Few Recent Projects and Publications (2002-2005)

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June, 2005

Quantum Integrable systems are *Interacting* + *Nonlinear* + *quantum* systems

but, they (due to high level of symmetry) allow

• I. Higher conserved operators $[C_a, C_b] = 0$, commuting mutually

And consequently, such quantum models and some related integrable statistical systems

• II. Can be solved *Exactly* & *Nonperturbatively*!

Famous Examples:

spin chains (xxx, xxz), vertex models, Toda chain, Nonlinear Schrödinger (NLS) equation , sine-Gordon model, Liouville equation, etc.

I. **Generation of new classes of integrable quantum and statistical models** *Integrability* of these quantum integrable models, known as Ultralocal models, is ensured by an underlying algebraic structure A (let us call it Yang-Baxter (YB) algebra. Since same A should be defined at each coordinate point, we must have $\Delta : A \rightarrow (A \otimes A \cdots \otimes A$ for the whole system. Therefore the YB algebra must be a *Hopf algebra*!

Discovery of the celebrated q-deformed (or quantum) Hopf algebra (in late 80's) $su_q(2)$:

$$\boxed{[\mathbf{S}^+, \mathbf{S}^-] = \sin(\mathbf{2}\alpha\mathbf{S}^\mathbf{3})/\sin\alpha, \quad [\mathbf{S}^\mathbf{3}, \mathbf{S}^\pm] = \pm\mathbf{S}^\pm,}$$

was accepted usually to be the most general YB algebra.

Our Result

We have discovered and developed a much effective and more general YB algebra (we call *Ancestor algebra*):

$$\boxed{[\mathbf{S}^+, \mathbf{S}^-] = \left(\hat{\mathbf{M}}^+ \sin(\mathbf{2}\alpha\mathbf{S}^\mathbf{3}) + \hat{\mathbf{M}}^- \cos(\mathbf{2}\alpha\mathbf{S}^\mathbf{3})\right)\frac{\mathbf{1}}{\sin\alpha}, \quad [\mathbf{S}^\mathbf{3}, \mathbf{S}^\pm] = \pm\mathbf{S}^\pm,}$$

where $\quad , [\hat{M}^\pm, \cdot] = 0$, are central-elements.

We have shown earlier that this Ancestor algebra (AA) can generate in an unified way

1) *All known integrable models* as well as 2) Construct some new quantum integrable models

During the last three years we have discovered *new* and *important* applications of the Ancestor algebra with theoretical as well as practical significance.

Firstly, we have established our AA as a *new* Hopf algebra $gl_{q,\hat{\mathbf{c}}}(2)$, given by operator $\hat{c}$-deformation of the quantum algebra.

Secondly, AA can generate as its subalgebras (at various eigenvalues of $\hat{\mathbf{M}}^{\pm}$) all known algebras like quantum algebras $su_q(2), gl_{p,q}(2)$, q-oscillator as well as $su(2), su(1,1)$ *bosonic* $(a, a^{\dagger})$, *canonical* $\hat{p}, \hat{q}$ etc. underlying integrable models.

Therefore, we could perform *two* different constructions, resulting two important classes of *new* models.

1) Idea of 1st construction:

Fix a subalgebra of AA (by fixing $\hat{\mathrm{M}}^{\pm}$) and with it an associated integrable model (say, NLS ,spin-chain , Sine-Gordon , Toda chain etc.). Use now the residual freedom of deforming operators $\hat{\mathrm{c}}$'s appearing in the Lax operator of the model to choose them as inhomogeneous parameters.

This would result new *inhomogeneious* extensions of the above quantum models as well as integrable Statistical models preserving their *integrability*! (Example: Inhomogeneious vertex models, NLS in ihomogeneious medium, SG with varying mass, spin with inhomogenious coupling etc).

2) Idea of 2nd construction:

Take different subalgebras of AA (by taking different $\hat{\mathrm{M}}^{\pm}$) and with it different integrable models, at different lattice sites (corrdinate poits). Due to Hopf algebra property of AA, this would result *new class of mixed integrable* *systems*!

(Examples:a) Combining ($su(2)$ + bosonic) $\longrightarrow$ Mixed spin-chain–NLS model

b) Combining ($su(2)$ + canonical) $\longrightarrow$ Mixed spin-chain–Toda chain model

c) Combining ($su_q(2)$ + q-oscillator) $\longrightarrow$ Combined sine-Gordon– derivative NLS (lattice) model, etc

Related Publications:

- Anjan Kundu, J Math Phys 44 (2003) 4589

  Unifying scheme for generating discrete integrable systems including inhomogeneous and hybrid models nlin.SI/0212004

- Anjan Kundu, Physica A 318 (2003) 144-153

  Generation of new classes of integrable quantum and statistical models cond-mat/0204495 (Statphys-Kolkata 02 invited talk)

- Anjan Kundu, 6th Chapter in *Classical & Quantum Integrable Systems* (Ed A Kundu, IOP, Bristol, 2003)

  Unifying approaches in integrable systems: Quantum & Statistical, Ultralocal & Nonultralocal

II.**New exactly solvable multi-atom matter-radiation systems**

Using above idea we have proposed New *Integrable matter-radiation models* of significant practical importance in
1) quantum optics
2) trapped ion model irradiated by laser beam
3)Cavity QED (microwave+optical)

Idea of construction :

Matter (= 2 level atoms , hence $s = \frac{1}{2}$ object): represented by $su(2)$ algebra

Radiation (=single mode boson) : represented by $(a, a^\dagger)$

Therefore we can repeat our *Mixed model construction* a) with N-spin model+ 1 boson, result $\longrightarrow$

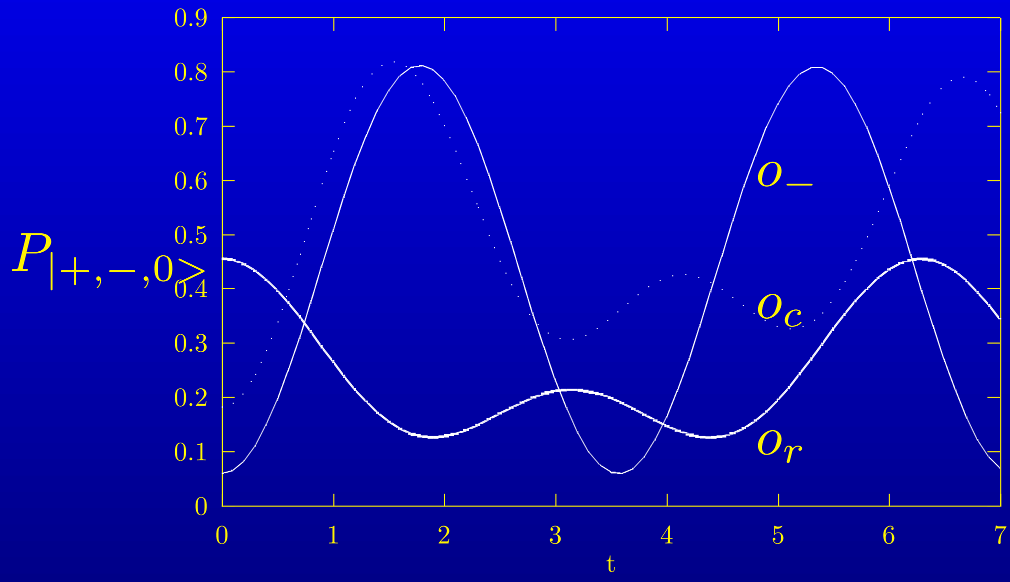
I. Integrable *Multi-atom Jaynes-Cummings model*:(JC)

II. Integrable*Multi-atom Buck-Sukumar model*:(BS)

III. Similarly constructing mixed integrable model with N-spin+Toda chain (canoncal=representing atomic oscillation) we get new *integrable Trapped-(multi) ion model* (TI)

IV. Repeating the above construction with q-deformed spin + q-bosons we can construct *new class of q-deformed integrable matter-radiation models*, e.g. integrable multi-atom q-BS, q-JC, q=TI models.

All such models by construction are exactly solvable. Therefore, we could make Experimental predictions through exact Bethe ansatz method: **Rabi-oscillation** (for 2-atom BS model)



Related Publications:

- Anjan Kundu, J.Phys. A 37 (2004) L281)

  Quantum integrability and Bethe ansatz solution for interacting matter-radiation systems

- Anjan Kundu, nlin.SI/0409032 (to be published in Theor Math Phys (Soviet

J.))

Quantum integrable multi atom matter-radiation models with and without rotating wave approximation

III. **Ultralocal solutions for quantum integrable nonultralocal models**

Apart from the standard or ultralocal integrable models discussed so far there exists another rich class of quantum integrable models, known as nonultralocal models, which include important models like nonabelian Toda chain, quantum mapping, Coulomb gas picture of CFT, current in WZWN model, integrable model on moduli space quantum mKdV model etc., for which the theory for their exact solutions could not be developed satisfactorily, due some inbuilt nonlocal structures.

To meet the challenge we have put forward a conjecture, that
such a model can always be connected to a corresponding ultralocal integrable model through some suitable gauge transformation

and hence can be solved exactly through Bethe ansatz. We have substantiated our claim starting again from the *ancestor model scheme* developed by us and explicitly constructing the required gauge transformation by exploiting the underlying algebraic structures. Thus we could derive such Ultralocal solutions for all nonultralocal integrable models, which was not known earlier, and discover alongwith new solvable quantum models like, noultralocal sine-Gordon and light-cone sine-Gordon models.

Related Publication:

Anjan Kundu, (Phys. Lett B 550 (2002) 128-34

Ultralocal solutions for nonultralocal integrable models hep-th/0208147

Other Publications:

- B. Basu-Mallick. T. Bhattacharyya and Anjan Kundu, Czech. J. Phys. 54 (2003) 5-12

Bound and scattering states of extended Calogero model with an additional PT invariant interaction

- Anjan Kundu (Editor) *Classical and Quantum Nonlinear Integrable Systems: Theory and Applications* (IOP Publishing, Bristol, UK), 2003

- Anjan Kundu, J. Phys. A35 (2002) L447-L453

  New series of integrable vertex models through a unifying approach cond-mat/0204470

- Anjan Kundu, J. Phys. A35 (2002) L125-L132

  Consistent refinement of Bethe strings for spin and electron models and a new non-Bethe solution cond-mat/0108175