providing me a memorable phase of life. It is a mammoth task to individually mention the names of my numerous friends (especially Ujjal Gayen, Subir Biswas and Mahatsab Mondal) whose friendship I cherish. I will also never forget the hospitality I enjoyed here in the hostel. My hearty thanks to all the hostel staffs such as Sureshda, Saktida and Somnathda. Last, but definitely not the least, I want to convey my sincere thanks to all the members of theory division with special mention to Avirup Ghosh, Atanu Kumar, Satyajit Seth, Najmul Haque and also teaching section, library section, computer section and post M.Sc. friends for their kind help and cooperation.
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