

Inequalities from socio-economic data

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Outline of the talk

- 1 Inequality in Socio-economic systems
 - Social inequality
- 2 Income and wealth
- 3 Cities and firms
- 4 Scaling with city size
 - Crime in a city
- 5 Bibliometrics
- 6 Voting
 - Proportional elections
 - Empirical data
 - First past the post elections
- 7 References

Social inequality

- existence of unequal opportunities and rewards for various social positions/statuses within the society
- unequal distributions of goods, wealth, opportunities, and even rewards and punishments.

Two main ways to measure:

- inequality of conditions
- inequality of opportunities

Inequality of conditions : unequal distribution of income, wealth and material goods.

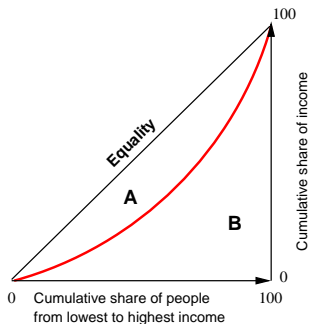
Inequality of opportunities : unequal distribution of "life chances" across individuals.

as in measures such as education, health, and treatment by the criminal justice system.

Measures of inequality

Gini coefficient measures the inequality among the values from a frequency (probability) distribution of a variable.

Lorenz curve



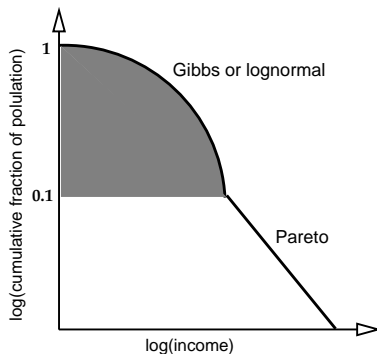
$$G = A/(A + B)$$

$G = 0$ is perfect equality, $G = 1$ is the maximum inequality.

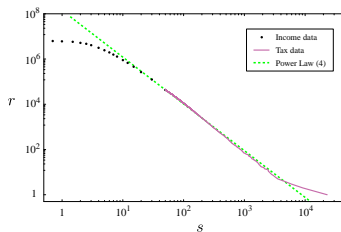
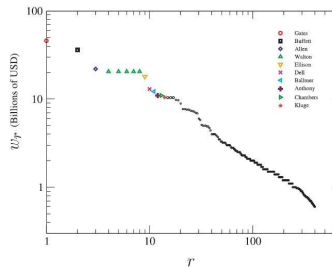
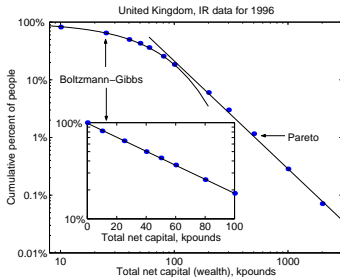
Income and wealth

Income and wealth: Empirical observations

General form of Income and wealth distributions



$$P(x) \sim \begin{cases} F(x) & \text{for } x < x_c, \\ \frac{\alpha x_c^\alpha}{x^{1+\alpha}} & \text{for } x \geq x_c, \end{cases}$$

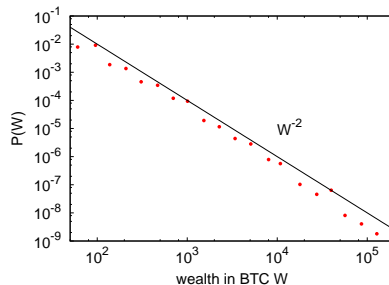
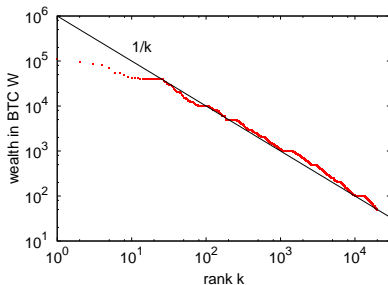


Drăgulescu+Yakovenko, Physica A 299 (2001) 213

Klass et al, EPJB 55 (2007) 143.

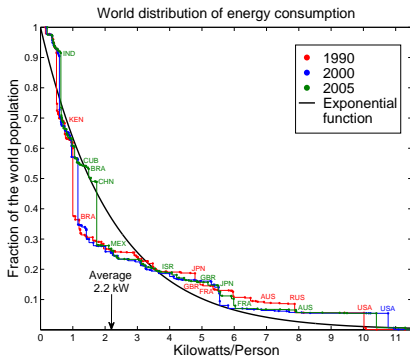
Aoyama et al, Fractals 8 (2000) 293

Wealth of Bitcoin accounts



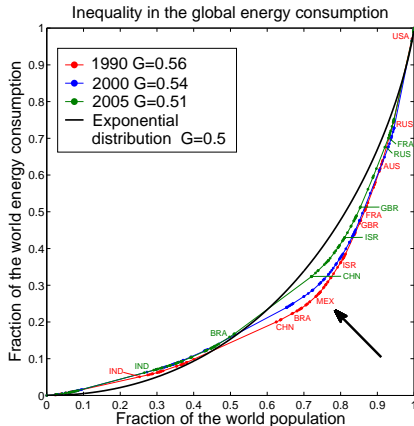
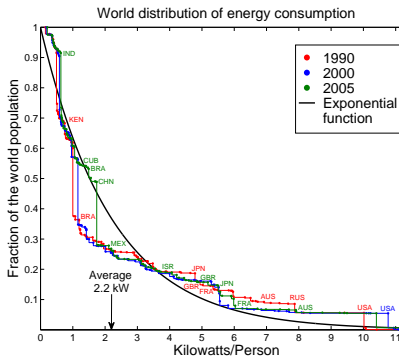
Global energy consumption

Energy consumption and global inequality

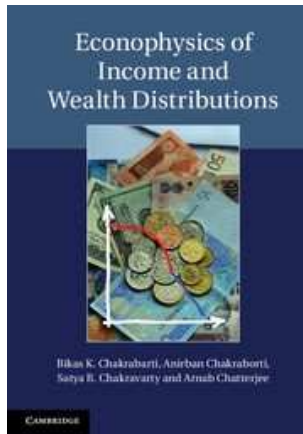


Global energy consumption

Energy consumption and global inequality



A good collection of empirical facts, data and models



BK Chakrabarti, A Chakraborti, SR Chakravarty, **AC**,
Econophysics of Income and Wealth Distributions, Cambridge Univ Press
(2013)

Cities & Firms

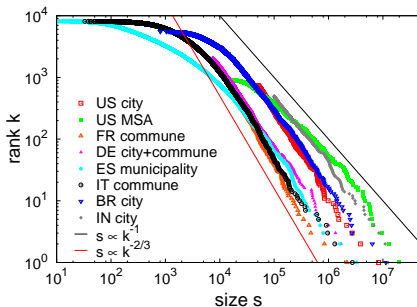
Zipf law for City size

City sizes (= population)

rank k of a city with population s goes as $s_k \sim k^{-\gamma}$.

Zipf exponent γ Distribution of size $P(s) \sim s^{-\nu}$

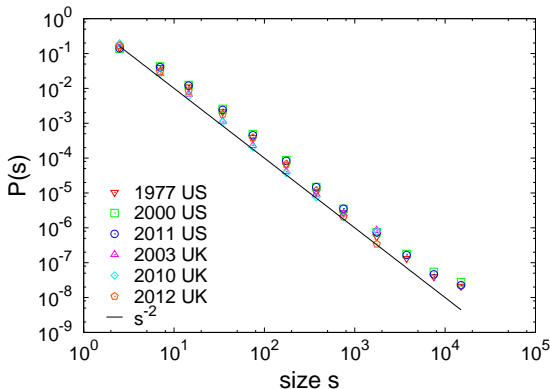
$$\nu = 1 + \frac{1}{\gamma}$$



Country	Year	ν	γ
USA	2012	2.34(5)	0.74
USA	2012	2.1(1)	0.91
France	2006	2.48(7)	0.67
Germany	2011	2.18(2)	0.85
Spain	2011	2.3(1)	0.77
Italy	2010	2.3(2)	0.77
Brasil	2012	2.14(4)	0.88
India	2011	2.6(4)	0.63

lack of universality in Zipf exponent

Zipf law for firms



Zipf exponent perhaps more stable (?)

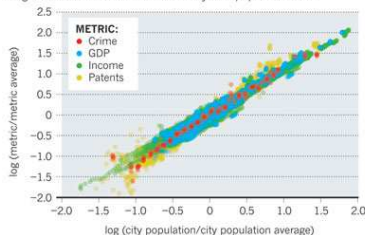
Scaling law for cities

Analysis of large urban datasets across decades and for several countries

- Size is the major determinant
- space required per capita shrinks: denser settlement, intense use of infrastructure
- pace of all socio-economic activities accelerate → higher productivity
- Socio-economic activities diversity and become more interdependent

PREDICTABLE CITIES

Data from 360 US metropolitan areas show that metrics such as wages and crime scale in the same way with population size.



As city size increases, several quantities increase by small factor more than linear

growth – **superlinear scaling** $Y = Y_0 N^\beta$ with $\beta > 1$.

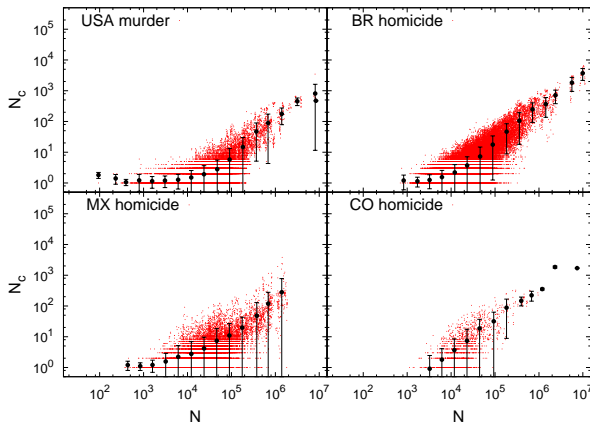
Bettencourt et al, *Proc Nat Acad Sci* (2007), *EPJB* (2008), *Plos One* (2010, 2012)

Bettencourt + West, *Nature* (2010); Bettencourt, *Science* (2013)

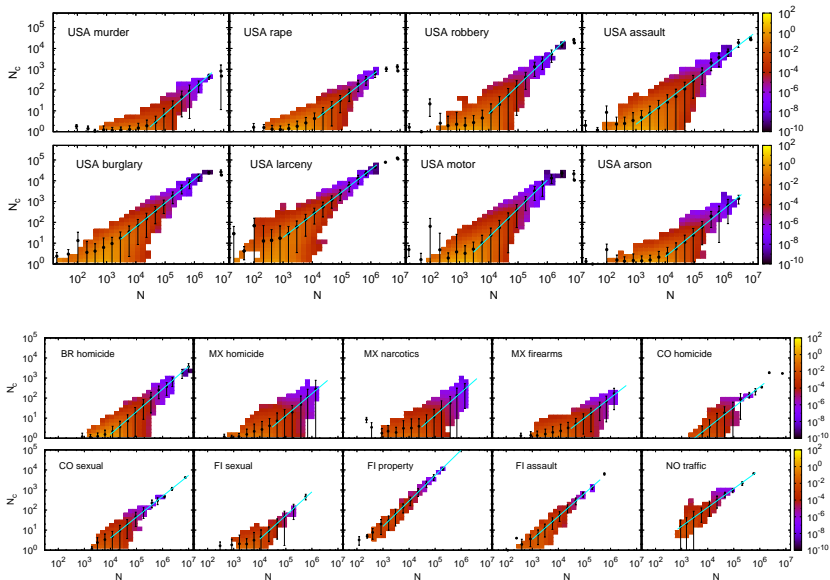
Crime in a city

How the number of crimes N_c scale with city size N ?

Violent crime: **Murder/Homicide**



Crime in a city



Universal trend:

Upto N^* , no scaling with city size, i.e, $N_c \propto N^0$, either small or practically constant, depending on the crime type.

Beyond N^* , which also depends on the crime category c , the scaling regime appears $N_c \propto N^{\beta_c}$.

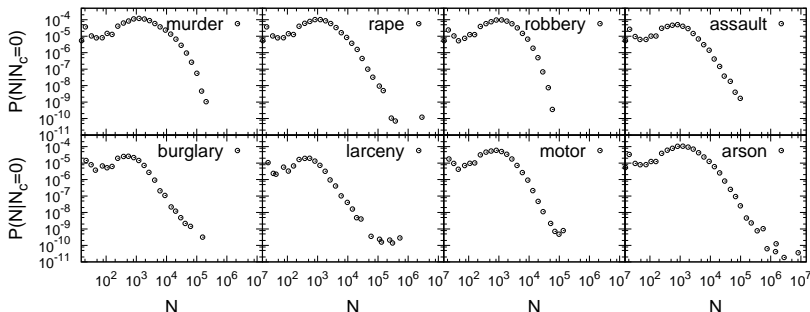
$$N_c = \begin{cases} AN^0, & N \leq N^* \\ N_0 N^{\beta_c}, & N > N^*, \end{cases}$$

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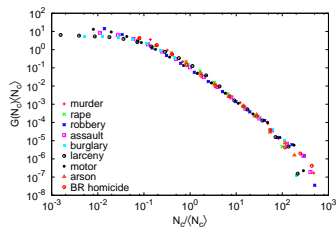
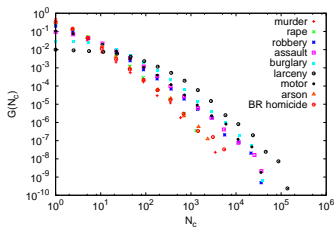
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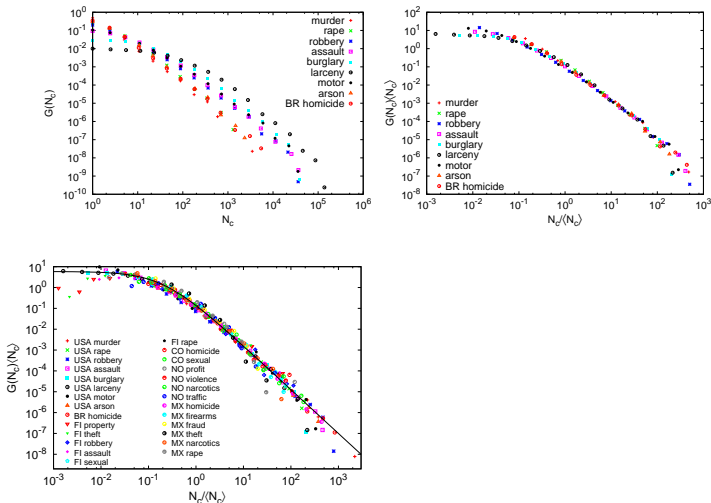
Crime categories from different countries, scaling regime and exponent

Country	crime	scaling region, in N	β_c
USA	larceny	1000 — 1600000	1.09(1)
	burglary	3000 — 1600000	1.14(2)
	robbery	10000 — 4000000	1.35(3)
	rape	15000 — 1600000	1.10(3)
	murder	30000 — 4000000	1.23(4)
	motor	3000 — 1000000	1.37(2)
	arson	10000 — 4000000	1.15(3)
	assault	1000 — 10000000	1.08(3)
Brasil	homicide	10000 — 10000000	1.12(2)
Mexico	homicide	30000 — 4000000	1.09(7)
	narcotics	30000 — 4000000	1.14(3)
	firearms	30000 — 4000000	1.01(4)
	fraud	30000 — 1000000	1.03(2)
Colombia	homicide	3000 — 1500000	1.02(3)
	sexual	10000 — 10000000	1.04(2)
Finland	sexual	10000 — 1000000	1.17(3)
	property	1000 — 1000000	1.26(1)
	assault	1000 — 400000	1.14(1)
	rape	18000 — 700000	1.22(3)
	theft	1500 — 700000	1.30(1)
	robbery	18000 — 700000	1.52(6)
Norway	profit	2500 — 700000	1.29(2)
	narcotics	2500 — 700000	1.15(4)
	violence	2500 — 700000	1.12(1)

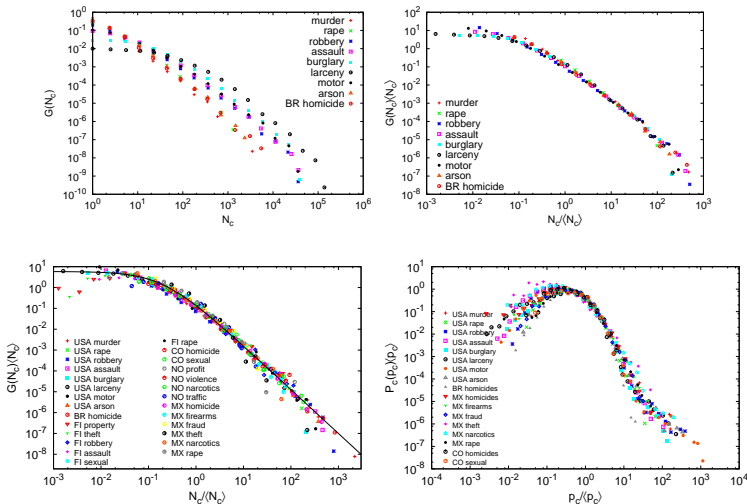
Number of crimes N_c and rate of crime $p_c = N_c/N$



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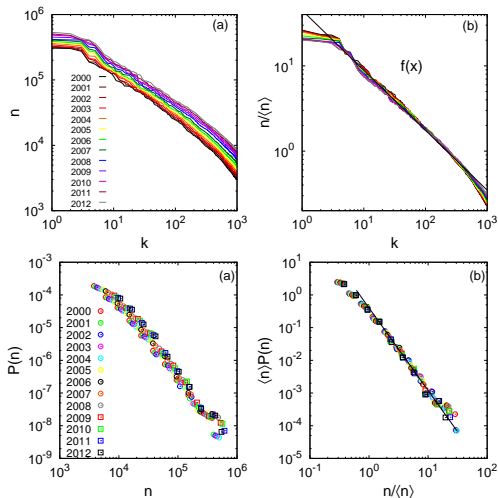
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Bibliometrics

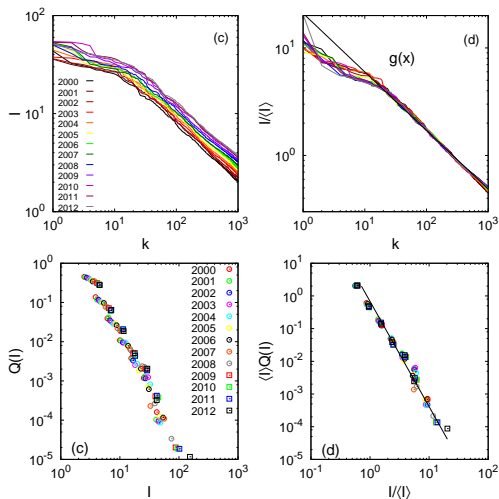
Annual Citations of Journals

Top 1000 journals ranked according to Annual Citations



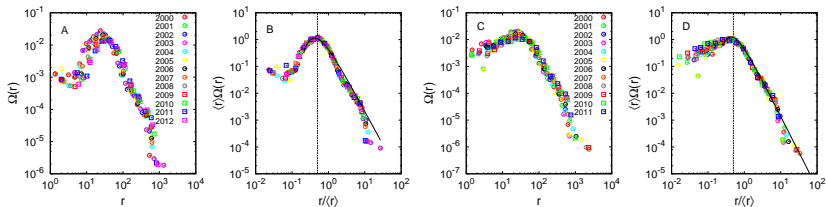
Journal Impact Factors

Top 1000 journals ranked according to Impact Factors



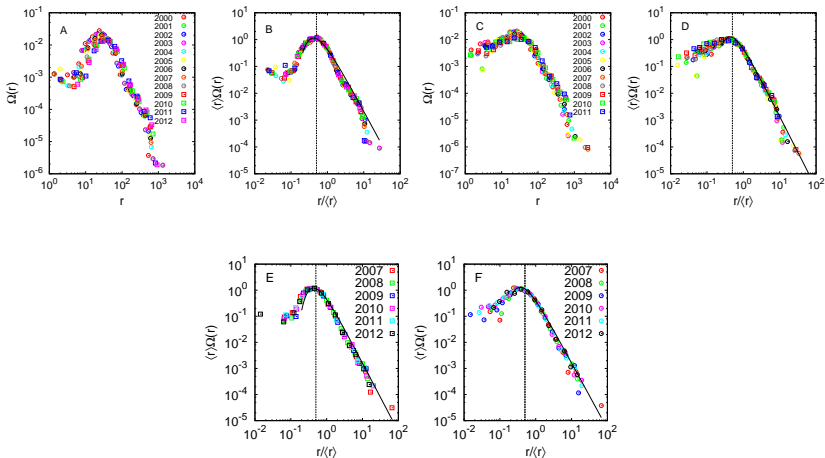
Journal Citation Rates

Annual citation rate $r(T) = n(T)/N(T)$.

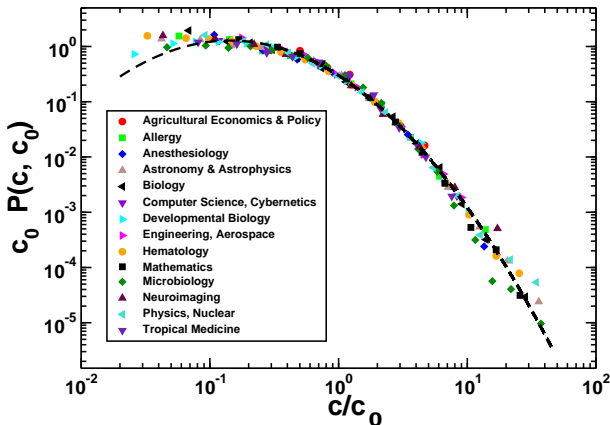


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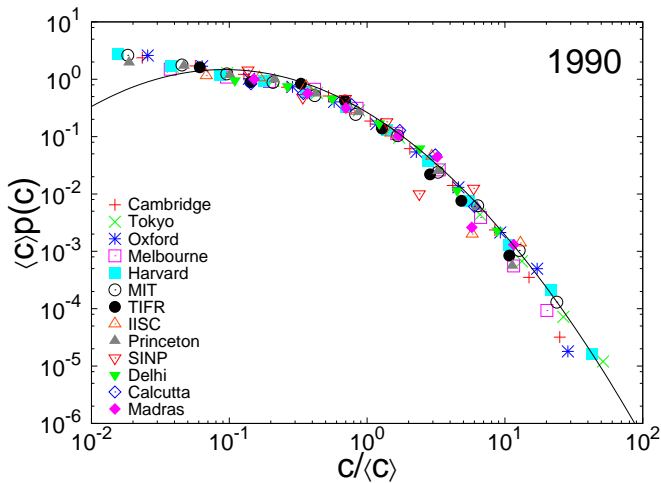


Journal citations

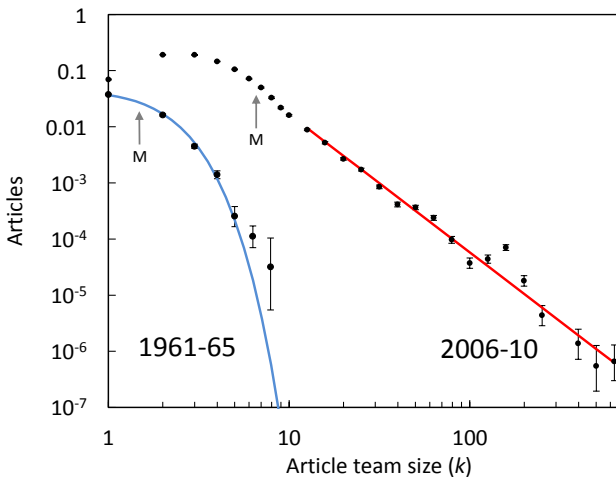


F. Radicchi, S. Fortunato, and C. Castellano. PNAS (2008)

Citations for institutions



Distribution of article team sizes in astronomy

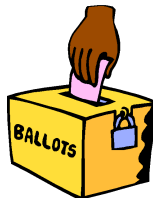


VOTING



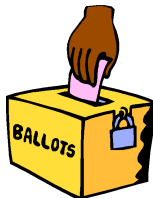
Voting

- Elections are among the largest social phenomena



Voting

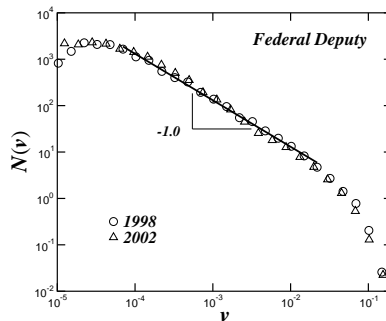
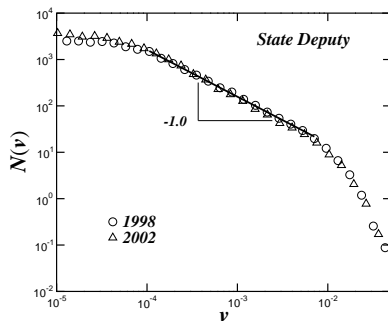
- Elections are among the largest social phenomena



- Election data has been well studied over years
 - Statistics of turnout rates
 - Detection of election anomalies
 - Polarization and tactical voting
 - Relation between party size and temporal correlations
 - Relation between number of candidates and number of voters
 - Emergence of third parties in bipartisan systems

Early reports

Fraction of votes v received by a candidate



Costa Filho et al. *Phys. Rev. E* **60** 1067-1068 (1999)

Physica A **322** 698-700 (2003)

Proportional elections

One of the widely used elections systems

Basic principle:

- All voters deserve representation and all political groups deserve to be represented in legislatures in proportion to their strength in the electorate.

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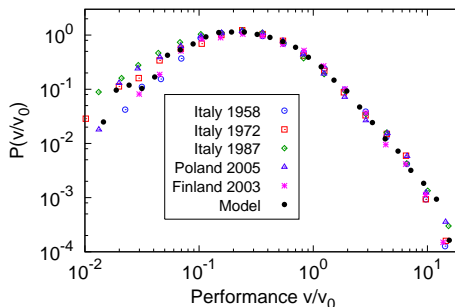
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- Number of seats allocated to a party in a district is proportional to the number of votes.
- **Open**, **Semi-open** and **Closed** lists.

Different quantities and Scalings

- **FC**: the number of votes v^i of a candidate is divided by the average number of votes $v_0 = N_i/Q^i$ of all candidates in his/her party list.
Compute distribution of v/v_0
- **CAAMd**: the fraction of votes received by a candidate in an electoral district.
Costa Filho, Almeida, Andrade and Moreira (CAAM)
Costa Filho et al, *Phys Rev E* **60** 1067-1068 (1999).
Compute distribution of v/N_D
- **CAAMn**: the fraction of votes received by a candidate compared at national level.
Compute distribution of v/N_T

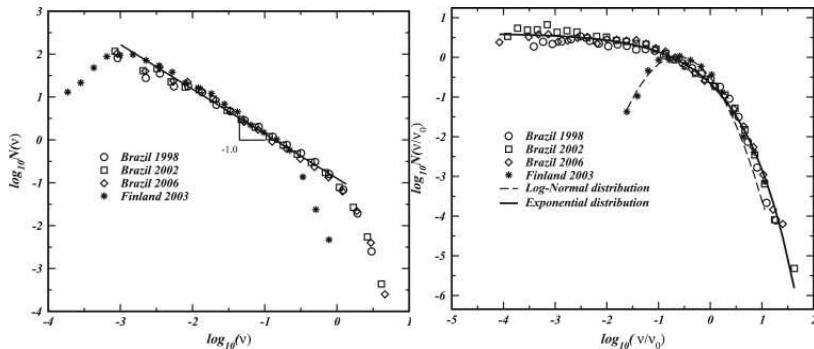


Universal scaling curve for open list proportional elections
Fits a log-normal very well

$$P(v/v_0) = \frac{v_0}{\sqrt{2\pi}\sigma v} \exp \left[-\frac{(\log(v/v_0) - \mu)^2}{2\sigma^2} \right]$$

with $\mu = -0.54$, $\sigma^2 = -2\mu = 1.08$

S Fortunato & C Castellano, *Phys. Rev. Lett.* **99** 138701 (2007)



This scaling is not convincing.
Does not speak of universality across a wide range.

LE Araripe & RN Costa Filho, *Physica A* **388** 4167-4170 (2009)

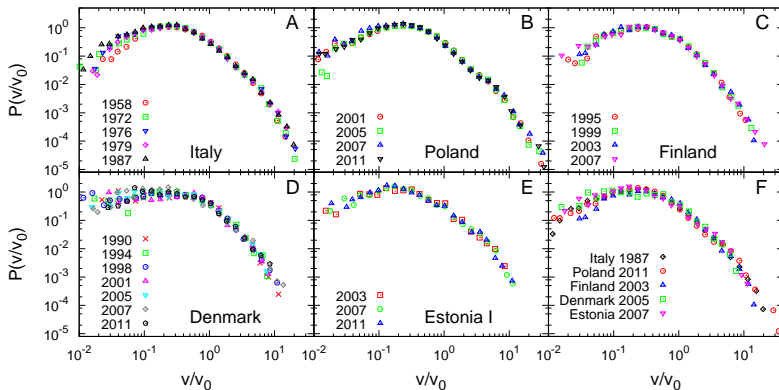
An exhaustive analysis of 53 datasets from 15 countries, to

- ... investigate and assess the universal behavior
- ... find the limitations/ exceptions to the universal behaviors
- ... provide a quantitative assessment

A Chatterjee, M Mitrović, S Fortunato, *Scientific Reports* **3** 1049 (2013).

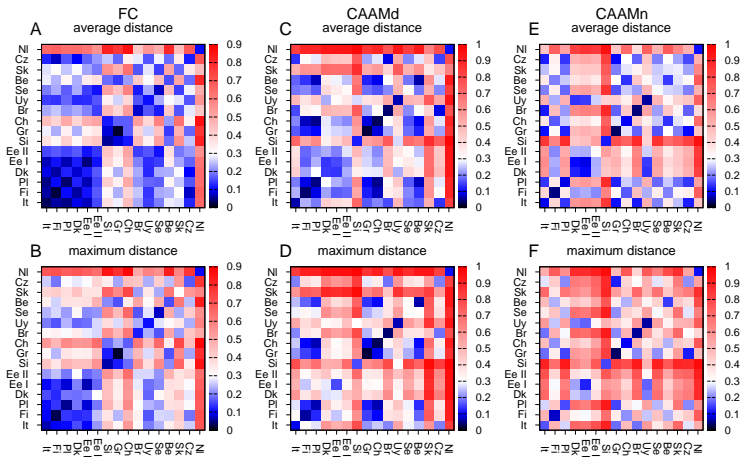
FC scaling

Electoral performance of candidates; proportional elections with open lists



Quantitative assessment of similarity between distributions at national level and between countries

Average K-S distance between datasets of different countries



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VIEW

4 COMMENTS



The Physics arXiv Blog

December 12, 2012

Sociophysicists Discover Universal Pattern of Voting Behaviour

The same voting patterns crop up in every country that shares a particular type of electoral system, say sociophysicists

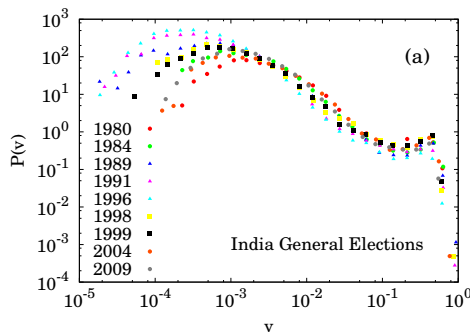
First past the post elections

In *first-past-the-post* (FPTP) or simple plurality, the candidate with the maximum votes is declared the winner, and hence referred to as the *winner takes all* system. It is used in Canada, the lower house of parliament (Lok Sabha) in India, the United Kingdom, in many of the former British colonies and the majority of elections in the USA.

Duverger's law: a political system using a FPTP system will eventually tend to become a two-party system in the long run.

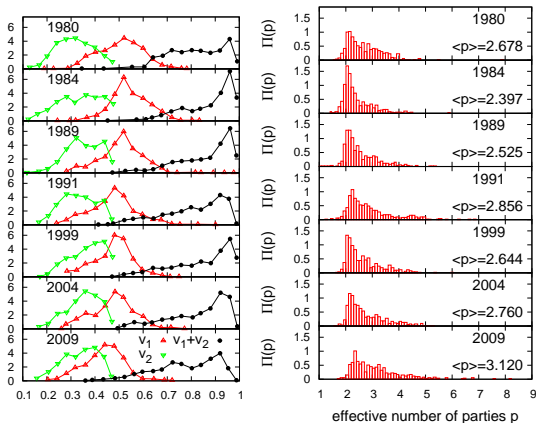
Historical evidences, however, do not prove this to be always true, as seen in many countries.

Analysis of election data from India (Parliamentary elections as well as state legislative elections), Bangladesh, Tanzania, United Kingdom, USA.



$P(v)$ curves very similar, with a lognormal body in the bulk and a second [leaderboard](#) peak around $v^* \approx 1/2$.

indicative of an emergent two-party/coalition competition?



Closed list proportional systems: very similar outcomes

work in preparation

resource allocation

to allocate resources properly

to make efficient use of resources to reduce inequality

A Chakraborti, D Challet, **AC**, M Marsili, Y-C Zhang, B K Chakrabarti,
arXiv:1305.2121.

Conclusions

- Socio-economic inequality
- Universal patterns exist in certain quantities, over time and space
- To understand society better, further quantities could be measured, and their interdependence be studied
- Efficient allocation of resources could be one of many ways to counteract socio-economic inequality.

References

Wealth distribution, city size

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book BK Chakrabarti, A Chakraborti, SR Chakravarty, AC, *Econophysics of Income and Wealth Distributions*, Cambridge Univ. Press (2013)

Crime in cities

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review A Chakraborti, D Challet, AC, M Marsili, Y-C Zhang, B K Chakrabarti, arXiv:1305.2121.

Acknowledging collaborations with

BK Chakrabarti, SS Manna, P Sen (Kolkata)

M Mitra, A Ghosh, S Biswas, A Khaleque, T Naskar (Kolkata)

M Marsili (Trieste), D De Martino (Trieste, Rome)

M Mitrović, S Fortunato (Espoo)

M Barthelemy (Paris)

AS Chakrabarti (Kolkata, Boston)