Light Front Dynamics: Structure and Scattering

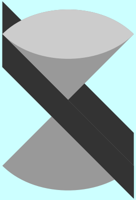
A. Harindranath

Brief Introduction to Light Front Dynamics
Applications to Deep Inelastic Scattering
Deeply Virtual Compton Scattering
Topological Sector of Two Dimensional ϕ^4 Theory in DLCQ
Transverse Lattice Quantum Chromo Dynamics
Basis Function Approach to Light Front Hamiltonian

Light Front Dynamics

Dirac, RMP (1949)

Approach to solve Quantum Field Theory - complementary to Lattice Field Theory



$x^{\pm} = x^0 \pm x^3,\ x^{\perp} = (x^1, x^2)$

x^+ "time",

x^- longitudinal coordinate

$x^2 = x^+ x^- - (x^{\perp})^2$

$[\ x^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2\ ]$

$k.x = \frac{1}{2}k^+ x^- + \frac{1}{2}k^- x^+ - k^{\perp} \cdot x^{\perp}$

For an on mass shell particle $k^2 = m^2 \rightarrow$ energy $k^- = \frac{(k^{\perp})^2 + m^2}{k^+}$.

longitudinal momentum $k^+ = k^0 + k^3 \geq 0$

Dependence on $k^{\perp}$ just like in the nonrelativistic dispersion relation

Longitudinal boost becomes scaling, transverse boosts become Galilean boosts. Thus boost become kinematical (non-relativistic).
In a relativistic theory, internal motion and the motion associated with center of mass can be separated $\Longrightarrow$ Can construct boost invariant wave functions.
Furthermore, the relativistic fermion is described by a two component field.
We start seeing the " remarkably non-relativistic character of the extreme relativistic limit " (Bjorken)

Detailed calculations $\Rightarrow$ structure functions in deep inelastic scattering are equal light front time correlation functions in relativistic physics just as structure functions in quasi elastic scattering are equal time correlation functions in non-relativistic physics.

Discrete Light Cone Quantization

Exploit the semi-positive definiteness of longitudinal momentum.
Compactify x^-: $-L \leq x^- \leq +L$.
With Anti Periodic Boundary condition (APBC),
$k^+ \to k_n^+ = \frac{n\pi}{L}, \quad n = 1, 3, 5, \ldots$
Longitudinal momentum $P^+ = \frac{2\pi}{L} K$ where K is the dimensionless longitudinal momentum.
For example, field expansion for a scalar field has the form
$\phi(x^-) = \frac{1}{\sqrt{4\pi}} \sum_n \frac{1}{\sqrt{n}} \left[a_n e^{-i \frac{n\pi}{L} x^-} + a_n^\dagger e^{i \frac{n\pi}{L} x^-} \right]$. Here $n = \frac{1}{2}, \frac{3}{2}, \ldots$.

Deep Inelastic Scattering

Basic formalism:

AH, Rajen Kundu, and Wei-Min Zhang, Deep Inelastic Structure Functions in Light-Front QCD: A Unified Description of Perturbative and Nonperturbative Dynamics, Phys. Rev. **D 59**, 094012 (1999). AH, Rajen Kundu, and Wei-Min Zhang, Deep Inelastic Structure Functions in Light-Front QCD: Radiative Corrections, Phys. Rev. **D 59**, 094013 (1999).

Some of the highlights:

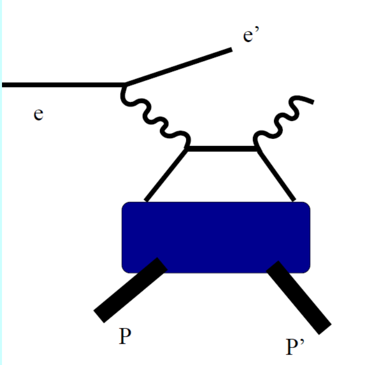
- A new sum rule for twist four longitudinal structure function
  AH, Rajen Kundu, Asmita Mukherjee and James P. Vary, Twist Four Longitudinal Structure Function in Light-Front QCD, Phys. Rev. **D 58**, 114022 (1998).
- Role of orbital angular momentum in the decomposition of proton spin
  AH and Rajen Kundu, On orbital angular momentum in deep inelastic scattering, Phys. Rev. **D 59**, 116013 (1999).
- A new sum rule for transverse spin
  AH, Asmita Mukherjee and Raghunath Ratabole, Transverse Spin in QCD and Transverse Polarized Deep Inelastic Scattering, Phys. Lett. **B 476**, 471 (2000).

Deeply Virtual Compton Scattering

- S.J. Brodsky, D. Chakrabarti, A. Harindranath, A. Mukherjee, J.P. Vary, Hadron Optics: Diffraction Patterns in Deeply Virtual Compton Scattering, Phys.Lett. **B 641**, 440, (2006).
- S.J. Brodsky, D. Chakrabarti, AH, A. Mukherjee, J.P. Vary, Hadron optics in three-dimensional invariant coordinate space from deeply virtual Compton scattering, Phys. Rev. **D 75**, 014003, (2007).

The process is $\gamma^*(q) + p(P) \rightarrow \gamma(q') + p(P')$, where the virtuality of the initial photon $Q^2 = -q^2$ is large and final photon is on-shell, ${q'}^2 = 0$.

Hard exclusive electroproduction of photons



$\Delta = P - P' = \left(\zeta P^+ , \vec{\Delta}_\perp , \frac{t+\vec{\Delta}_\perp^2}{\zeta P^+}\right)$ where $t = \Delta^2$.

The virtual Compton amplitude

$M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) \;=\; i \int d^4y\, e^{-iq\cdot y} \langle P' | T J^\mu(y) J^\nu(0) | P \rangle$,

μ and ν denote the polarizations of the initial and final photons. In the limit $Q^2 \to \infty$ at fixed ζ and t the Compton amplitude

$$M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) \;= \epsilon^I_\mu\, \epsilon^{*J}_\nu\, M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) \;=$$

$$-e_q^2 \int_{\zeta-1}^{1} \mathrm{d}z\; t^{IJ}(z,\zeta) \int \frac{dy^-}{8\pi}\; e^{izP^+y^-/2} \langle P' | \bar{\psi}(0)\, \gamma^+\, \psi(y)\, | P \rangle \Big|_{y^+=0, y_\perp=0}$$

Three kinematical regions:

- $\zeta - 1 < z < 0$: Antifermion with momentum fraction $\zeta - z$ is removed, reinserted with momentum fraction $-z$
- $0 < z < \zeta$: Photon scatters off from a virtual antifermion-fermion pair in the initial hadron
- $\zeta < z < 1$: A fermion with momentum fraction z is removed and re-inserted with momentum fraction $z - \zeta$

Generalized Parton Distributions

The generalized (off forward) parton distributions H, E are defined through matrix elements of the bilinear vector and axial vector currents on the light-cone. For the vector current

$$F_{\lambda,\lambda'} = \int \frac{dy^-}{8\pi}\, e^{izP^+y^-/2}\, \langle P',\lambda'|\bar{\psi}(0)\,\gamma^+\,\psi(y)\,|P,\lambda\rangle\Big|_{y^+=0,y_\perp=0}$$
$$= \frac{1}{2\overline{P}^+}\,\overline{U}(P',\lambda')\left[H(z,\zeta,t)\,\gamma^+ + E(z,\zeta,t)\,\frac{i}{2M}\,\sigma^{+\alpha}(-\Delta_\alpha)\right]U(P,\lambda)$$

In the forward limit ($\zeta = 0 = \Delta_\perp$), off-forward parton distribution $\Rightarrow$ parton distribution in Deep Inelastic Scattering.

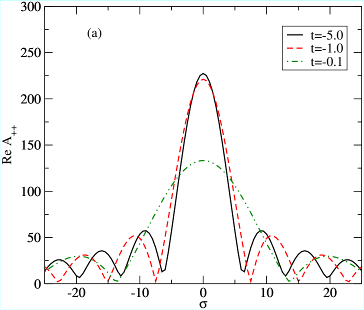
Integral of the off-forward parton distribution over the momentum fraction yields the elastic form factor.

To obtain the DVCS amplitude in longitudinal coordinate space, we take a Fourier transform in ζ as,

$$A(\sigma, t) = \frac{1}{2\pi} \int d\zeta \; e^{i\sigma\zeta} \; M(\zeta, \Delta_\perp)$$

where $\sigma = \frac{1}{2} P^+ b^-$ is the longitudinal distance on the light-cone and the FTs are performed at a fixed invariant momentum transfer squared $-t$.

Finite range of ζ integration acts as a slit of finite width $\Rightarrow$ occurrence of diffraction pattern in the FT. Position of the first minimum inversely proportional to $\zeta_{\max}$. Since $\zeta_{\max}$ increases with $-t$, position of the first minimum moves to a smaller value of σ when $-t$ increases.



Fourier spectrum of the real part of the DVCS amplitude of an electron vs. σ when the electron helicity is not flipped.

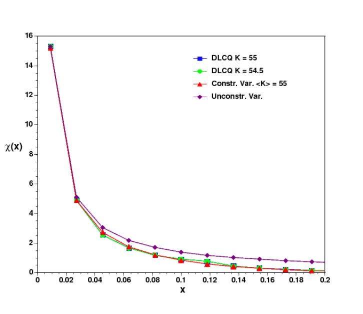
Topological sector of two dimensional ϕ^4 theory on the light front

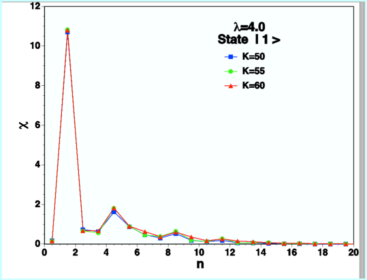
- Dipankar Chakrabarti, AH, Lubomir Martinovic and James P. Vary, Kinks in Discrete Light Cone Quantization, Phys. Lett. **B 582**, 196 (2004).
- Dipankar Chakrabarti, AH, Lubomir Martinovic, Grigorii B. Pivovarov and James P. Vary, Ab Initio Results for the Broken Phase of Scalar Light Front Field Theory, Phys. Lett. **B 617**, 92 (2005).
- Dipankar Chakrabarti, AH and James P. Vary, A Transition in the Spectrum of the Topological Sector of ϕ_2^4 Theory at Strong Coupling, Phys. Rev. **D 71**, 125012 (2005).

Using DLCQ, masses of the lowest few excitations, parton distribution functions and Fourier transforms of the form factor are calculated. Also vacuum energy density and the value of the condensate.

odd sector		even sector	
K	dimension	K	dimension
15.5	295	16	336
31.5	12839	32	14219
39.5	61316	40	67243
44.5	151518	45	165498
49.5	358000	50	389253
54.5	813177	55	880962

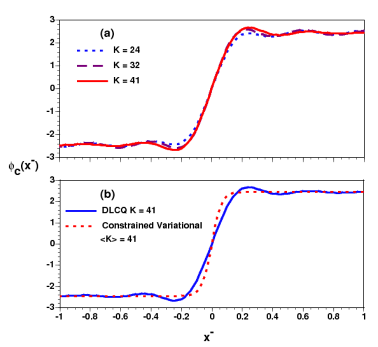
Dimensionality of the Hamiltonian matrix in odd and even particle sectors with APBC.



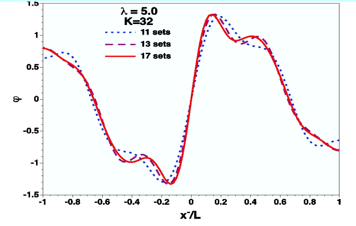


Parton distribution function $\chi(x)$ for even ($K = 55.0$) and odd ($K = 54.5$) sectors for $\lambda = 1$ compared with unconstrained and constrained ($\langle K \rangle = 55$) variational results.

Parton distribution function χ versus *n*, the half-odd integer representing light front momentum with APBC, for the lowest excitation for $K = 50, 55$, and 60, $\lambda = 4$.



Fourier Transform of the kink form factor at λ=1; (a) results for $K = 24, 32$, and 41 each obtained with DLCQ eigenstates from 11 values of K centered on the designated K value; (b) comparison of DLCQ profile at K=41 with constrained variational result with $\langle K \rangle = 41$.



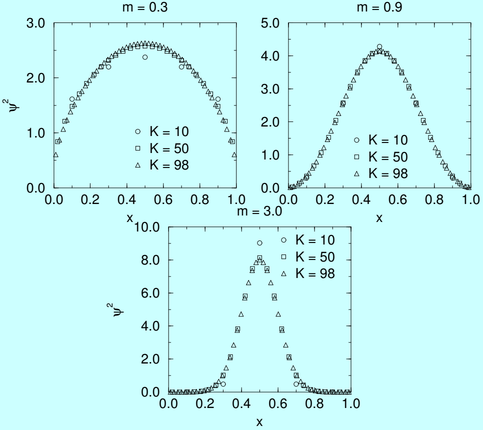
Fourier Transform of the kink form factor at λ=5, $K = 32$. The number of adjoining momentum transfer terms included in the summation is indicated.

Transverse Lattice QCD

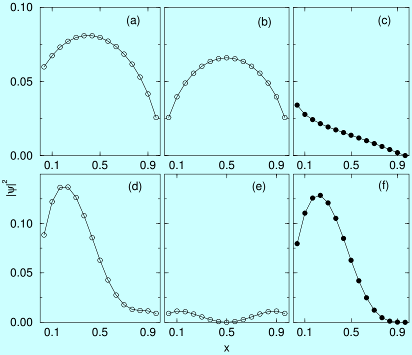
- Dipankar Chakrabarti, Asit K. De, AH, Fermions on the Light Front Transverse Lattice, Phys. Rev. **D 67** 076004 (2003).
- Dipankar Chakrabarti, AH, and James P. Vary, A Study of Quark-Antiquark States in Transverse Lattice QCD Using Alternative Fermion Formulations, Phys. Rev. **D 69**, 034502 (2004) .

Why Transverse Lattice QCD?

- Infinitely many degrees of freedom - Need to put cutoffs - Lattice provides a gauge invariant cutoff
- Hamiltonian provides the most direct route to wavefunctions - Keep time direction continuous
- Theory is inherently nonlocal in x^- $\Rightarrow$ Keep x^- continuous
- Conventional ultraviolet divergences come from small $x^\perp$ - Discretize transverse space
- Retain minimal gauge invariance - Fix the gauge $A^+ = 0$ - gauge invariance associated with x^- independent gauge transformations



Quark distribution function $|\,\psi(x)\,|^2$ of the ground state in the $q\bar{q}$ approximation for quark mass = 0.3, 0.9 and 3.0 with coupling constant $g = 1.0$



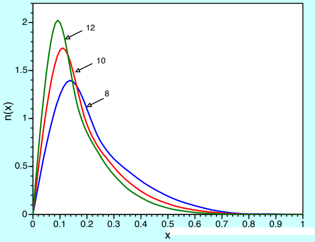
(a) Quark distribution function $|\psi(x)|^2$ of the ground state in the one link approximation, (b) $q\bar{q}$ contribution to the ground state, (c) $q\bar{q}$ link contribution to the ground state. (d) Quark distribution function $|\psi(x)|^2$ of the fifth eigenstate in the one link approximation, (e) $q\bar{q}$ contribution to the fifth eigenstate, (f) $q\bar{q}$ link contribution to the fifth eigenstate.

Basis function approach to light front Hamiltonian

Hamiltonian light-front field theory in a basis function approach, J.P. Vary, H. Honkanen, Jun Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, Phys. Rev. **C 81**, 035205 (2010).

Features:

- Light front gauge $A^+ = 0$
- DLCQ in the longitudinal direction (momentum space)
- To handle many different length scales, two dimensional Harmonic oscillator wavefunctions in the transverse plane (coordinate space)



Light front momentum distribution functions for bosons at three different values of $N_{max} = K$ that are labeled.

SUMMARY

- Applications to Deep Inelastic Scattering New sum rules in unpolarized and polarized deep inelastic scattering, elucidation of the role of orbital angular momentum in deep inelastic scattering
- Deeply Virtual Compton Scattering Hadron structure in longitudinal coordinate space
- Topological Sector of Two Dimensional ϕ^4 Theory in DLCQ Masses, parton distribution functions and Fourier transforms of form factors of topological structures, value of the condensate
- Transverse Lattice Quantum Chromo Dynamics Elucidation of fermion doubling, spectra and parton distribution functions
- Basis Function Approach to Light Front Hamiltonian Ability to address multiple length scales, multiparton states, parton distribution functions