The continuum shell model description of the ^{17}F(p,γ)^{18}Ne radiative capture reaction

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Plan of the talk

I. Introduction

Motivation for the work

II. Shell Model Embedded in the Continuum

Shell Model and how to treat the continuum

The spectra of ^{18}Ne

III. ^{17}F(p,γ)^{18}Ne radiative capture

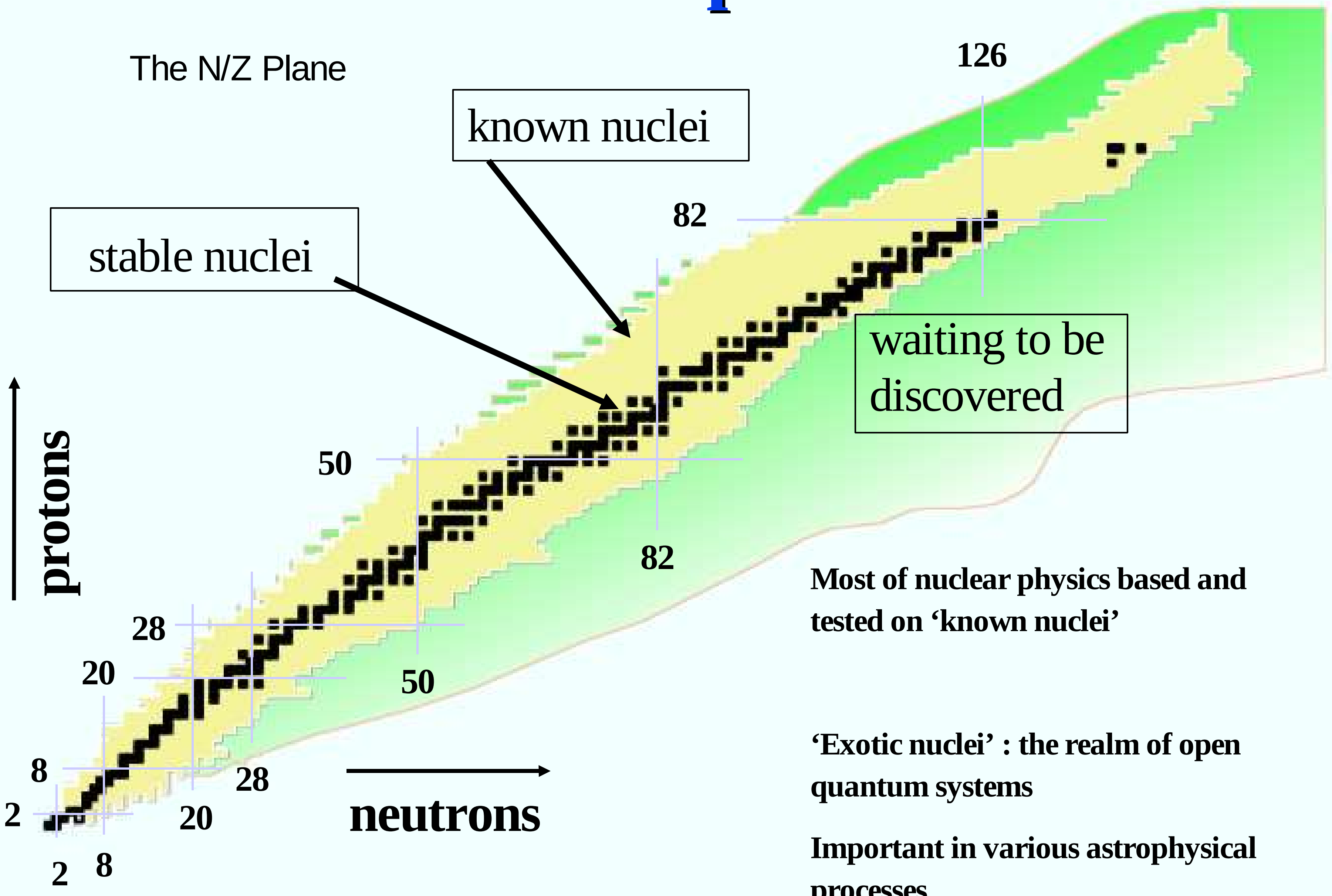
Capture from both the ground and excited states of ^{17}F

The astrophysical S-factors, reaction rate

IV. Indirect Methods in Nuclear Astrophysics

Nuclear Landscape

The N/Z Plane



known nuclei

stable nuclei

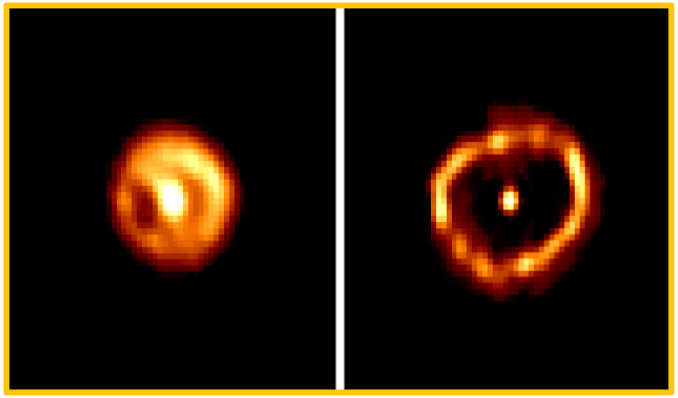
waiting to be discovered

Most of nuclear physics based and tested on ‘known nuclei’

‘Exotic nuclei’ : the realm of open quantum systems

Important in various astrophysical processes

Explosive stellar nucleosynthesis



Nova Cygni Erupted 2/92

Left : 5/93

Right : 6/94

http://hubblesite.org

- Novae, X-ray bursts, Supernovae type 1a

- Binary system – compact object (white dwarf or neutron star) and main sequence or red giant star

Nuclei far from stability (radioactive nuclei) are involved

NOVA

- Accretion of hydrogen rich material on surface of white dwarf that had C burning

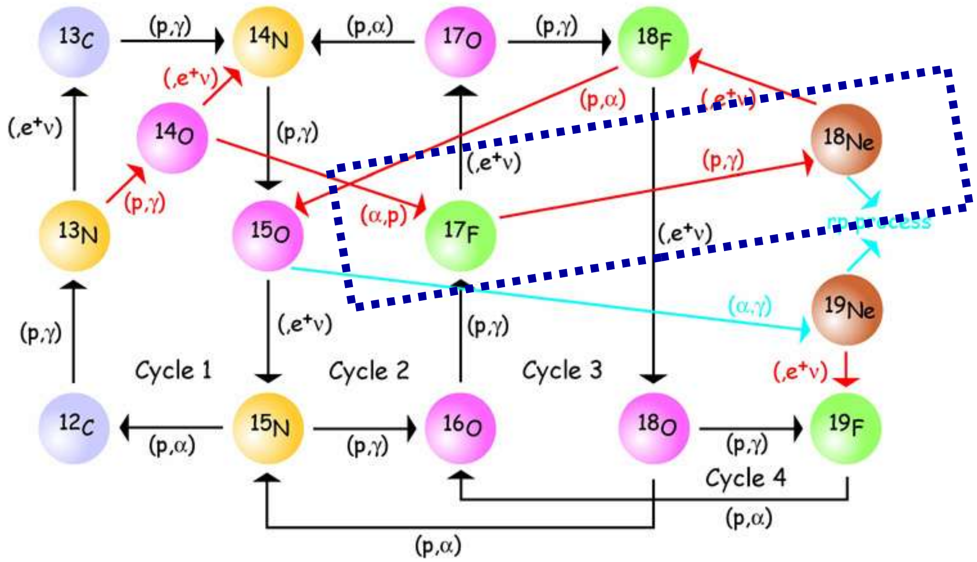
- Thermonuclear runaway – lots of energy

- High temperatures and short timescales

Importance of the ^{17}F(p,γ)^{18}Ne radiative capture reaction

CNO

cycle



Small ^{17}F(p,γ)^{18}Ne reaction rate

- favour production of ^{15}O, ^{15}N
- explains overabundance of ^{15}N in nova ejecta?

D.W. Bardayan *et al.* PRC **62** (2000) 055804

S. Parete-Koon *et al.* ApJ **598** (2003) 1239

Large ^{17}F(p,γ)^{18}Ne reaction rate

- could alter ^{18}F/^{17}F abundance ratio
- breakout from the CNO cycle; subsequent routes of nucleosynthesis

Multiconfigurational Shell Model (1953)

Closed quantum system

→ all nucleons are in bound orbits

Open quantum system

Gamow Shell Model (GSM) : (2002)

→ complex energy plane:

bound states, resonances and complex energy scattering states

→ no restriction on the number of particles in the continuum

(N. Michel *et al* , 2002)

Continuum Shell Model (CSM): (1976)

→ shell model with the real energy continuum:

bound states, resonances and real energy scattering states

→ only one nucleon in the continuum

→ realistic applications with Shell Model Embedded in the Continuum

(K. Bennaceur *et al* , 1999)

Shell Model Embedded in the Continuum (SMEC) : The essential steps

- division of Hilbert space in different subspaces: (H. Feshbach)

$Q = [A]$

$P = [A-1] \otimes [1]$

$T = [A-2] \otimes [2]$

…… …..

nucleons in (quasi) bound states

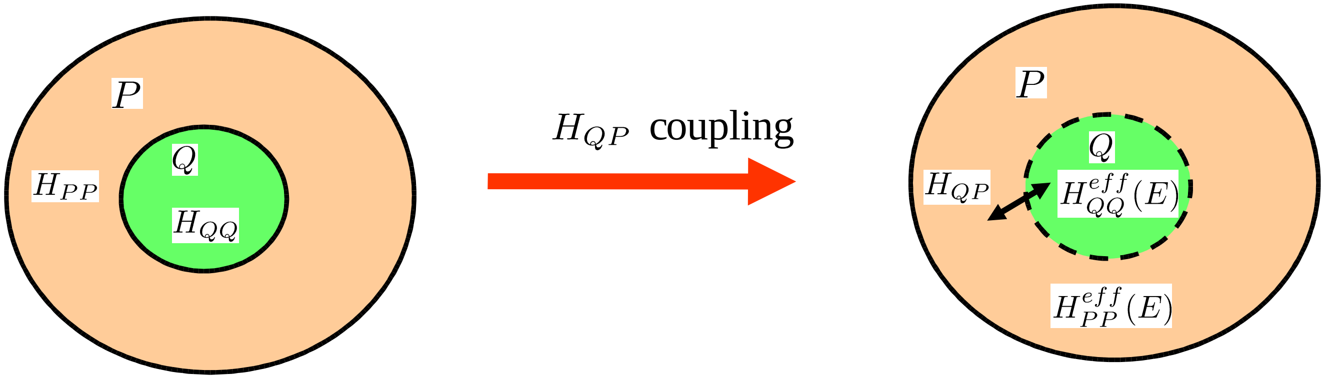
nucleon(s) in continuum states

- completeness relation:

$$Q + P + T + \ldots = I_d$$

- choice of the subspaces according to the considered case

$$Q + P = I_d$$



Closed Quantum System (CQS)

Open Quantum System (OQS)

- $H_{QQ} \equiv H_{SM}$ → shell model effective interaction → $H_{QQ}^{eff}(E) = H_{QQ} + H_{QP}G_P^{+}(E)H_{PQ}$

correction due to continuum coupling

- $H_{PP} \equiv H_{cc'}$ → coupled channel hamiltonian → $H_{PP}^{eff}(E) = H_{PP} + H_{PQ}G_Q^{+}(E)H_{QP}$

Q subspace : Solve the SM problem. Get SM eigenstates $|\Phi_i\rangle$

P subspace : Solve for scattering states. Get solutions $|\xi\rangle$

Using $P + Q = I_d$

continuation of SM state in P : $|\omega_i\rangle$

- diagonalisation of $H_{QQ}^{eff}(E)$ in the shell model basis $|\Phi_i\rangle$

$$\langle\Phi_i|H_{QQ}^{eff}(E)|\Phi_j\rangle = E_i^{SM}\delta_{ij} + \langle\Phi_i|H_{QP}\underbrace{G_P^{(+)}(E)H_{PQ}|\Phi_j\rangle}_{|\omega_j\rangle}$$

H_{QQ}^{eff} is hermitian below the particle emission threshold and non-hermitian above it

eigenvalues : $\tilde{E}_i(E) - \frac{1}{2}i\tilde{\Gamma}_i(E)$

Above particle emission threshold

$$\tilde{E}_i(E) = E, \quad \tilde{\Gamma}_i(E) = \Gamma$$

Below particle emission threshold

$$\tilde{E}_i(E) = E, \quad \tilde{\Gamma}_i(E) = 0$$

Total wavefunction : $|\Psi\rangle = |\xi\rangle + (Q + G_P^{(+)}(E)H_{PQ})\dfrac{1}{E^+ - H_{QQ}^{eff}(E)}H_{QP}|\xi\rangle$

non-resonant solution

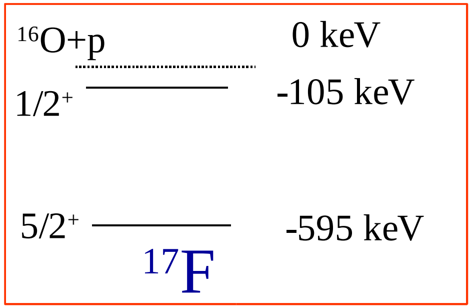
resonant part

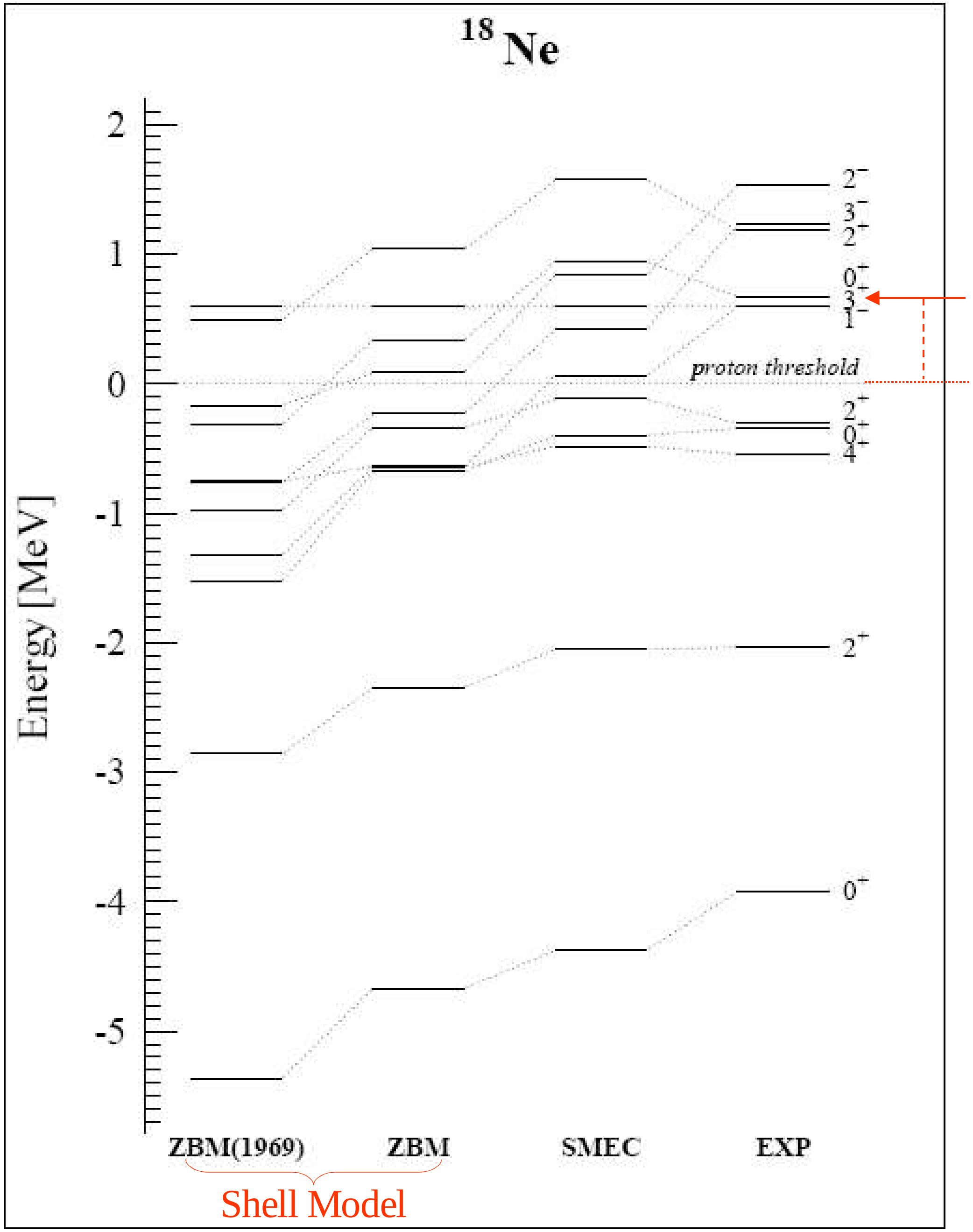
Shell Model Interaction : ZBM (1969)

Zuker-Buck-McGrory, PRL **23** (1969) 983

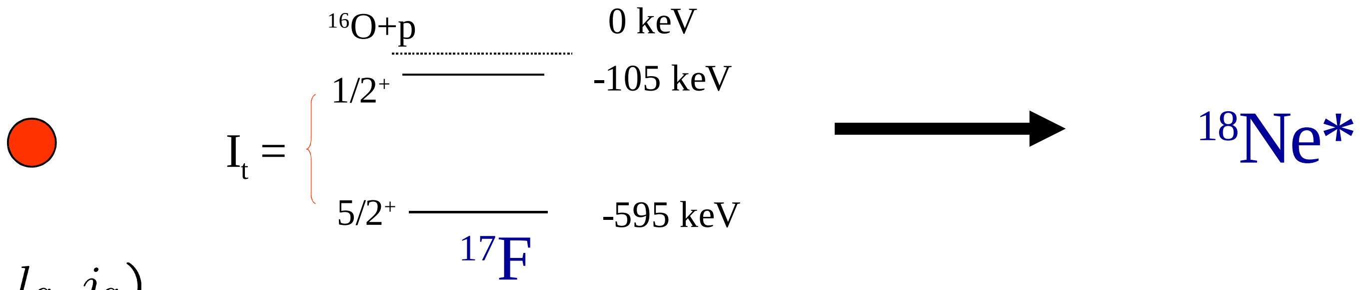
Model Space : ^{12}C core, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$

3π, 2ν : ^{17}F and 4π, 2ν : ^{18}Ne





Radiative capture



(s_a, l_a, j_a)

projectile

target

Target excitation : some amount of energy of the projectile used to ‘excite’ the target

Initial channel : $[{}^{17}\mathrm{F} + \mathrm{p}]^{J_i^{\pi_i}}$

Projectile angular momentum : $j_a = l_a \otimes s_a$

Initial channel spin : $J_i = j_a \otimes I_t$

Parity : $\pi_i = \pi_{I_t} \times (-1)^{l_a}$

Final channel : $[{}^{18}\mathrm{Ne}]^{J_f^{\pi_f}}$

Final channel spin parity : $J_f^{\pi_f}$

$(\underbrace{s_b, l_b, j_b}), \underbrace{I_{t_b}}$

final state target spin

spin and angular momenta of captured nucleon (projectile)

Astrophysical S – factor

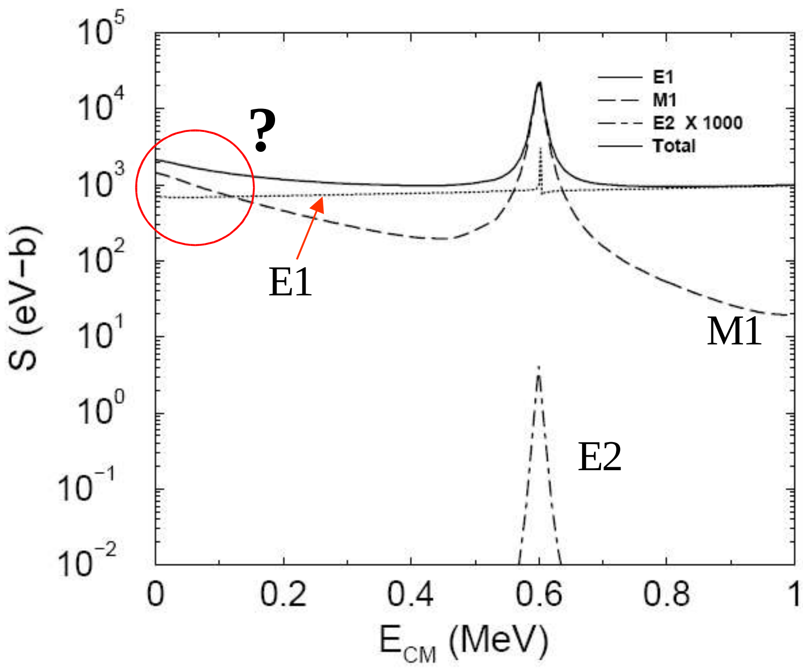
$$\boxed{S(E) = E\, e^{2\pi\eta}\, \sigma(E)}$$

Radiative capture cross section

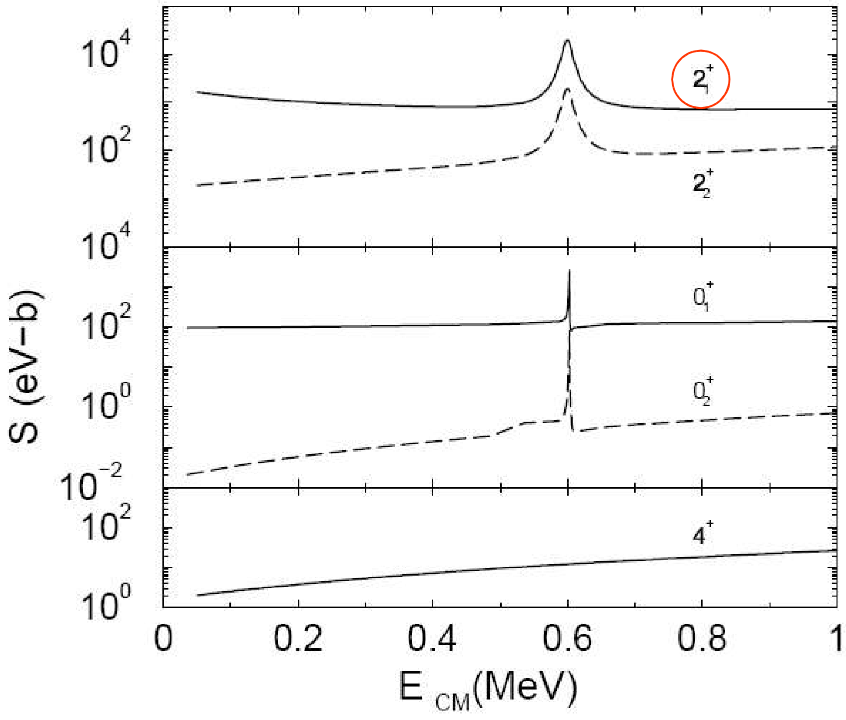
The astrophysical S-factor for ^{17}F(p,γ)^{18}Ne

R. C., J. Okolowicz and M. Ploszajczak, Nucl. Phys. A **764** (2006) 528, *arXiv: nucl-th/ 0509026*

Total contributions from E1, M1, E2



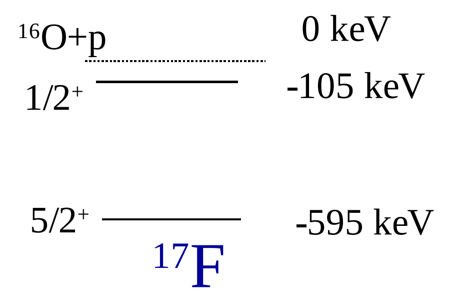
At very low energy $S_{M1} > S_{E1}$



Transitions to different final states of ^{18}Ne

Transitions to $\mathbf{2^+_1}$ dominate

The S_{M1} factor and individual contributions (I)

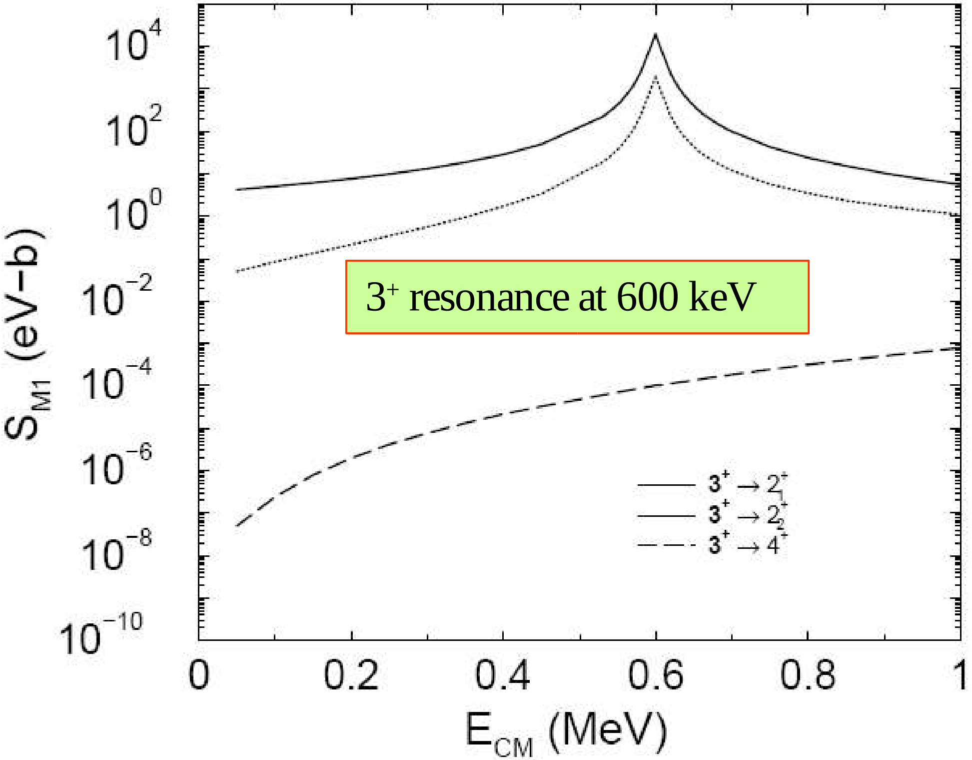


^{16}O+p 0 keV

$1/2^+$ -105 keV

$5/2^+$ -595 keV

^{17}F



3^+ resonance at 600 keV

$$j = l \otimes 1/2 \qquad J_f = j \otimes I_t$$

$$\pi_f = \pi_{I_t} \times (-1)^l$$

l	j	I_t	J_f
$0\ (s)$	$1/2$	$5/2^+$	2^+
$2\ (d)$	$3/2$ $5/2$		
4	$7/2$ $9/2$		
$2\ (d)$	$5/2$ $3/2$	$1/2^+$	2^+

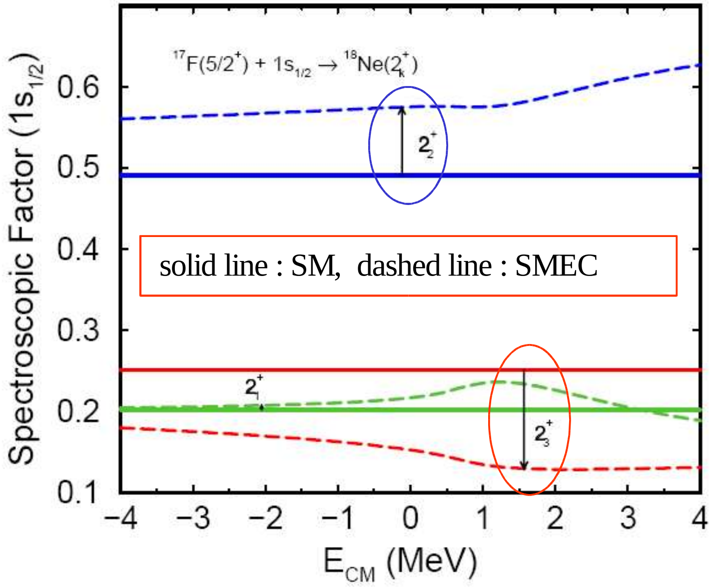
Negligible contribution from the excited state of ^{17}F

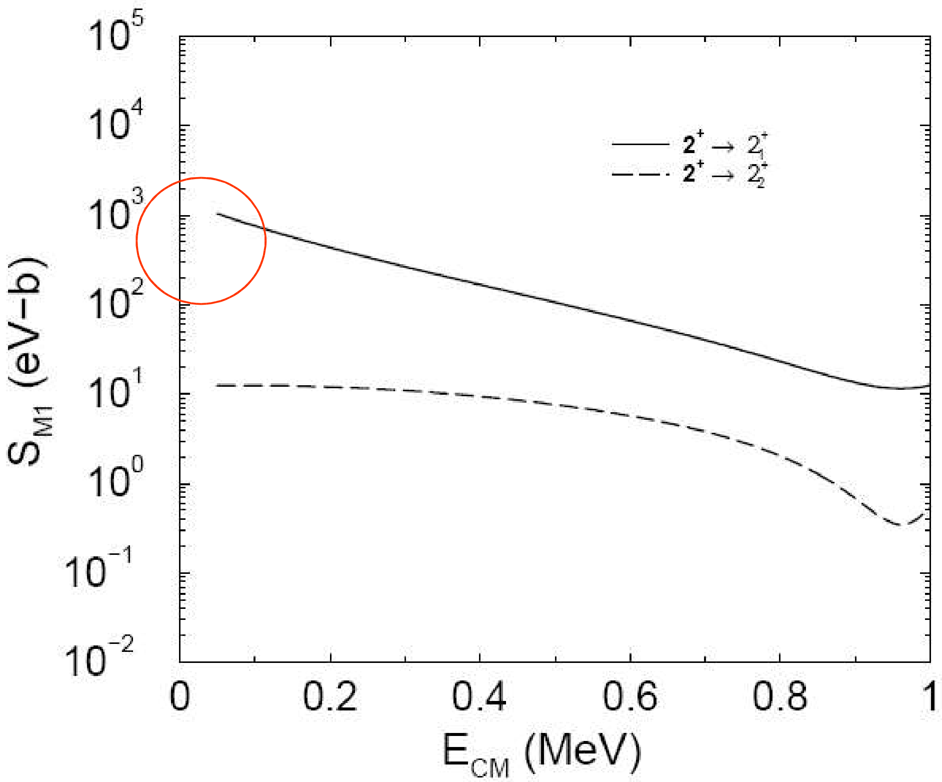
- ext. st. channel opens only at 490 keV.

- ext. st. coupling to 2^+ involves only ***l* = 2** (d - state) waves (protons)

The S_{M1} factor and individual contributions (II)

- Strong interference between 2^+_2, 2^+_3 eigenvalues of H_{QQ}^{eff}
- Significant part of 2^+_3 resonance strength shifted into region of low energy continuum

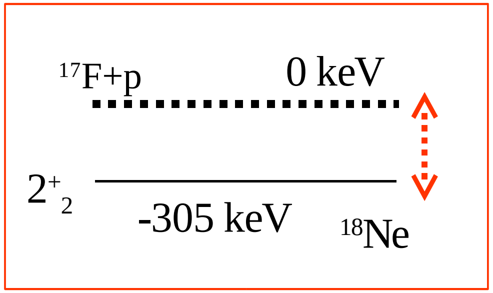




Strong ($2^+_{cont} \longrightarrow 2^+_1$) M1 transition

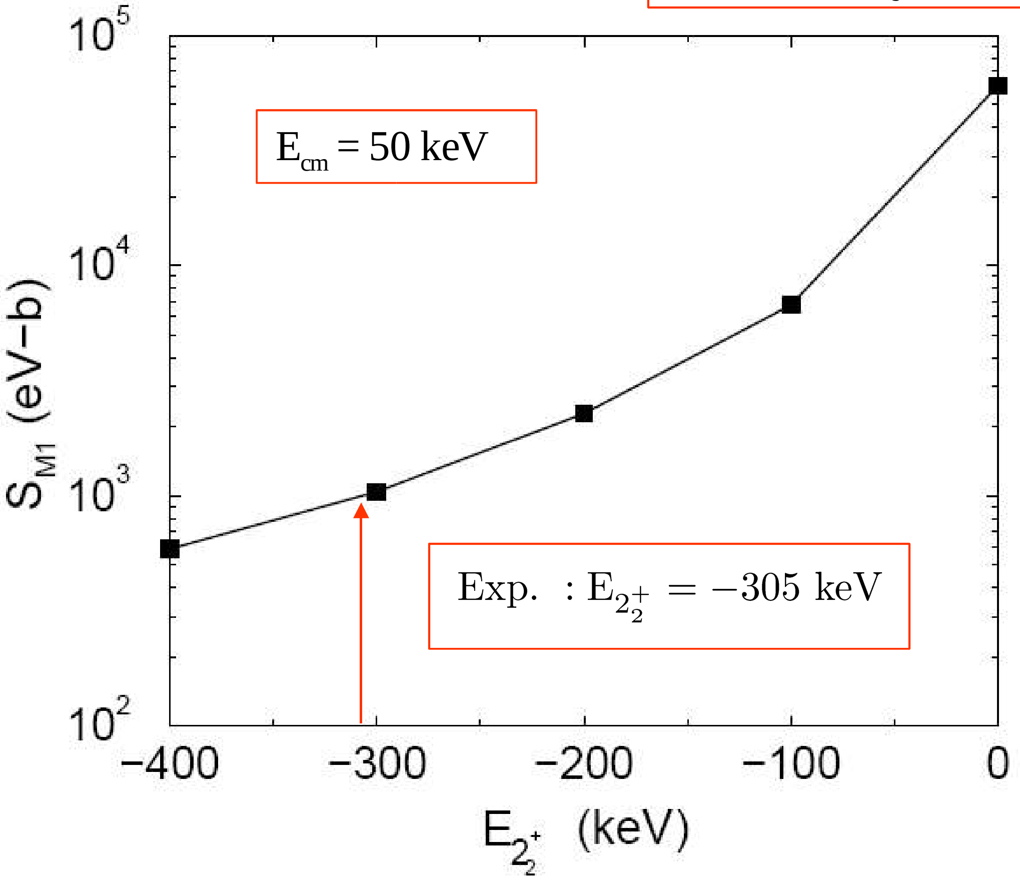
2^+ continuum at low ext. energy is **strongly correlated** by the proximity of a **weakly bound 2^+_2 state** which introduces a resonant-like feature (‘resonant halo’) in the 2^+ continuum owing to its **large ‘s’ – state spec. factor.**

The S_{M1} factor for 2^+ 2^+_1 transition as a function of ‘position of 2^+_2



- Fixed strength of the continuum (*Q*– *P*) coupling
- This effect is not due to taking a larger radial extent of the s.p. wave functions
- Same WS generating radial s.p. wave functions

  (s.p. *s*-state ε_s=-2034 keV)



- Interference effect in Open Quantum Systems

Careful while extrapolating S-factor to low energies just by considering its value at higher energies.

^{17}F(p,γ)^{18}Ne reaction rates

Reaction rate per particle pair :

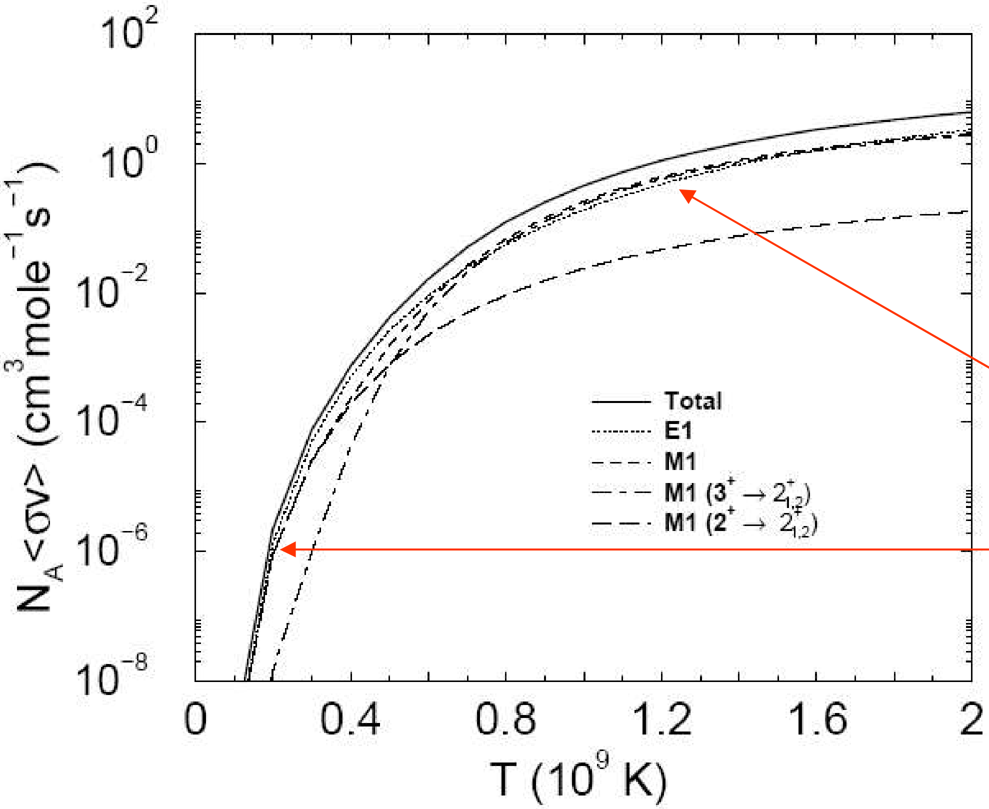
$$\langle\sigma v\rangle = \left(\frac{8}{\pi\mu(kT)^3}\right)^{1/2}\int_0^\infty S[E]exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right]dE$$

$b = (2\mu)^{1/2}\pi e^2 Z_1 Z_2/\hbar$ related to barrier penetrability

T → temperature

k → Boltzmann constant

μ → reduced mass



M1

Above 0.6GK, 3^+ to $2^+_{1,2}$ dominates

Below 0.4GK, 2^+ to $2^+_{1,2}$ dominates (Novae temperatures)

Need to disentangle **E1** and **M1** at very low energies

^{17}F(p,γ)^{18}Ne reaction rates

At low temperatures Gamow window spans low energies < 0.4 MeV

Ext. st. channel opens only at 495 keV

Couples mainly to *d* – wave protons

g.s. + ext. state of ^{17}F

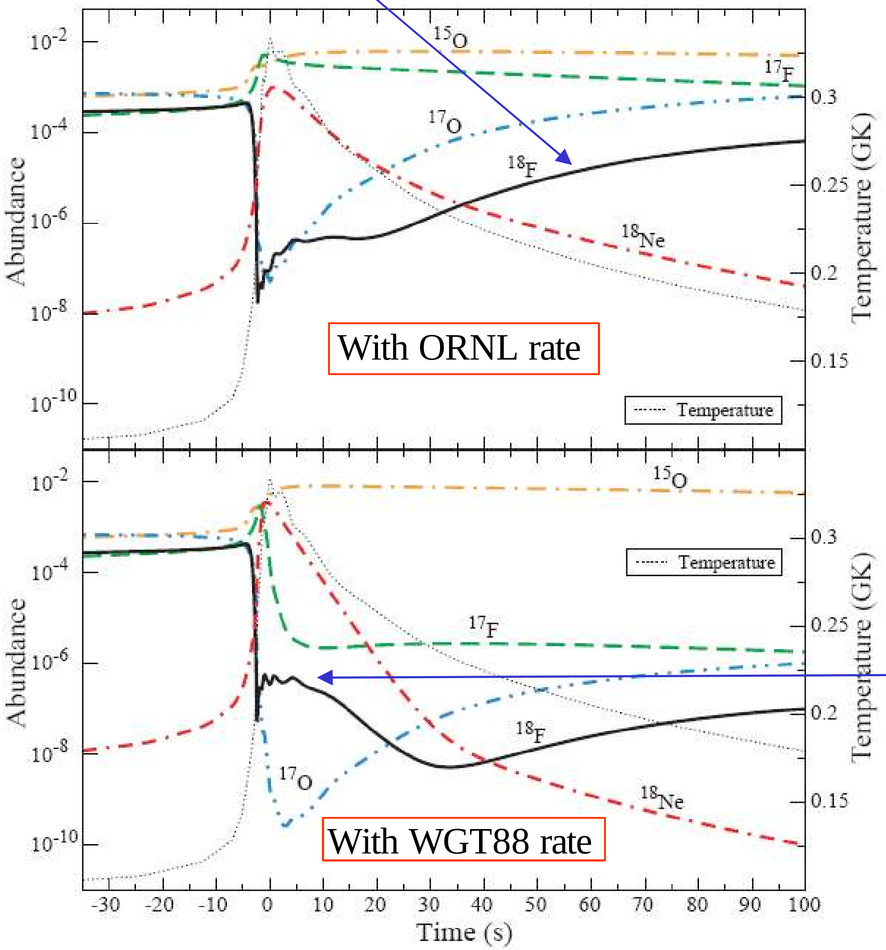
T(10^9K)	SMEC	SMEC (g.s.)	Bardayan et al.
0.1	1.317×10^{-9}	1.317×10^{-9}	$(2.68 \pm 0.38) \times 10^{-9}$
0.2	2.209×10^{-6}	2.209×10^{-6}	$(5.15 \pm 0.75) \times 10^{-6}$
0.3	7.736×10^{-5}	7.736×10^{-5}	$(1.97 \pm 0.29) \times 10^{-4}$
0.4	7.458×10^{-4}	7.453×10^{-4}	$(2.29 \pm 0.40) \times 10^{-3}$
0.5	4.124×10^{-3}	4.116×10^{-3}	$(1.77 \pm 0.49) \times 10^{-2}$
0.6	1.646×10^{-2}	1.640×10^{-2}	$(9.29 \pm 3.28) \times 10^{-1}$
0.7	5.023×10^{-2}	5.000×10^{-2}	$(3.32 \pm 1.30) \times 10^{-1}$
0.8	1.227×10^{-1}	1.221×10^{-1}	$(8.80 \pm 3.61) \times 10^{-1}$
0.9	2.516×10^{-1}	2.503×10^{-1}	$(1.88 \pm 0.78) \times 10^{0}$
1.0	4.516×10^{-1}	4.491×10^{-1}	$(3.43 \pm 1.44) \times 10^{0}$
1.5	2.643×10^{0}	2.629×10^{0}	$(1.97 \pm 0.78) \times 10^{1}$
2.0	6.185×10^{0}	6.155×10^{0}	$(4.62 \pm 1.64) \times 10^{1}$

D.W. Bardayan *et al.* PRC **62** (2000) 055804

Scenarios in Nova nucleosynthesis based on rates of Bardayan *et al.*

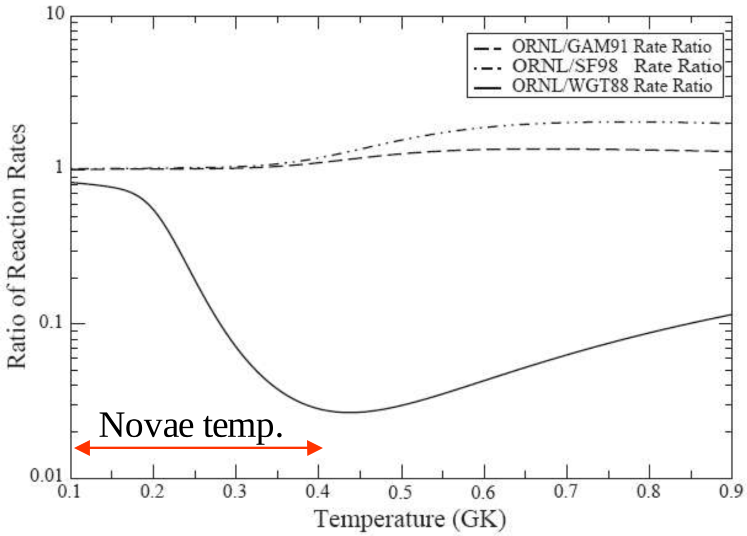
1-D hydrodynamical calculations for outbursts on white dwarfs. (S. Parete-Koon *et al.* ApJ **598** (2003) 1239)

More ^{18}F survives because of cooler temperatures



With ORNL rate

With WGT88 rate



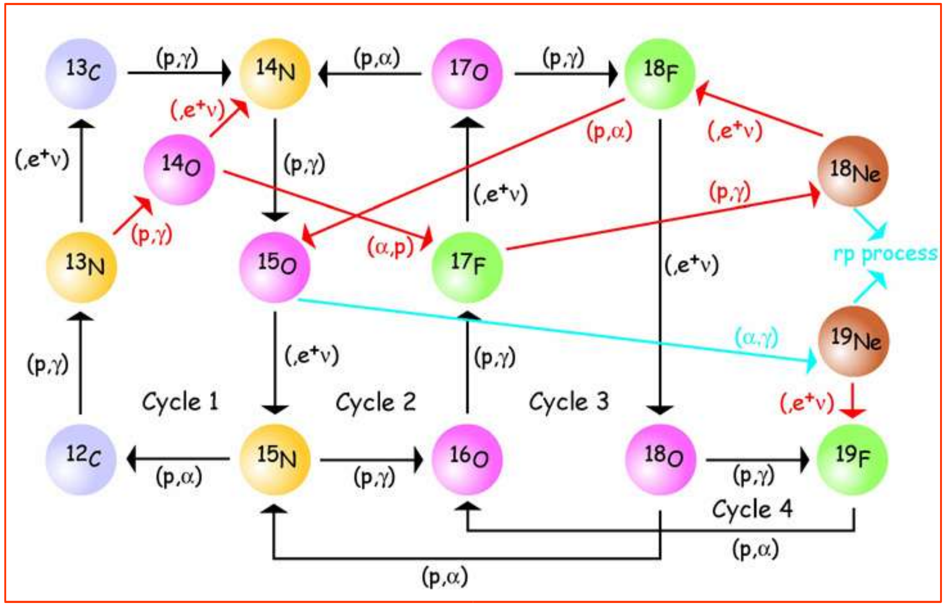
Novae temp.

ORNL : D.W. Bardayan *et al.* PRC **62** (2000) 055804

WGT88 : M. Wiescher, *et al.* ApJ **326** (1988) 384

early surplus of ^{18}F destroyed by ^{18}F(p,α)^{15}O because of high temperatures

Scenarios in Nova nuclosynthesis based on our reaction rate (qualitative)

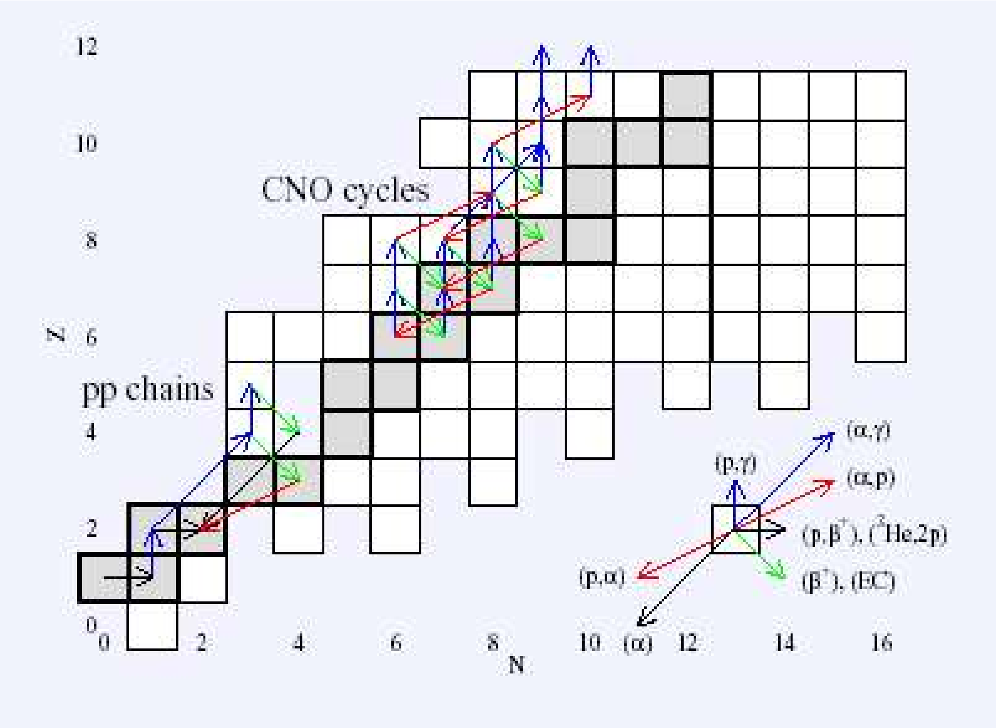


Consequences of lower $^{17}F(p,\gamma)^{18}Ne$ reaction rate in Nova nucleosynthesis

1. Survival of ^{18}F and subsequent beta decay : 511 keV γ-ray from e-e^{+} annihilation
   Detect by γ-ray astronomy ? Constraints on nova models?

2. More ^{17}F survives – pathway to ^{15}N via $^{17}F(,e^{+}\nu)^{17}O(p,\alpha)^{14}N(p,\gamma)^{15}O(,e^{+}\nu)^{15}N$
   Explanation for the overabundance of ^{15}N in nova ejecta ?

Indirect Methods in Nuclear Astrophysics. Why do we need them?



Nuclear Astrophysics :

Nuclear reaction rates at small energies are needed in many astrophysical models (stellar nucleosynthesis, novae, supernovae ….) for various processes (pp-chains, CNO cycle,

r, p, s, rp, ..)

Novae : Temperature ≤ 0.4 GK
^{17}F(p,γ)^{18}Ne peak reaction rates comes at around E_{cm} = 400 keV.

Direct measurements are preferable, but are often difficult.

Cross sections are small, unstable nuclei are involved, low yields...

Alternative indirect methods, depending on the type of reaction, offers a way forward.

Coulomb Dissociation Method, Asymptotic Normalization Coefficient, Trojan Horse Method.

Coulomb dissociation method

Nucleosynthesis :

$^{1}H(n,\gamma)^{2}H(n,\gamma)^{3}H(d,n)^{4}He(^{3}H,\gamma)^{7}Li(n,\gamma)^{8}Li$

$^{8}Li(\alpha,n)^{11}B(n,\gamma)^{12}B$ …. (higher masses)

Competition between these reaction pathways

$^{8}Li(n,\gamma)^{9}Li(\beta^{-},\nu)^{9}Be(p,\alpha)^{6}Li$ (turns reaction flow back)

$^{8}Li(n,\gamma)^{9}Li$ c.s. still uncertain

Find the Coulomb dissociation c.s. of

$$^{9}Li \longrightarrow {}^{8}Li + n$$

and relate back to the capture c.s.

Larger $^{8}Li(n,\gamma)^{9}Li$ c.s. could lead to almost **50%** reduction in abundance of $A \geq 12$ isotopes

R.A. Malaney and W.A. Fowler, *The Origin and Distribution of the Elements,* World Scientific (1988) p.76

Coulomb dissociation with three charged particles in the final state :

(Work in Progress) R.C., R. Shyam

Applications to the breakup of ^{8}B ($^{7}Be + p$) on heavy targets

Can relate to the $^{7}Be(p,\gamma)^{8}B$ solar capture reaction

Important for the neutrino oscillation problem

Conclusions and Perspectives

1. Shell Model Embedded in the Continuum – Realistic Shell Model solutions are coupled to the environment of one-particle decay channels. Recent extention to describe two nucleons in the scattering continuum and application for the description of the two-proton radioactivity.

3. Energy dependence of eigenfunctions (spectroscopic factors) and eigenvalues of the effective Hamiltonian and spectroscopic factors

5. Radiative proton capture cross section calculated from both $5/2^+$ g.s. and $1/2^+$ first excited state of ^{17}F using fully antisymmetrized wave functions in both initial and final channels.

7. Capture from the excited target state has negligible effect on the cross section in ^{17}F(p,γ)^{18}Ne. This effect may be more important in : ^{26}Al(p,γ)^{27}Si, ^{22}Na(p,γ)^{23}Mg ....

9. Lower ^{17}F(p,γ)^{18}Ne reaction rate could be an explanation for the over abundance of ^{15}N in nova ejecta ?

11. Indirect methods in Nuclear Astrophysics, like the Coulomb dissociation method..., offers a way forward when the direct method is too difficult or is not feasible.