

Derivative- δ function anyon gas

Anjan Kundu

Theory Group & CAMCS, Saha Institute of Nuclear Physics

Calcutta, INDIA

anjan.kundusaha.ac.in

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Anyons

- 3D: (Boson, Fermion)
- 2D: (Boson, Fermion, anyon: \rightarrow (B, F))
- 1D: (Boson (B), Fermion (F), anyon (A) (?)) [As projection 2D \rightarrow 1D]

In 1D : Exactly solvable interacting Bosonic, Fermionic models

Example: δ -function Bose gas, CS model

SUSY t-J model , Hubbard model etc.

Dynamical interaction \rightarrow GES Already like fractional or anyonic statistics!

Bosonic interaction \rightarrow Fermion like (at certain interaction limit)

1D anyon enhances this behavior by adding more parameters and interesting consequences

Note:

in 1D: anyon seems to have different possible definitions:

1) B-like (B at same point, A at different points)

- 2) F-like (F at same point, A at different points)
 3) A-like (A at same point, A at different points) Some of 1D solvable models \rightarrow 1),2) anyon models (δ -anyon gas (B to A 1)), t-J anyon model (F to A 2))

Question ?

- Can all 1D solvable models be A 1,2)-extended.
- Can any solvable models be A 3)-extended

Our aim:

Is to report for A1)-extension of an exactly solvable B-model: δ' -function interacting anyon gas .

Field model :

Derivative NLS field model

$$H = \int dx (\psi_x^\dagger \psi_x + i\epsilon((\psi^\dagger)^2 \psi \psi_x))$$

- Quantum Integrable system B-field
- QYBE with trigonometric R -matrix

• algebraic Bethe ansatz (ABA) solvable [K+BM, JMP'93]

anyonic DNLS ?

with Anyonic (B-type) field:

$$\tilde{\psi}(x_1)\tilde{\psi}^\dagger(x_2) = e^{-i\kappa\epsilon(x_1-x_2)}$$

$$\tilde{\psi}^\dagger(x_2)\tilde{\psi}(x_1)$$

etc. where

$$\epsilon(x-y) = \pm 1 \text{ for } x > y, x < y$$

and B-type CR at $x = y$

However the quantum system becomes Nonultralocal

$$[(I \otimes L_j(\mu)), (L_k(\lambda) \otimes I)] \neq 0, j \neq k$$

Beyond QIST solvability!

Can we solve it through Braided YBE ?

Bosonic realization takes anyonic DNLS to the Gerdjikov-Ivanov (B)-model with higher nonlinearities

$$H = \int dx (\psi_x^\dagger \psi_x + (\kappa + \epsilon)(i\rho\psi^\dagger \psi_x + \kappa\psi^\dagger \rho^2 \psi)), \quad \rho \equiv (\psi^\dagger \psi),$$

Integrable only at the classical level !!

Therefore we restrict to

N-particle sector of anyonic DNLS model

$$H_N = - \sum_k^N \partial_{x_k}^2 + i\epsilon \sum_{\langle k,l \rangle} \delta(x_k - x_l) (\partial_{x_k} + \partial_{x_l})$$

B-model is solved by coordinate BA [Shnirman+ PRA'94]

- Consider anyonic gas interacting via δ' potential.
- Eigenvalue problem:

$$\tilde{H}_N \tilde{\Phi}(x_1, \dots, x_N) = E_N \tilde{\Phi}(x_1, \dots, x_N)$$

Bethe ansatz :

$$\tilde{\Phi}(x_1, \dots, x_N) = \Phi_A(x_1, \dots, x_N) \Phi_B(x_1, \dots, x_N)$$

Here Φ_B BA form

$$\Phi_B(x_1, \dots, x_N) = \sum_P A(P) e^{i \sum_j x_j k_{Pj}}$$

Φ_A is anyonic part

$$\Phi_A(x_1, \dots, x_N) = e^{-i \frac{\kappa}{2} \sum_{i < j} \epsilon(x_i - x_j)}$$

with wave function $\tilde{\Phi}$ showing anyon type symmetry

$$\tilde{\Phi}(x_1, \dots, x_i, \dots, x_j, \dots, x_N) = e^{-i\kappa \left(\sum_{k=i+1}^j \epsilon(x_i - x_k) - \sum_{k=i+1}^{j-1} \epsilon(x_j - x_k) \right)} \tilde{\Phi}(x_1, \dots, x_j, \dots)$$

Acting on it by H_N : 1. Energy spectrum :

$$E_N = \sum_{n=1}^N (k_n^2)$$

2. Discontinuity in the derivative

$$(\partial_{x_l} - \partial_{x_k}) \tilde{\Phi}|_+ - (\partial_{x_l} - \partial_{x_k}) \tilde{\Phi}|_- = i\epsilon(\partial_{x_l} + \partial_{x_k}) \tilde{\Phi}|_0$$

with notation $|\pm = |_{x_l=x_k^\pm}$ and $|_0 = |_{x_l=x_k}$.

Taking care of anyonic relation for Φ_A and eliminating it get for Φ_B

$$((\partial_{x_l} - \partial_{x_k})\Phi_B(x_1, \dots, x_N))|_0 = i\tilde{\epsilon}((\partial_{x_l} + \partial_{x_k})\Phi_B(x_1, \dots, x_N))|_0,$$

with original coupling constant ϵ modified to $\tilde{\epsilon} = \frac{\epsilon}{2 \cos \frac{\kappa}{2}}$ through anyonic interaction κ .

3. Scattering matrix given through BA:

$$A(\dots, k_m, k_n, \dots) = S_{nm}A(\dots, k_n, k_m, \dots),$$

is derived in trigonometric functions

$$S_{nm} = \frac{\sin(\frac{\theta_n^0 - \theta_m^0}{2} - \phi)}{\sin(\frac{\theta_n^0 - \theta_m^0}{2} + \phi)}$$

where

$$k_n = e^{i\theta_n^0}, \quad \tan \phi = \tilde{\epsilon} = \frac{\epsilon}{2 \cos \frac{\kappa}{2}}.$$

Limits:

1) $\phi = 0, \pm\pi$ gives $\tan \phi = \tilde{\epsilon} = 0$ hence $S_{nm} = 1$ (free Bosonic)

2) $\phi = \pm\frac{\pi}{2}$ hence $S_{nm} = -1$ (free Fermionic)

gives $\tilde{\epsilon} = \pm\infty$ (possible with ϵ finite by adjusting $\kappa = \pi$))

3) S_{nm} -periodic trig.

function 4) θ, ϕ small \rightarrow rational limit.

5) Richer picture than the δ -anyon gas (solved) [K'99,B+G'06,G'06,Kor'07]

6) GES more interesting

Bethe ansatz equation

For fixing rapidity parameters

Periodic BC: $\tilde{\Phi}(x_1 = 0, x_2, \dots, x_N) = \tilde{\Phi}(L, x_2, \dots, x_N)$

$$e^{ik_n L} = -e^{i\kappa(N-1)} \prod_{m=1}^N \frac{\sin\left(\frac{\theta_n^0 - \theta_m^0}{2} - \phi\right)}{\sin\left(\frac{\theta_n^0 - \theta_m^0}{2} + \phi\right)}$$

Twisted by anyonic factor κ , also $\phi(\epsilon, \kappa)$.

- *Thermodynamic limit , TBA , low-lying excitation, finite T analysis etc.*

Peculiarity of BE for anyon :

No unique Bethe equation, Depends on our choice of BC +position:

$$\tilde{\Phi}(x_1, x_2, x_j, \dots, x_N) = e^{i\beta\kappa(N-1)} \tilde{\Phi}(x_1, \dots, x_j + L, \dots, x_N) \dots \dots (A1)$$

1) periodic: $\beta = 0$ or 2) twisted $\beta = \pm 1$

2) position of the particle j

Anyonic Bethe equation :

$$e^{ik_j L} = T \prod_{k=1, s \neq j}^N \frac{k_j - k_s + i\tilde{\epsilon}}{k_j - k_s - i\tilde{\epsilon}}, \quad T = e^{i\kappa((N-2j+1) - \beta(N-1))} \dots \dots \dots (A2)$$

Note:

1. at $\beta = 0$ (periodic BC), $j = 1$ (usual convention) ,

twisting $T = e^{i\kappa(N-1)}$ (as we obtained)

2. $\beta = 0$ (periodic BC) but $2j = N + 1$, or $\beta = 1$ (twisted BC) and $j = 1$ (usual convention),

we get $T = 1$, i.e. no twisting in the BE !

etc. Due to projection of 2D: \rightarrow 1D, such ambiguity reflects the 2D wrapping of particle 2 around 1 or 1 around 2, etc.

Thank You !