Nonlinear dynamics of stock markets during critical periods

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Contents

Background & Literature

- Stock market crashes : a subject of intimate study in financial economics literature
- From (Fisher, 1930) to Lauterbauch et al. (2012):
  - reasons, nature and impact of stock market crashes have been analysed in a various ways by various authors

Background & Literature

- Behavioural
  - Galbraith (1954) , Kindleberger (1978)
  - (Shiller R. J., 1987)tries to provide a behavioural finance perspective to stock market crashes
  - (Bereave & Veronesi, 2003): focus on micro-behaviour of traders and information asymmetry
  - Li et al.(2009) identify a Bayesian investor's belief evolution when facing a structural break in economy and links it to the bubbles and crashes

Background & Literature

- Empirical studies on financial market crashes
  - (Barro, 1990)
  - Ding, Granger, & Engle(1993)
  - (Longin, 1996)
  - (Choudhry, 1996),
  - (Yang & Min, 2003)
  - Lauterbauch et al. (2012)

Background & Literature

- 1987 Black Monday
  - (Shiller R. J., 1987)
  - (Seyhun, 1990)
  - (Harris, 1989)
  - (Mitchell & Netter, 1989)
  - (Bates, 1991)
- Tech bubble
  - (Griffin & Topaloglu, 2011)

Recent Developments

- Apart from traditional econometric modelling
- Considerable research work towards modelling financial crashes based on analogies from physics
- Close to a crash the market behaves like a thermodynamic system which undergoes phase transition
- crashes as critical points in a system
- log-periodic fluctuations in stock market indices
- earthquake-stock market analogy
  - (Feigenbaum & Freund, 1996), (Feigenbaum & Freund, 1998)

Recent Developments

- Slow build up of long-range correlations between traders
- leading to a collapse of the stock market
- A crash : interpreted as a critical point
  - Sornette D. A., 1996)
  - (Sornette, Johansen, & Bouchaud, 1996)
- An analogy between crashes and phase transition
  - (Sornette & Johansen, 1999)
  - (Johansen & Sornette, 1999),
  - (Vandewalle, Ausloos, Boveroux, & Minguet, 1998), (Vandewalle, Boveroux, Minguet, &Ausloos, 1998)
  - (T. Kaizoji, D. Sornette ,2009)

Recent Developments

- Standard econometric models it is not possible to detect the bubble in advance or predict a crash even in approximate terms
- Technique evolved from nonlinear dynamics and physics : Recurrence Plot
- possible to graphically represent the dynamic evolution of a system

Motivation

- Recurrence Plot(RP) can detect critical phases in the system and changes in the same
- Inspired by this (Fabretti & Ausloos, 2005)first show
  - that using recurrence plot and its quantification one can detect endogenous crashes
- (Guhathakurta, Bhattacharya, & Roychowdhury, 2009), (Guhathakurta, Bhattacharya, & Roychowdhury, 2010a), (Guhathakurta, Bhattacharya, & Roychowdhury, 2010b) further established the tool
- (Bastos & Caiado, 2011) reinforced their findings.

Present Work

- Extends the original findings of Guhathakurta et al. (2010b) by analysing eight different financial crashes at different times
- Purpose
  - Identify the critical phases
- understand the dynamics of the stock market during such periods
- Using the recurrence statistics, we show
  - it is possible to detect critical periods in advance for all the cases where there was a known bubble building up in the market

Crashes

- Black Monday (Oct 19, 1987) Crash, Hong Kong AOI
- The Friday the 13th mini-crash (Oct 13,1989) Dow Jones Industrial Average (DJIA)
- Japanese bubble (1986-91), NIKKEI
- October 27, 1997 mini-crash, Hang Seng
- 11 Sept NYSE , 2001 crash, DJIA
- Stock market downturn of 2002, DJIA
- China 2007 Crash, CSI300
- 2010 Flash crash, DJIA

Data & Software

- Our analysis covers eight different stock market crashes which occurred in different stock exchanges across the world.
- The data for analysis was the closing value of the respective stock indices.
- All the analysis was carried out on MATLAB platform.

RECURRENCE PLOT

Definition: A recurrence plot (RP) is a graph that shows all those times at which a state of the dynamical system recurs. In other words, the RP reveals all the times when the phase space trajectory visits roughly the same area in the phase space.

RECURRENCE PLOT

- A phase space, introduced by Willard Gibbs in 1901, is a space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space.
- In a phase space, every degree of freedom or parameter of the system is represented as an axis of a multidimensional space.

RECURRENCE PLOT

- For every possible state of the system, or allowed combination of values of the system's parameters, a point is plotted in the multidimensional space.
- Often this succession of plotted points is analogous to the system's state evolving over time.
- In the end, the phase diagram represents all that the system can be, and its shape can easily elucidate qualities of the system that might not be obvious otherwise.

RECURRENCE PLOT

- Eckmann et al. (1987) have introduced a tool which can visualize the recurrence of states xi in a phase space
- Usually, a phase space does not have a dimension (two or three) which allows it to be pictured
- Eckmann's tool enables us to investigate the m-dimensional phase space trajectory through a two-dimensional representation of its recurrences.

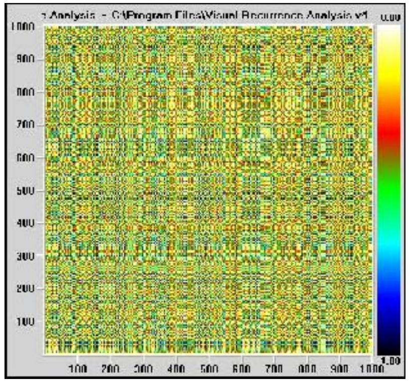
RECURRENCE PLOT

- Such recurrence of a state at time i at a different time j is marked within a two-dimensional squared matrix with ones and zeros dots (black and white dots in the plot), where both axes are time axes
- This representation is called recurrence plot (RP). Such an RP can be mathematically expressed as
- $R_{i,j} = \Theta(\varepsilon_i - || x_i - x_j ||), \quad x_i \in \Re^m, \quad i, j = 1 \ldots N,$
- where $R_{i,j}$ is the recurrence plot, N is the number of considered states x_i, ε_i is a threshold distance, $|| \cdot ||$ a norm and $\Theta(\cdot)$ the Heaviside function
- The Heaviside step function is given by:
- $\Theta(x) = 0$ if $x<0$
- $\Theta(x) = 1$ if $x \geq 0$

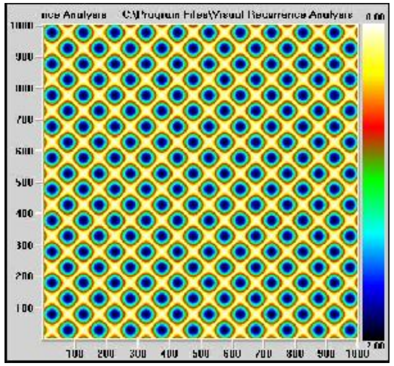
RECURRENCE PLOT

- Structures in Recurrence Plots
- Homogeneous RPs are typical of stationary and autonomous systems. Ex-a random time series
- Oscillating systems have RPs with diagonal oriented, periodic recurrent structures (diagonal lines, checkerboard structures
- Abrupt changes in the dynamics as well as extreme events cause bands in the RP

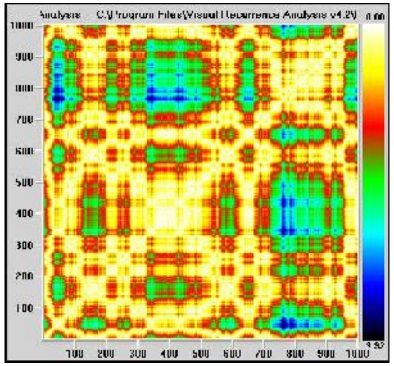
RECURRENCE PLOT



(A)



(B)



(C)

(A) homogeneous (uniformly distributed noise),

(B) periodic (super-positioned harmonic oscillations)

(C) disrupted (Brownian motion).

RECURRENCE QUANTIFICATION ANALYSIS

%DET: the percentage of recurrent points forming line segments parallel to the main diagonal. The presence of these lines reveals the existence of a deterministic structure.

$$DET = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)},$$

RECURRENCE QUANTIFICATION ANALYSIS

The *average diagonal line length L* gives the average time that two segments of the trajectory are close to each other, and can be interpreted as the mean prediction time.

$$L = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} P(l)},$$

RECURRENCE QUANTIFICATION ANALYSIS

Analogously to the definition of the determinism in Eq. (5), the fraction of recurrence points forming vertical structures in the RP is defined as

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}$$

and is called laminarity.

RECURRENCE QUANTIFICATION ANALYSIS

The average length of vertical structures is given by

$$TT = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=v_{min}}^{N} P(v)}$$

and is called trapping time.

Both *LAM and TT have been proven to be useful for describing the dynamics of discrete* systems and studying chaos-chaos transitions. RQA as the whole is a very powerful technique for quantifying differences in the dynamics of complex systems

RECURRENCE QUANTIFICATION ANALYSIS

- One key question in empirical research concerns the confidence bounds of the calculated RQA measures
- Schinkel et al. have suggested a method to estimate the confidence of the RQA measures [(Schinkel, Marwan, Dimigen, & Kurths, 2009)]
- We have used 95% confidence level for the computation of these measures.

Results

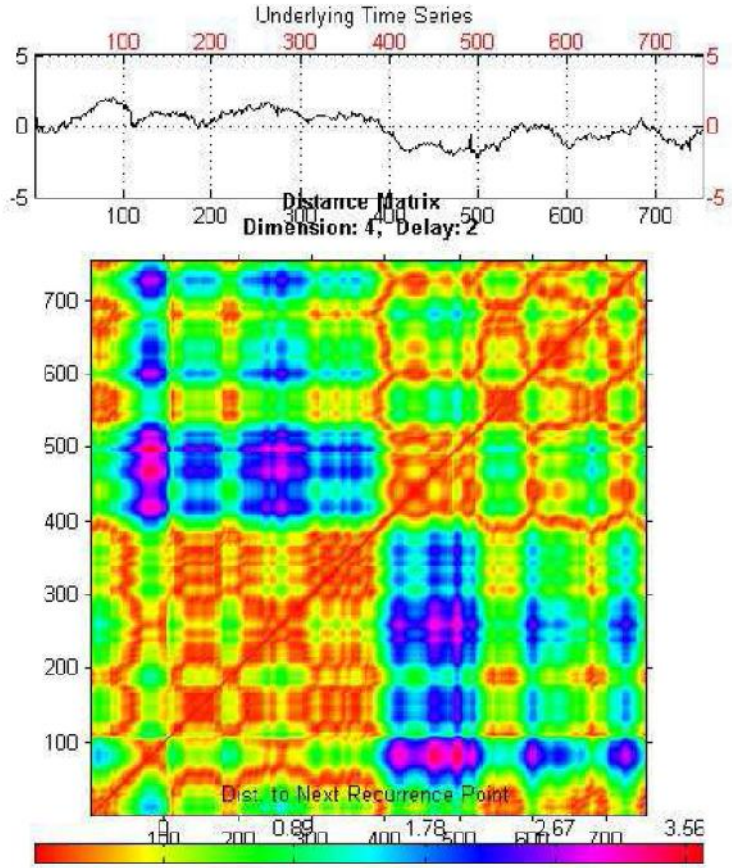


Fig 1a Recurrence plot of Black Monday (Oct 19, 1987) Crash

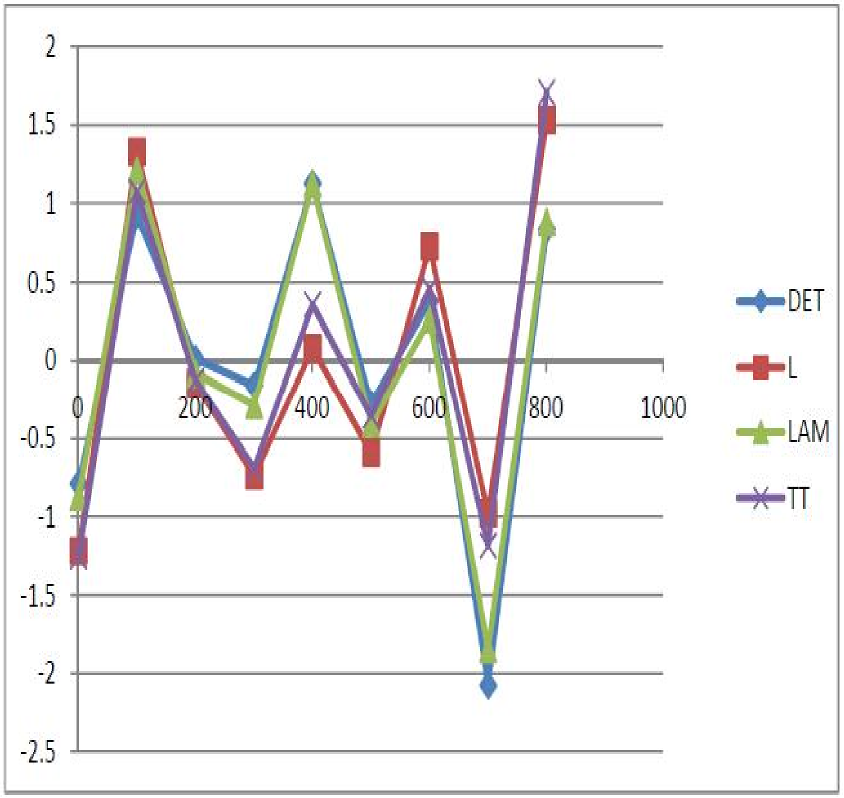


Fig 1b RQA values of Black Monday (Oct 19, 1987) Crash

Results

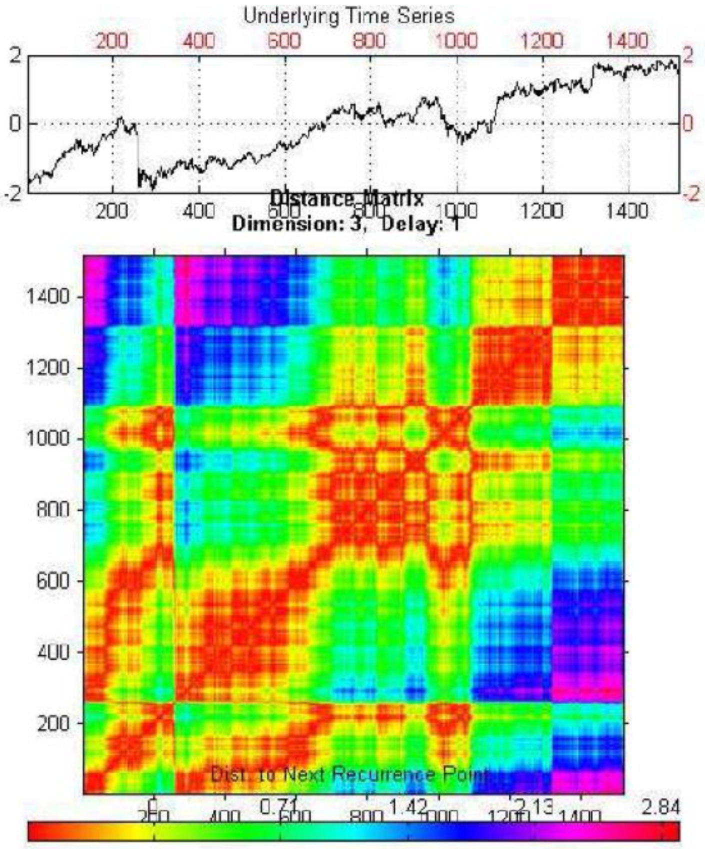


Fig 2a Recurrence Plot of the Friday the 13th mini-crash (Oct 13, 1989)

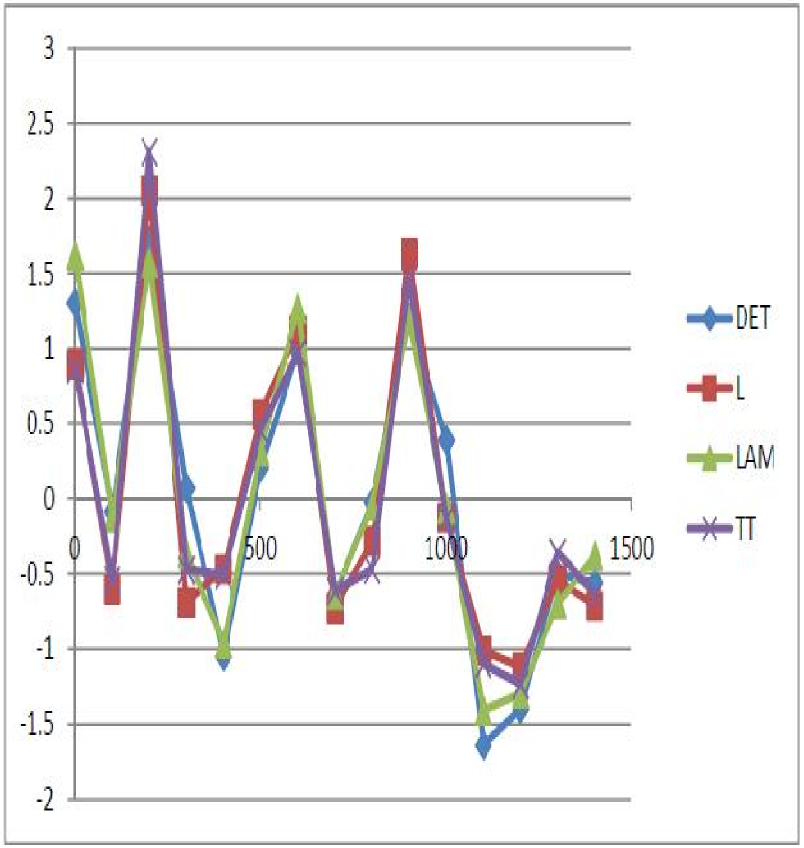


Fig 2b RQA values of the Friday the 13th mini-crash (Oct 13, 1989)

Results

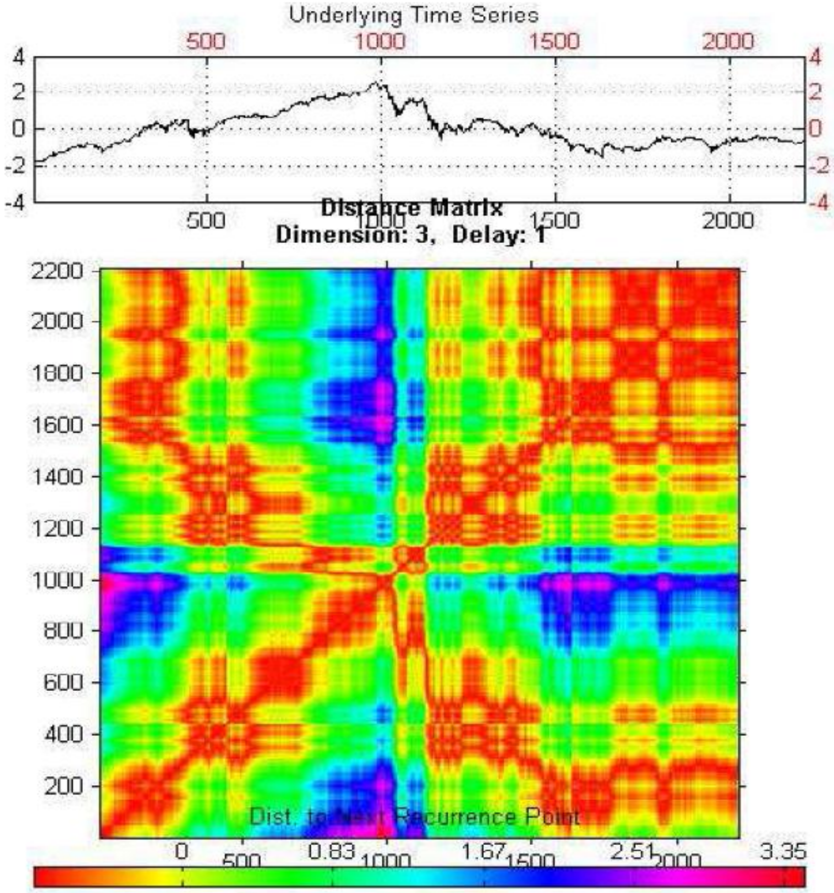


Fig 3a Recurrence Plot of Japanese bubble (1986-91)

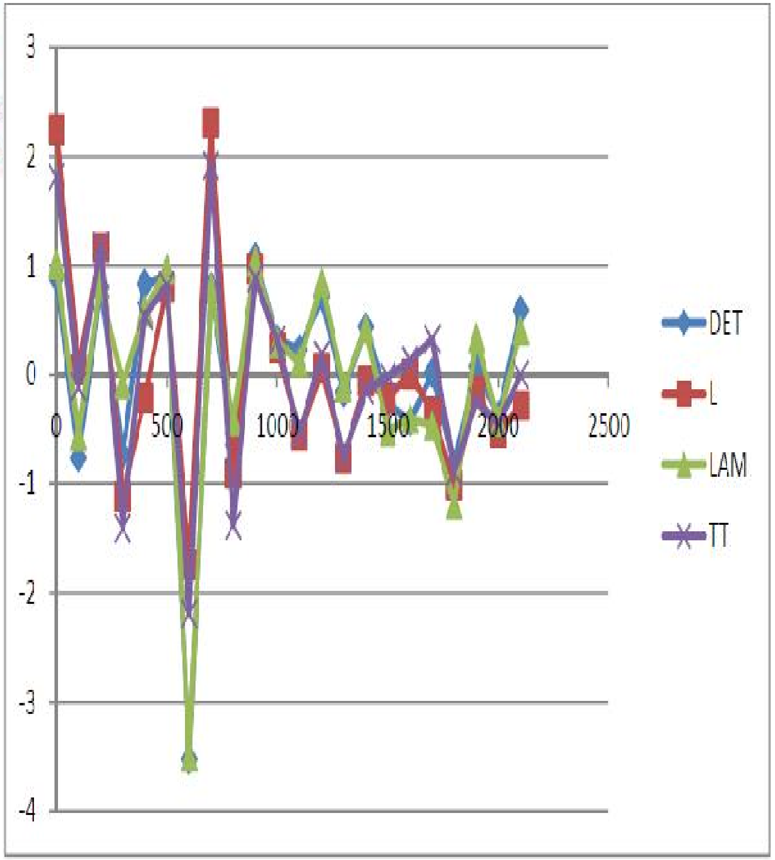


Fig 3b RQA values of Japanese bubble (1986-91)

Results

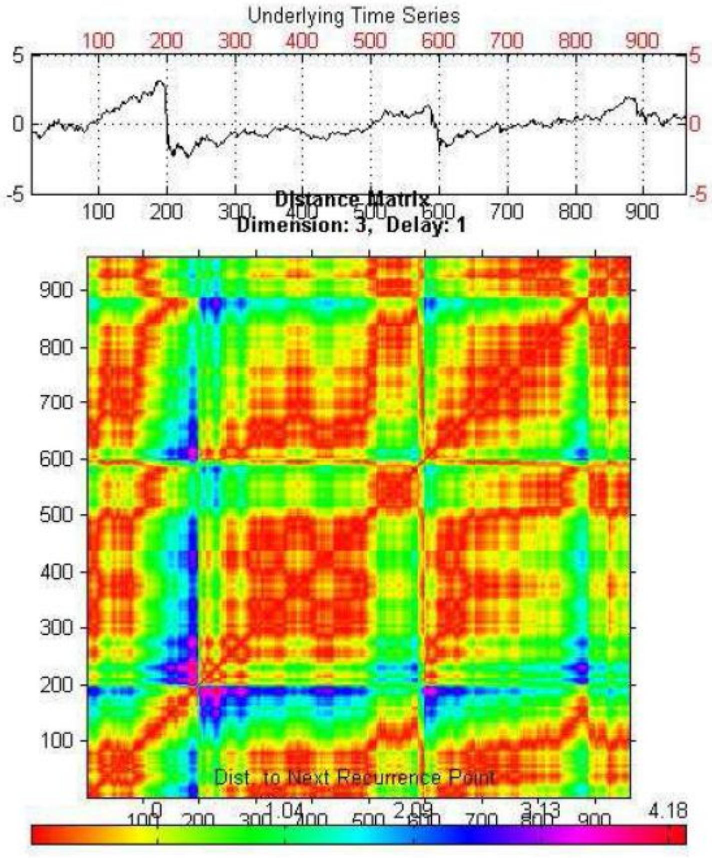


Fig 4a Recurrence Plot of October 27, 1997 mini-crash

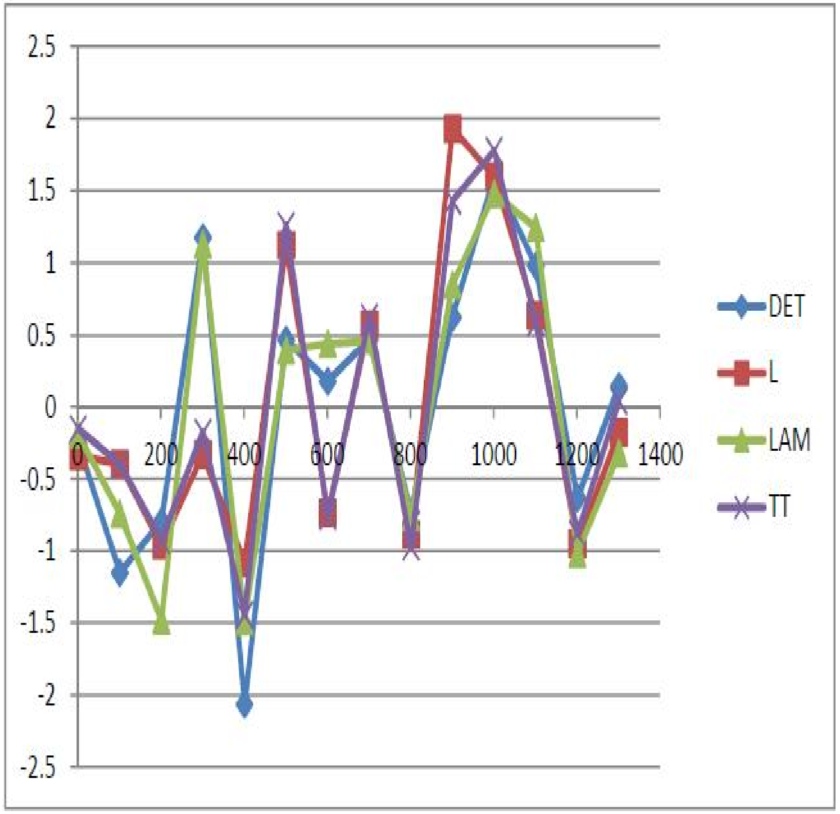


Fig 4b RQA values of October 27, 1997 mini-crash

Results

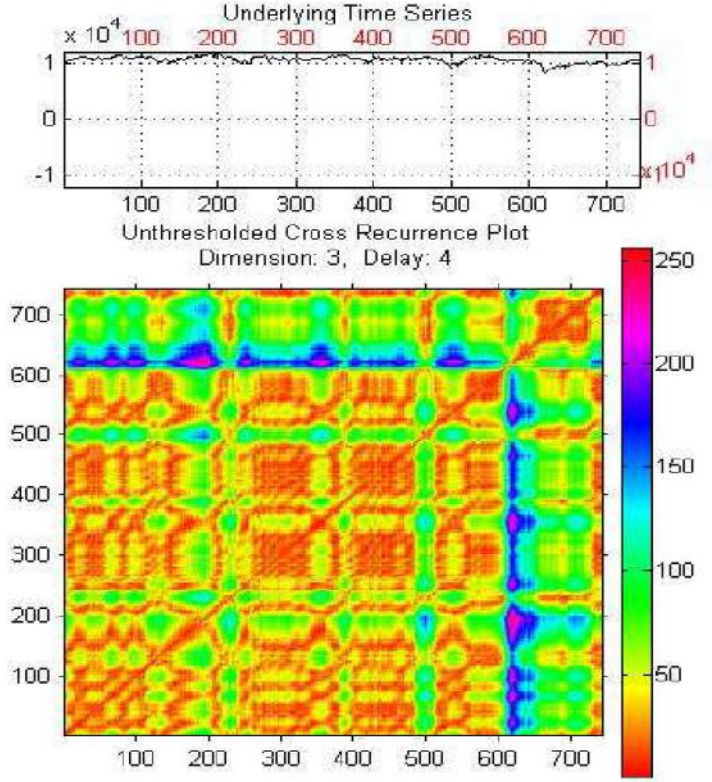


Fig 5a Recurrence Plot of Sept 11 , 2001 NYSE crash

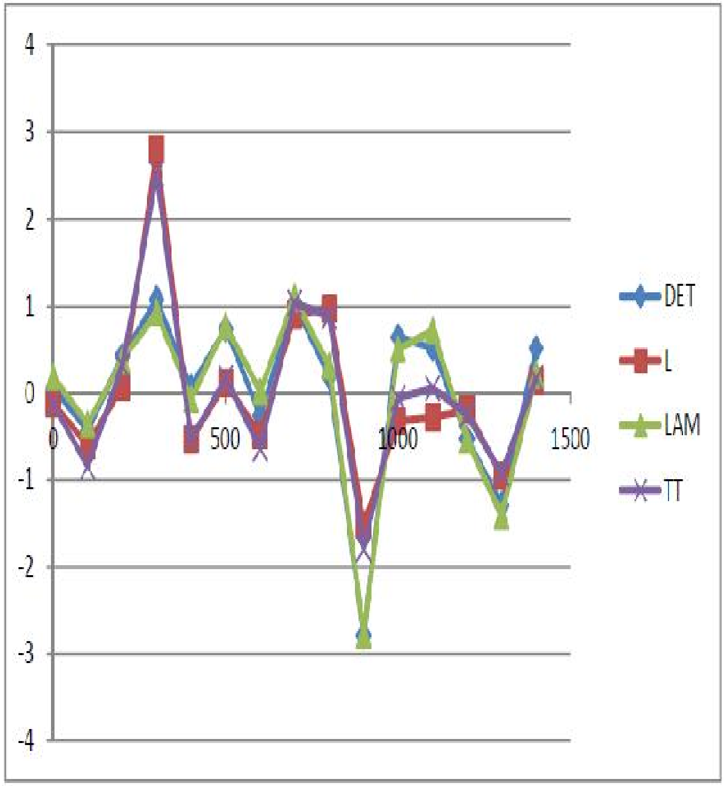


Fig 5b RQA values of Sept 11 , 2001 NYSE crash

Results

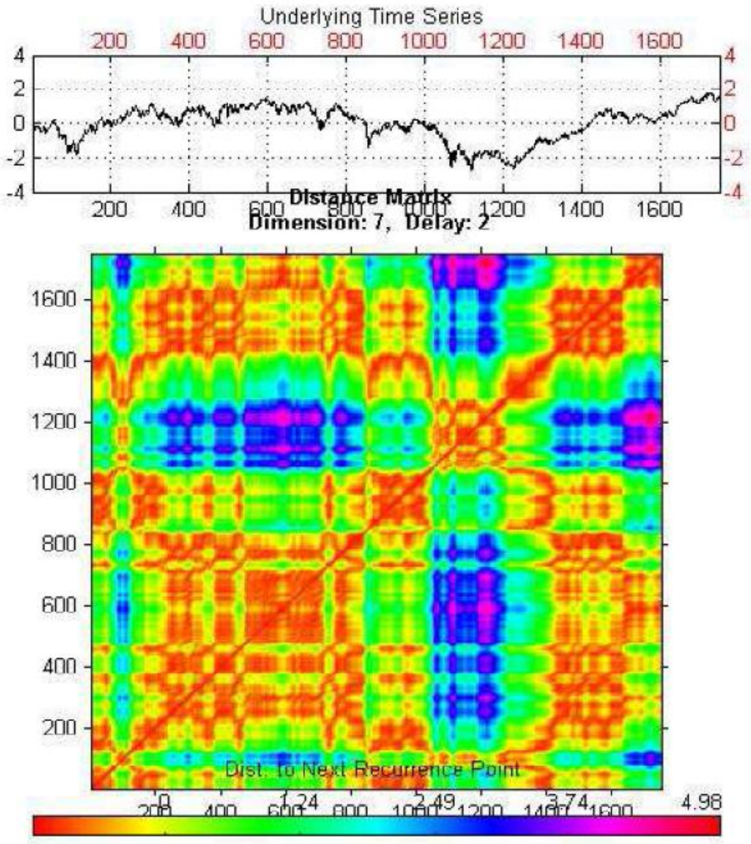


Fig 6a 2002 Recurrence Plot of downturn of DJIA

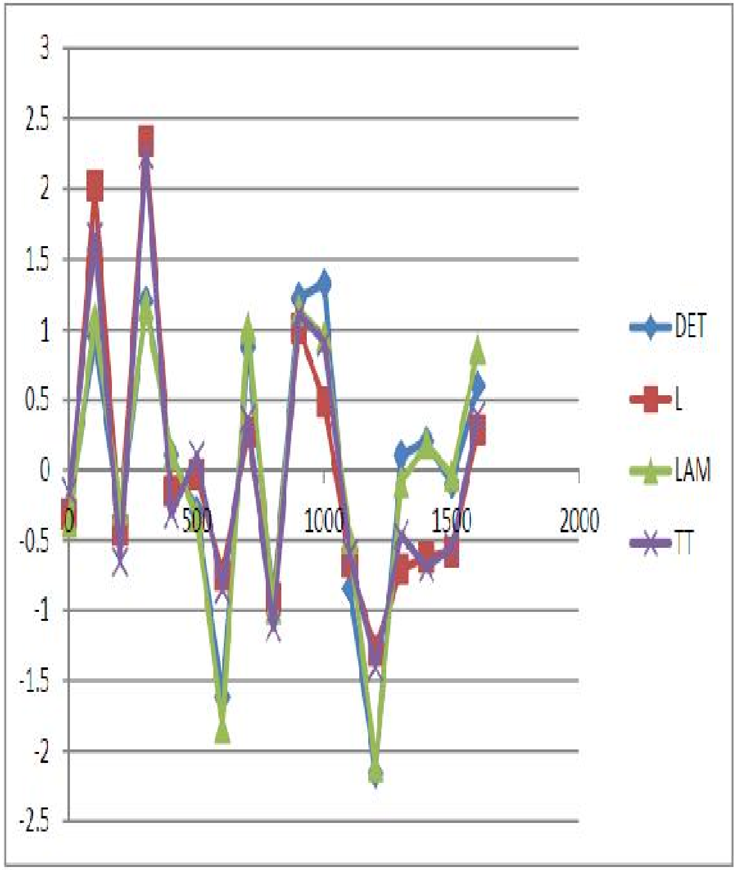


Fig6b ROA values of downturn of DJIA

Results

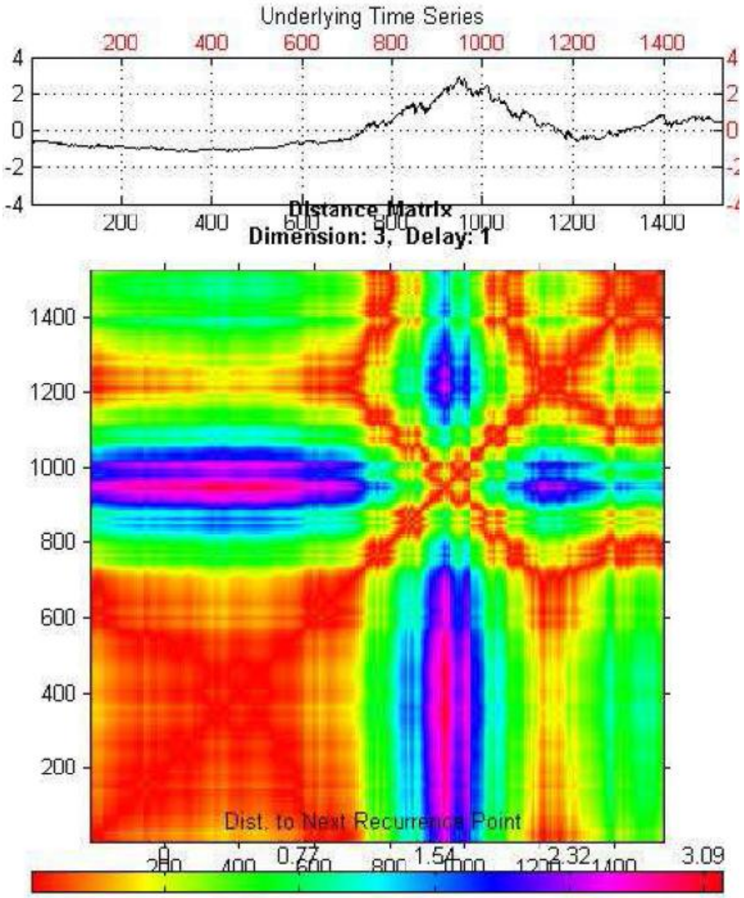


Fig 7a Recurrence plot of China 2007 Crash

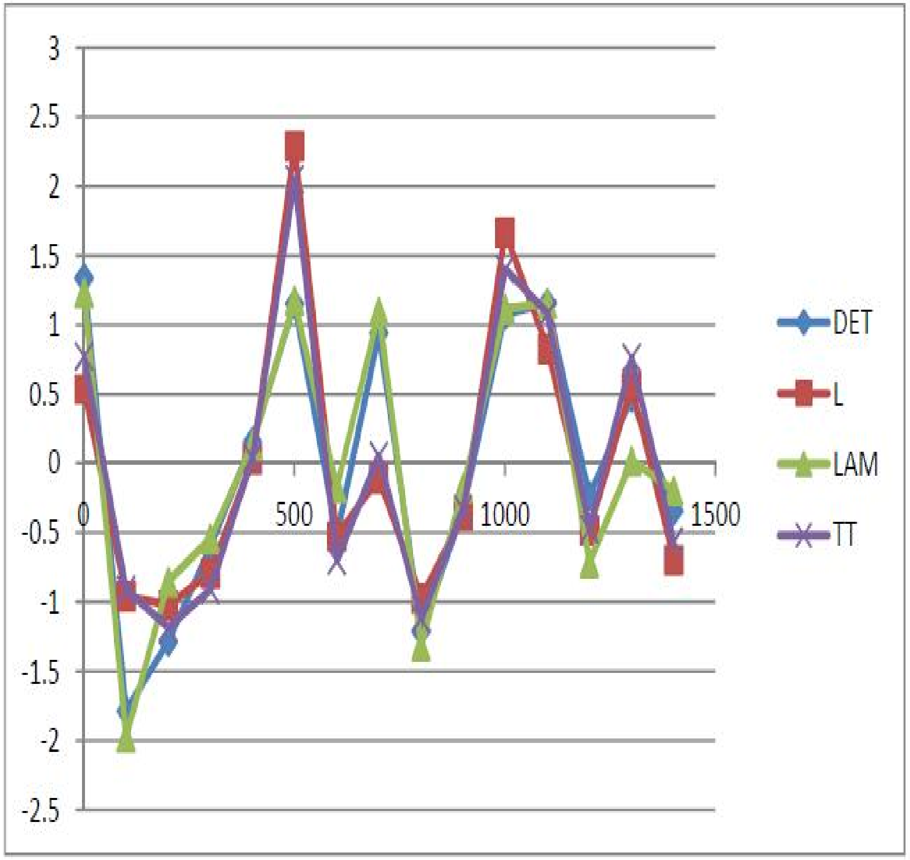


Fig 7b RQA values of China 2007 Crash

Results

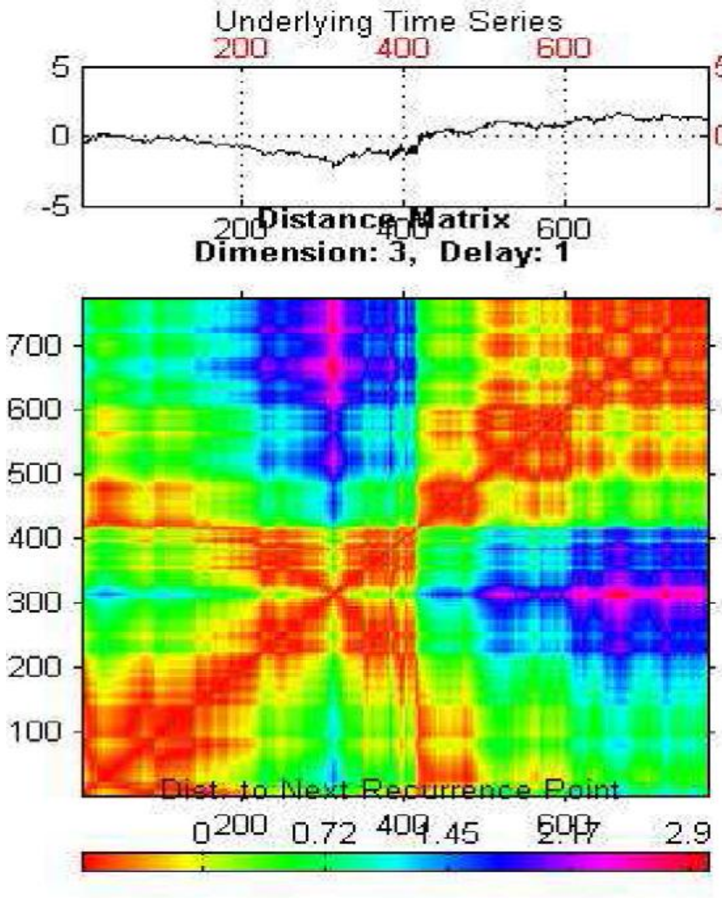


Fig 8a Recurrence Plot of 2010 Flash Crash

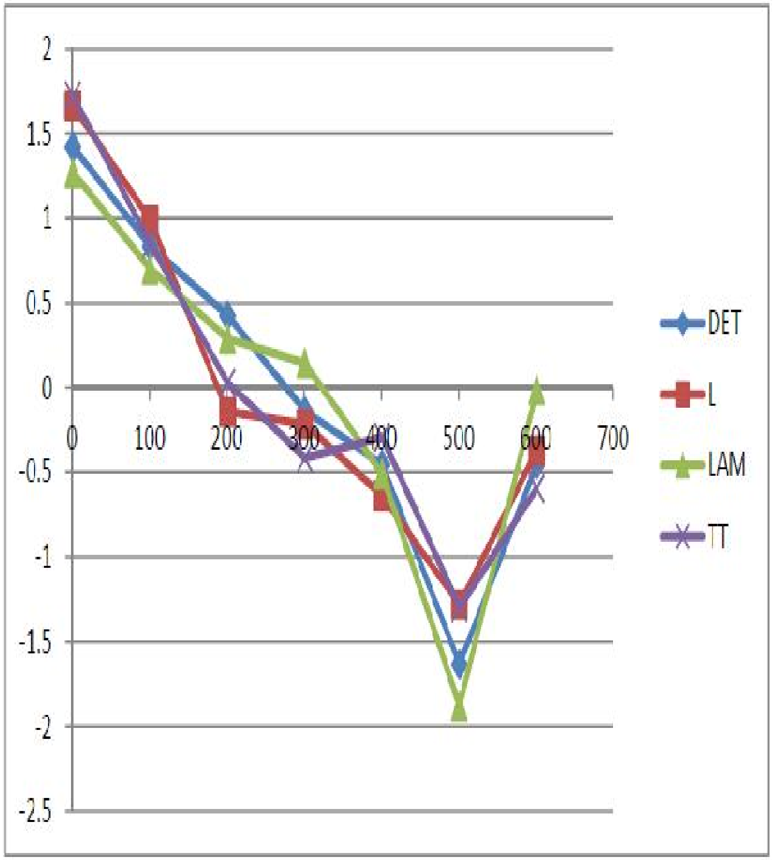


Fig 8b RQA values of 2010 Flash Crash

Discussion

- The Recurrence Plot of any time series reveals the dynamic patterns of the time evolution of the data set
- The colour bands changed over the entire regime indicating a transition in dynamic state
- The colour code : a clue about the level of determinism in the system
- Whenever we see a dark band (Blue and Magenta) emanating we can understand the dynamic system is no longer exhibiting pure stochastic behaviour but a trend is emerging instead
- With respect to stock market data, we can take this as a bubble setting in.

Discussion

- Whether the RP can display such dark bands before the rash dates
- A close inspection: that in all the cases except for the 9/11 crash and Flash crash ,known exogenous crashes, we see that a dark band is emerging
- Average length of such bands is lasting about 100 trading days - about four months of active trading
- The bubble always gathered momentum three to four months before the crash occurred
- Corresponding RQA statistics also reinforces our findings

Discussion

- The presence of dark band : formation of trend in the time signal
- Phenomenological interpretation of the same may be emergence of herd behaviour
- Shiller R. (2000)
  - speculative bubbles are motivated by "precipitating factors"
  - amplification mechanisms" that take the form of price-to-price feedback.

Discussion

- A long recurrence : can happen because of the feedback effects
- One may also refer to(Bakshi & Madan, 1998) , (Bakshi & Madan, 1999), (Bakshi, Madan, & Panayotov, 2010)
- strong relation between higher moments and crashes
- indicating the trending effect

Conclusion

- Reinforces the findings of Guhathakurta et al. (2010)
- Corresponding to the epoch of 100 days before the crash there was a marked change in the nonlinear dynamics of the system
- recurrence plot can be confidently used in identifying bubble
- Crash may be expected if it continues for three or more months

Conclusion

- RP alone can not predict cashes
- Raises doubt about the ability of the phase transition model as proposed by (Sornette D. A., 1996) to precisely predict the crashes
- But definitely, this tool may be used
  - to identify changes in market dynamics
  - can serve as a warning bell.
- Future works may evolve around modelling bubbles as regime changes

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