Unequal Distribution of Wealth

in Artificial Market Economies

Thomas Lux

Department of Economics
University of Kiel

International Workshop on Econophysics of Wealth Distribution,
Saha Institute Kolkota, 15 - 19 March 2005

Email: lux@bwl.uni-kiel.de, *http://www.bwl.uni-kiel.de/vwlinstitute/gwrp/german/team/lux.html*

Content

- A Two-Country Monetary Exchange Model and the Role of Wealth

  (as it turns out this role might be very limited)

- Exchange Models and Emerging Wealth Distribution : A Market-Based Approach

Two-Country Monetary Exchange Model
(Kareken/Wallace Economy)

- Two generations, two countries, agents live for two periods
- Two assets: money holdings in home and foreign currency
- No production, given endowments, one homogeneous good
- -> no international trade, only capital movements, young agents save and decide about capital allocation, spend their savings when old
- Flexible exchange rates
- Identical agents (identical utility function)

Agents‘ Optimization Problem

max U(c(t), c(t+1))

subject to:

$$c(t) \le w_1 - s(t) = w_1 - \frac{m_1(t)}{p_1(t)} - \frac{m_2(t)}{p_2(t)}$$

$$c(t+1) \le w_2 + \frac{m_1(t)}{p_1(t+1)} + \frac{m_2(t)}{p_2(t+1)}$$

w_1, w_2 : endowments

m_1, m_2 : money demand

p_1, p_2 : price levels

Strategic choice variables: $c(t)$ and $f(t) = \frac{m_1(t)/p_1(t)}{s(t)}$

Prices: $p_1(t) = \frac{H_1}{\sum_i f_i(t)\, s_i(t)}$, $p_2(t) = \frac{H_2}{\sum_i (1-f_i(t))\, s_i(t)}$, $e(t) = \frac{p_1(t)}{p_2(t)}$

H_1, H_2 : money supply , $i = 1, 2, \ldots, N$: agents

Equilibria:

$$\frac{p_1(t)}{p_1(t+1)} = \frac{p_2(t)}{p_2(t+1)} \Leftrightarrow e(t+1) = e(t)$$

Consequences:

(1) equilibrium exchange rate is indeterminate, $e^* \in (0, \infty)$

(2) equilibrium portfolio composition is indeterminate, $f^* \in [0, 1]$

(3) equilibrium consumption from maximization of $U(c(t), w_1 + w_2 - c(t))$

Selection of equilibrum?
Out-of-equilibrium dynamics?

Learning of agents via *genetic algorithms*:

- each agent‘s choice variables are encoded in a *chromosome*

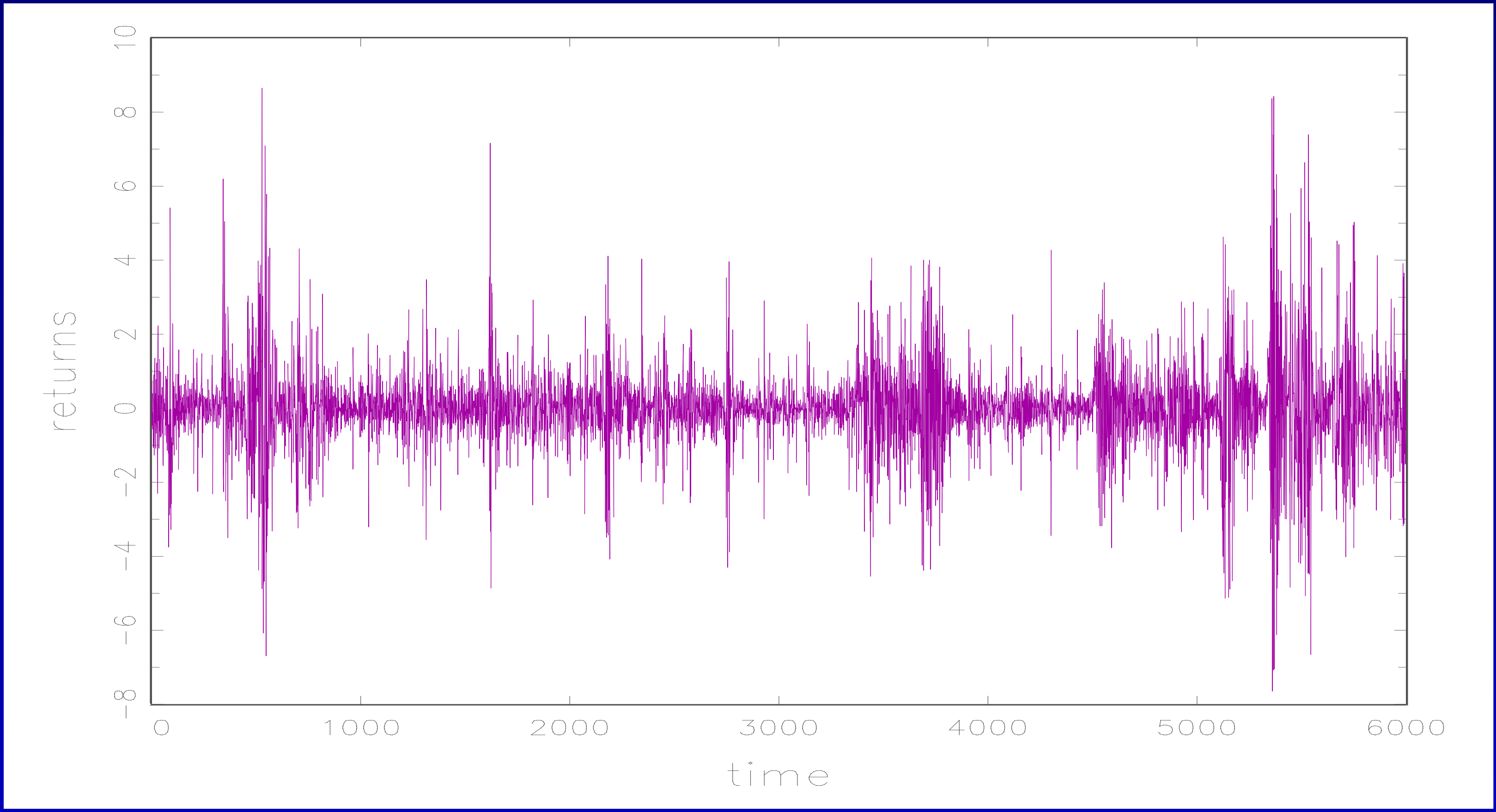
- after lifespan of each generation (2 periods), a new generation is formed via genetic operations:

  (i) *reproduction* according to fitness (utility)
  (ii) *crossover*: recombination of genetic material
  (iii) *mutation*
  (iv) *election*: new chromosomes replace existing ones only if at least as fit as parents

(Lux and Schornstein, J. of Mathematical Ec., 2005)

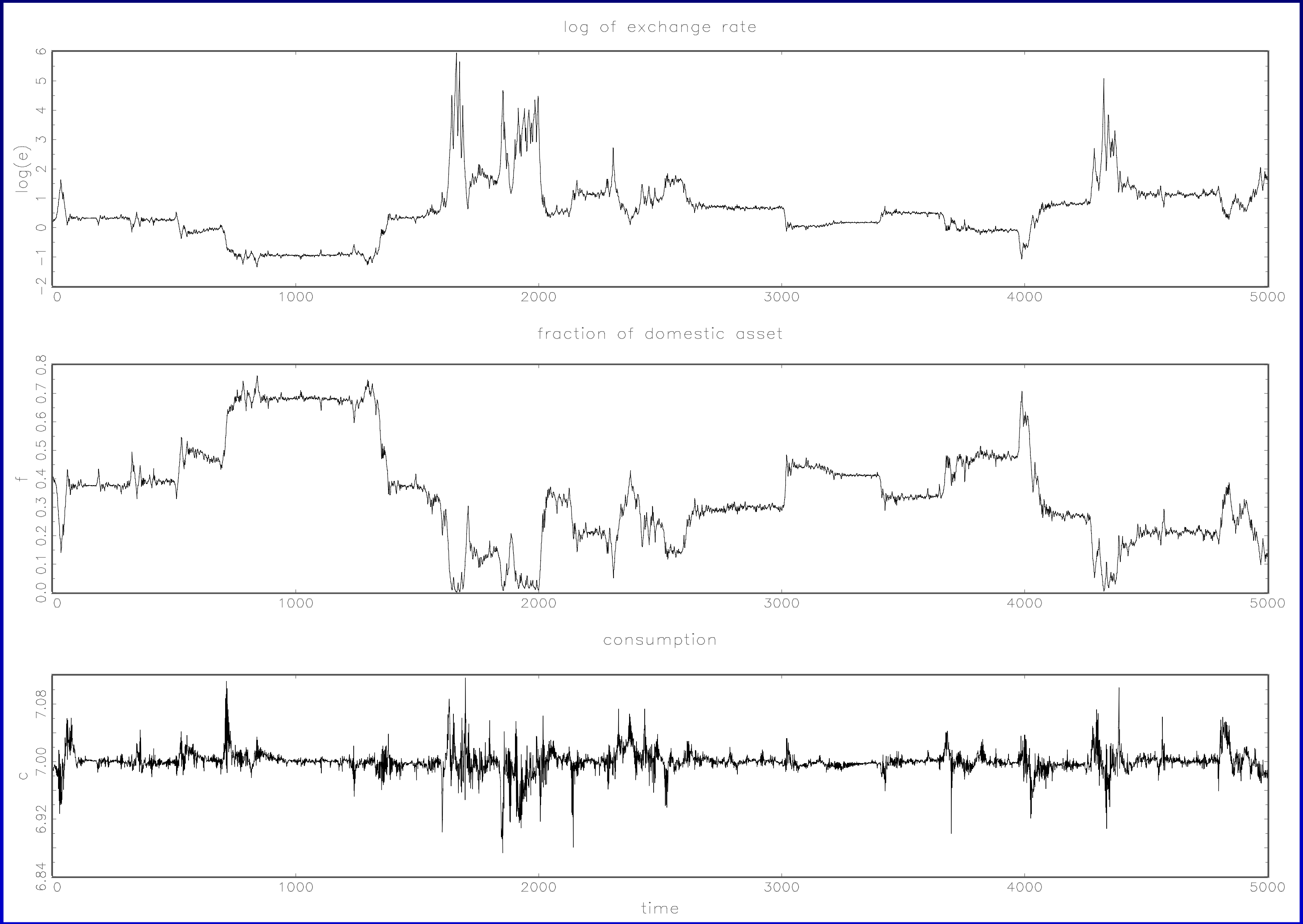
Example with realistic time series properties of returns: Binary coded GAs:

50 agents, $p_{mut} = 0.01$, $w_1 = 10$, $w_2 = 4$, $U = c(t)c(t+1)$

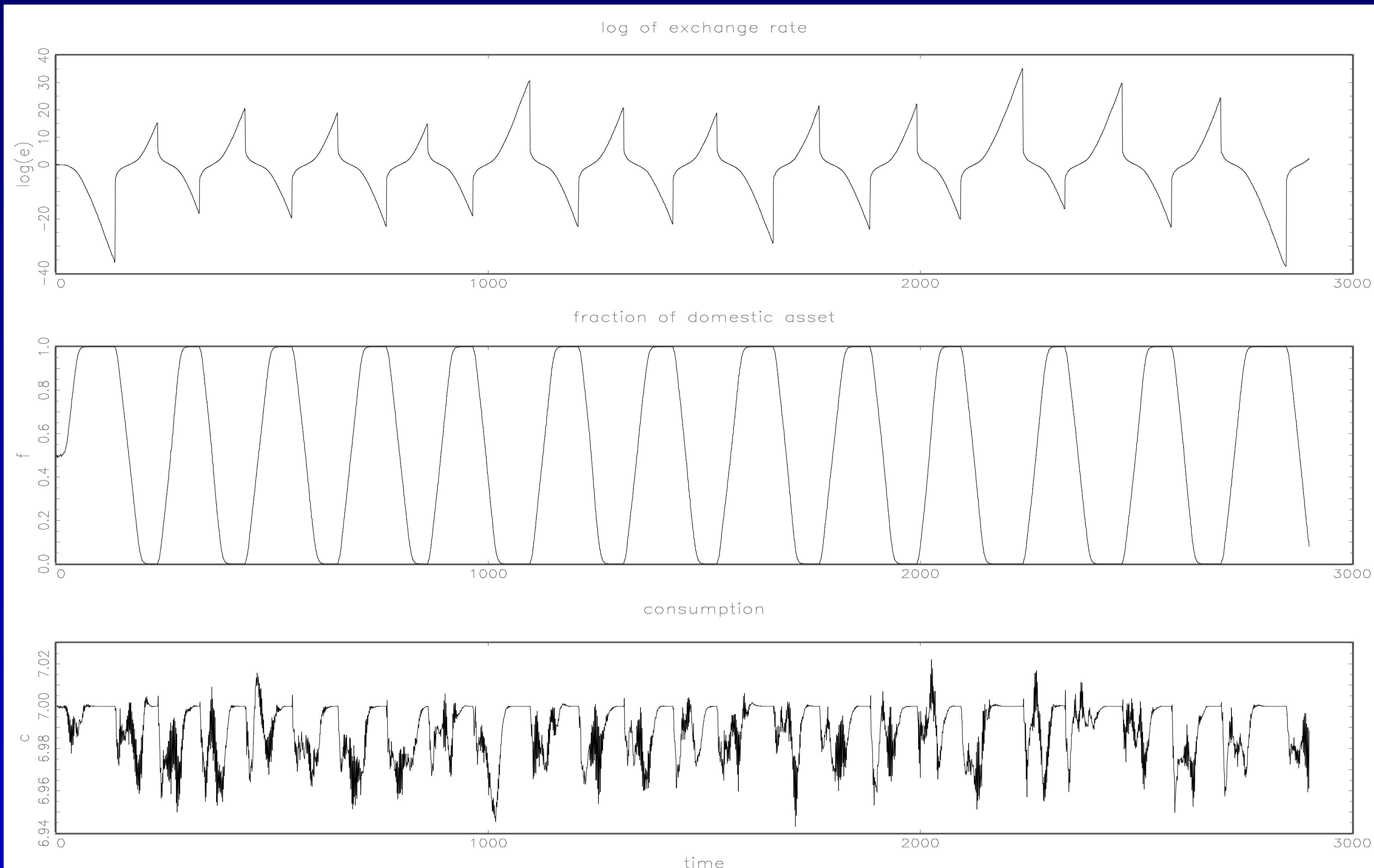


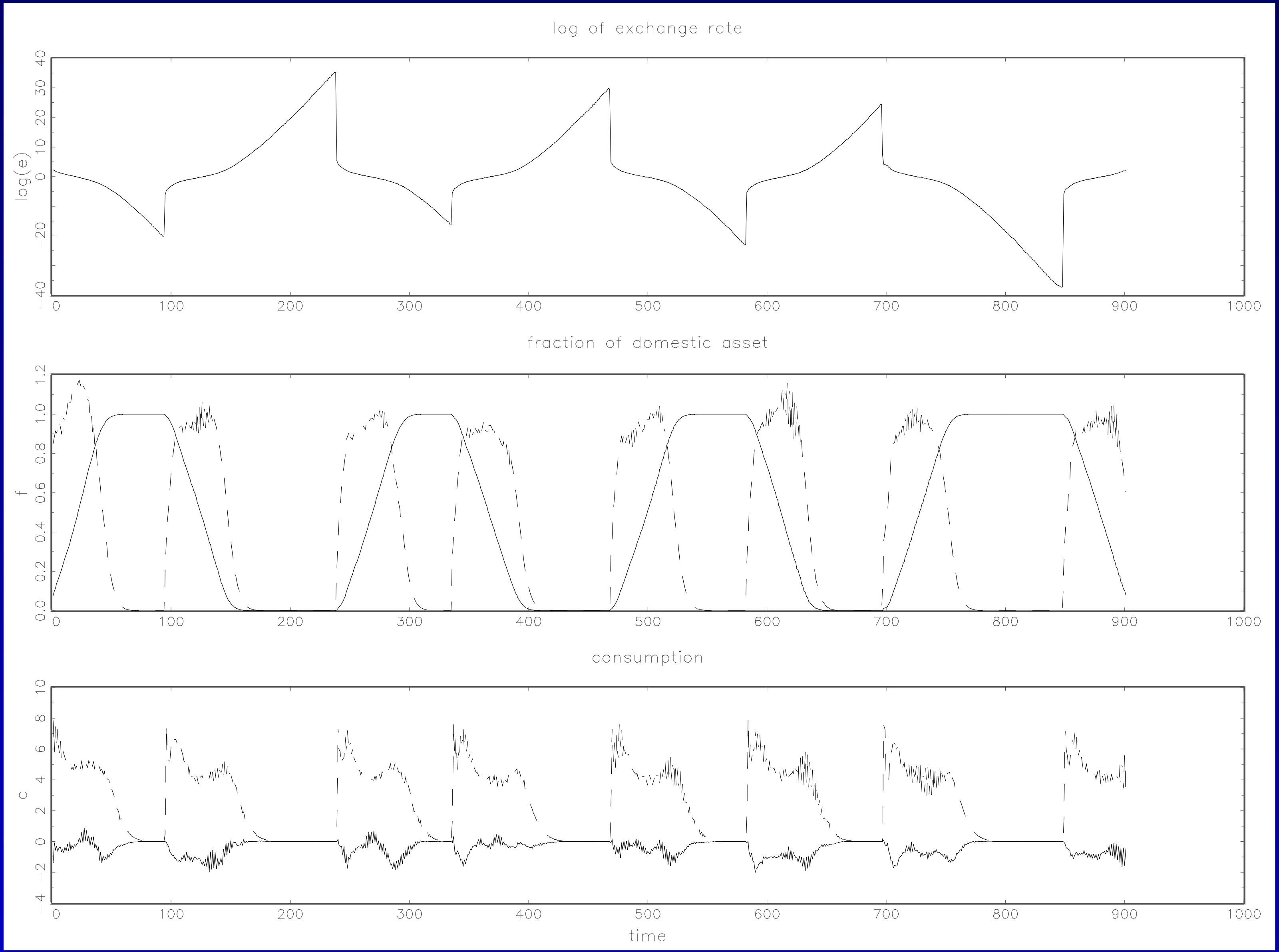
Question: sensitivity with respect to genetic algorithm parameters and number of agents

Influence of number of agents: real coding, N = 200



Influence of number of agents: real coding, N = 20,000





First and second moments

The Large Economy Limit

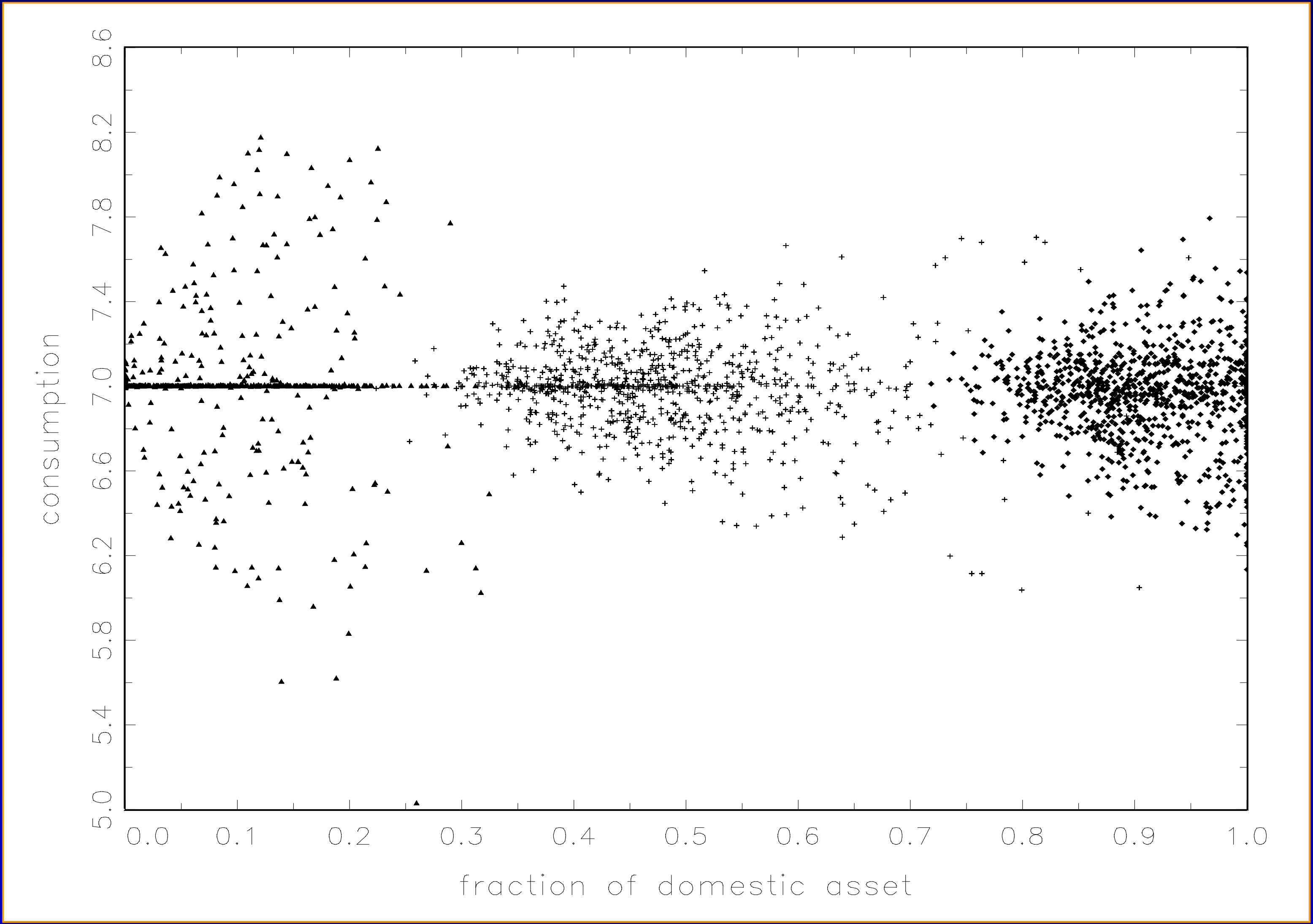
GA learning leads to gradual adjustment of choice parameters towards momentary optimum:

$$c^*(t) = \frac{1}{2}\left(w_1 + \frac{w_2}{f(t)\frac{p_1(t)}{p_1(t+1)} + (1-f(t))\frac{p_2(t)}{p_2(t+1)}}\right)$$

$$f^*(t) = \begin{cases} 1 & \text{if } \frac{p_1(t)}{p_1(t+1)} > \frac{p_2(t)}{p_2(t+1)} \\ 0 & \text{if } \frac{p_1(t)}{p_1(t+1)} < \frac{p_2(t)}{p_2(t+1)} \end{cases}$$

with $U = c(t)\, c(t+1)$

-> cyclic dynamics between corner equilibria $f = 0$ and $f = 1$

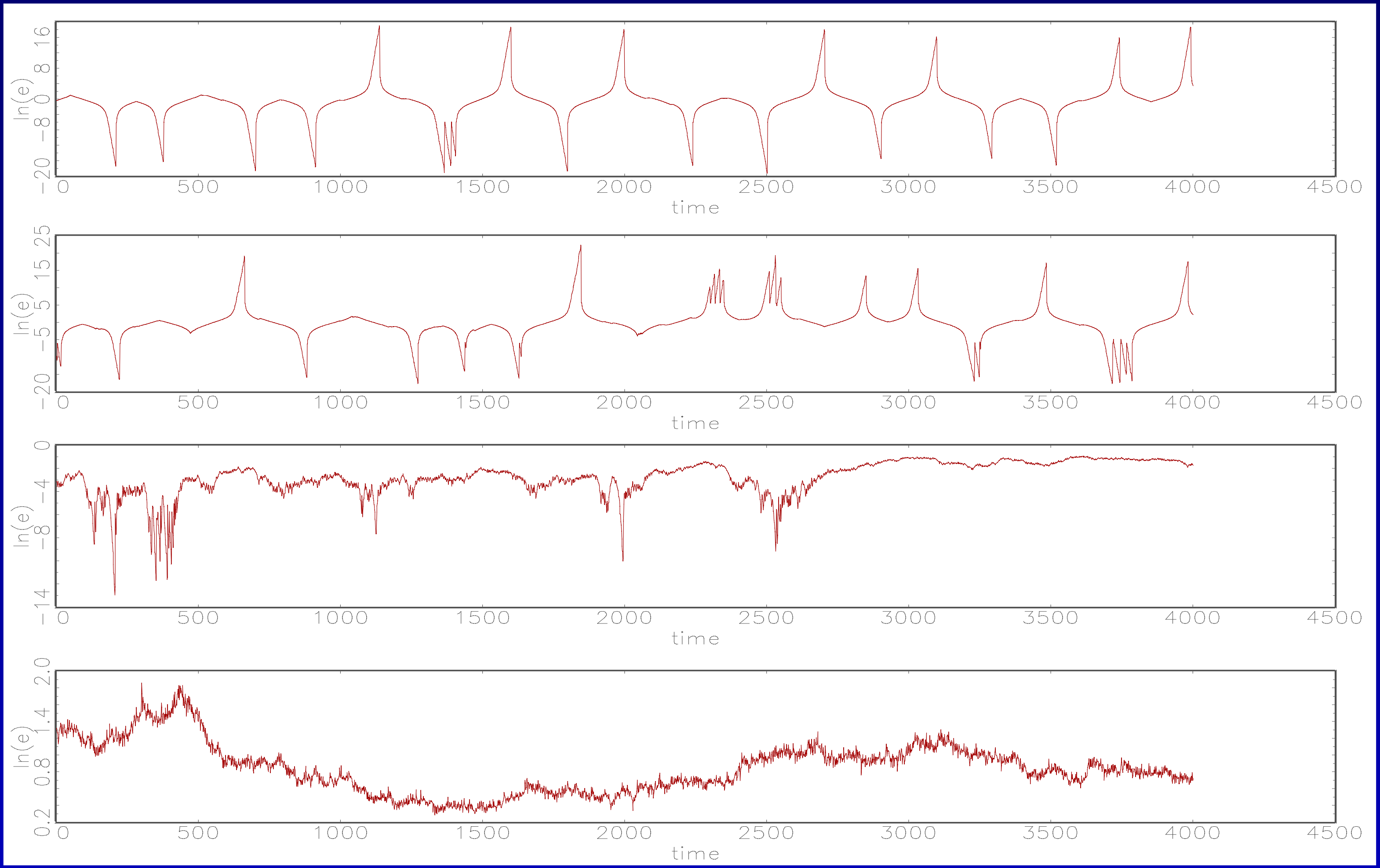


A snapshot of the evolution of the population like in many agent-based models of financial markets, the interesting dynamics gets lost with increasing numbers of agents

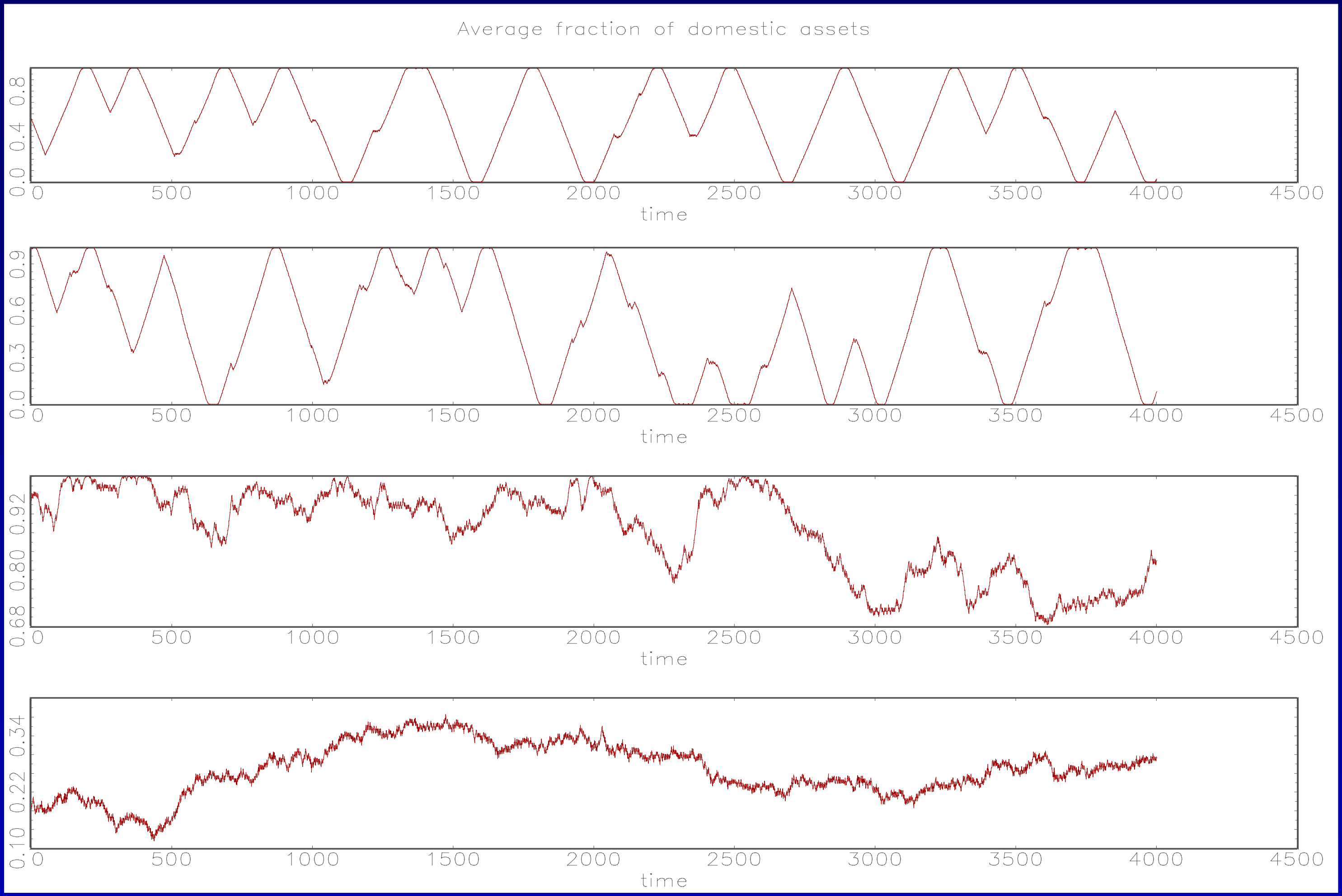
Question: can we safe it with an unequal distribution of wealth (endowments) or some similar assumption?*

*Gabaix et al.: power laws of returns are due to power law of size distribution of investors, Solomon: *vice versa*

Unequal distribution of endowments

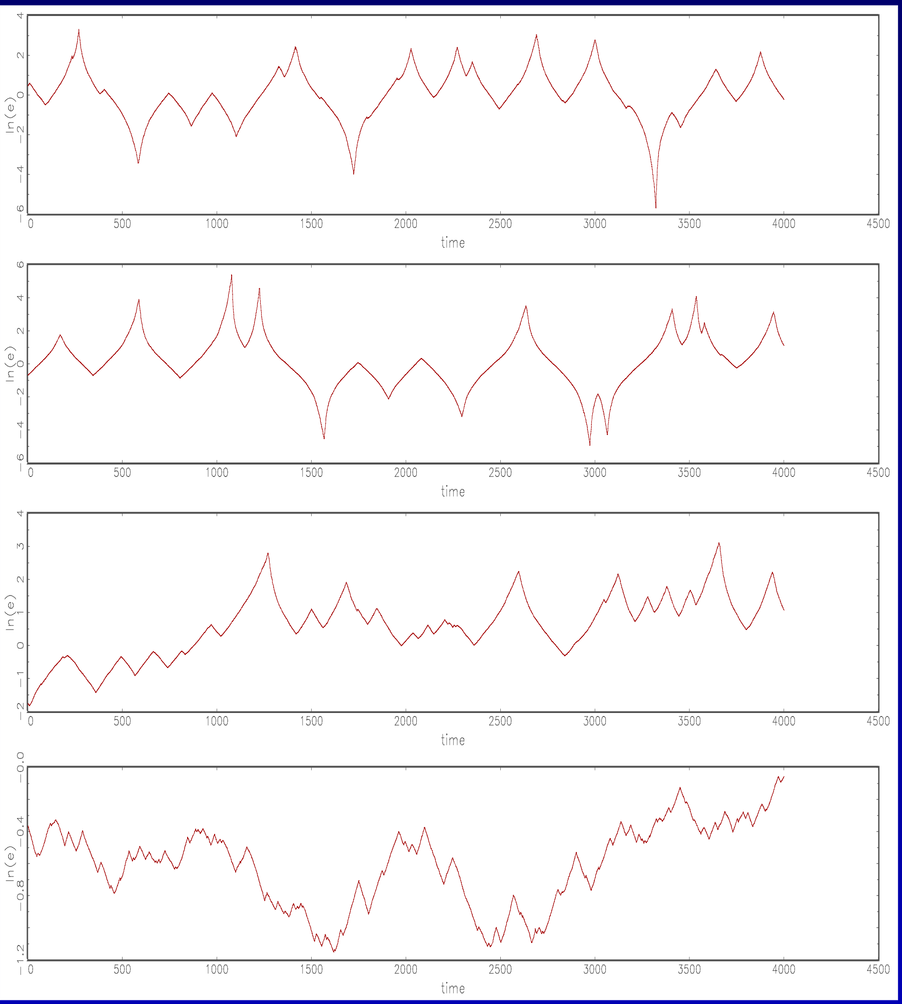


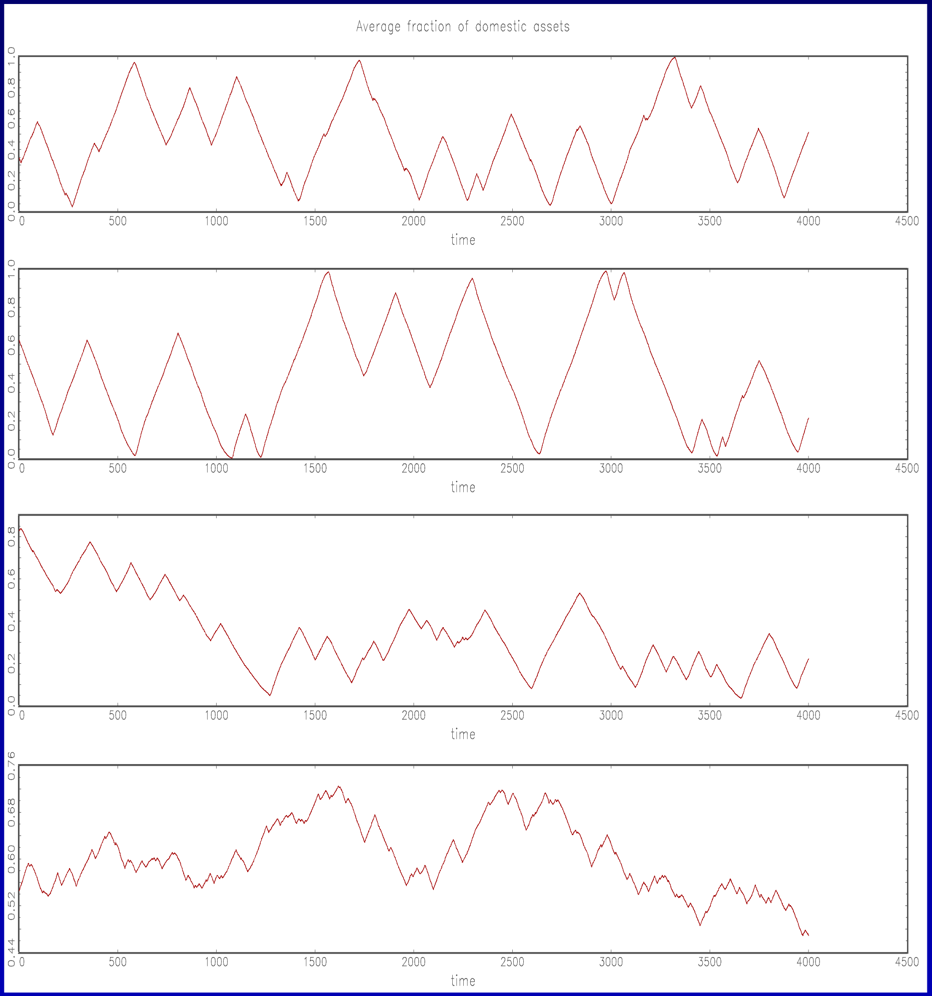
Large economies with Pareto distribution of endowments:
From bottom to top: $\alpha = 0.5, 1, 1.5, 2.5$



Large economies with Pareto distribution of endowments:
From bottom to top: $\alpha = 0.5, 1, 1.5, 2.5$

→ except for relatively trivial cases, the distribution of wealth is not reflected in market outcomes





Effects of segmentation (continent czcle ideas): population of 2000 individuals with groups of 10, 20, 50, 100 (from bottom to top)

Exchange Models and Emerging Wealth Distribution

- wanted: an interacting agent exchange model with realistic emergent properties

- an early example along the lines of recent econophysics models:
(Angle: The surplus theory of social stratification and the size distribution of personal wealth, *Social Forces*, 1986, J. of Math. Sociology, 1992,1993,1996)

  - agents have random encounters in which a transfer of a fixed proportion ω of wealth from one to the other happens (interacting particles)

  - the richer has a probability $p > 0.5$ to be the winner ($D_t = 1$ with prob p, $D_t = 0$ with prob. 1- p)

  - stochastic evolution of wealth:

$$
\begin{aligned}
w_{i,t} &= w_{i,t-1} + D_t \omega w_{j,t-1} - (1 - D_t)\omega w_{i,t-1}, \\
w_{j,t} &= w_{j,t-1} + (1 - D_t)\omega w_{i,t-1} - D_t \omega w_{j,t-1},
\end{aligned}
$$

Angle's Surplus Theory of Social Stratification

- archeological evidence: hunter/gatherer societies are egalitarian, inequality appears as soon as there is some *surplus* over subsistence production
- the *surplus* becomes the subject of agents' competition, every agent tries to extract wealth from others
- expropriation of others happens via:
  - theft
  - extortion
  - taxation
  - exchange coerced by unequal power between participants
  - *genuinely voluntary exchange*
  - gift

Problems:

- This is not a model of a modern society: no role for mutually advantageous exchange (which is a key property of economic activities) ~ theft and fraud
- no voluntary participation in this process
- encounters resemble a box fight rather than economic activity

An Alternative Avenue: A Simple Exchange Economy with Changing Preferences

(following Silver et al. Statistical Equilibrium Wealth Distributions…, JET 106, 2002)

- again: two goods (x, y)
- all agents have Cobb-Douglas preferences:

$$U_{i,t} = x_{i,t}^{f_{i,t}} y_{i,t}^{1.f_{i,t}}$$

- changing preferences $f_{i,t}$ lead to demand/supply:

$$x_{i,t} = f_{i,t}(x_{i,t-1} + p y_{i,t-1})$$

$$y_{i,t} = (1 - f_{i,t})(\frac{x_{i,t-1}}{p} + y_{i,t-1})$$

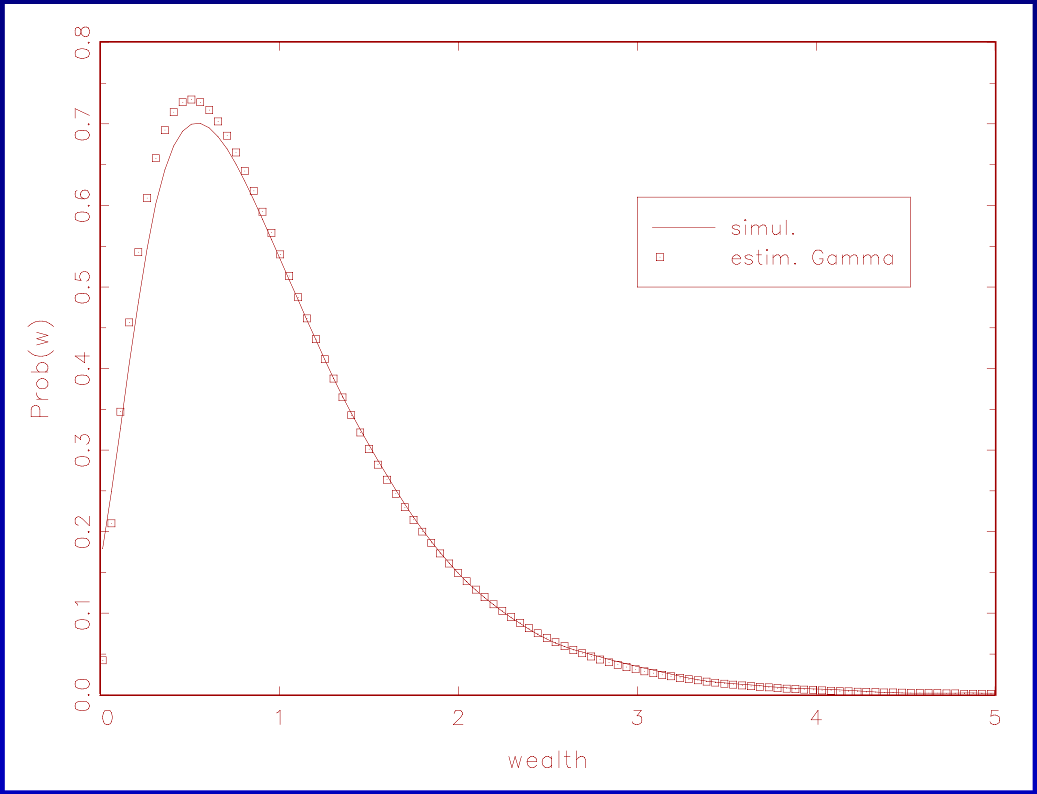
- summing up demand and supply, we compute the relative price p that clears both markets

$$p = \frac{\sum_i (1 - f_{i,t}) x_{i,t-1}}{\sum_i f_{i,t} y_{i,t-1}}$$

- evolution of wealth of agents (in units of one good)

$$w_{i,t} = x_{i,t} + p y_{i,t}$$

The baseline case: an exchange economy with two goods and changing preferences, $f_i(t)\sim U[0,1]$ -> in each period, agents prefer new combinations of goods and have to exchange their possessings.



Estimated:

Gamma(2, 0.5)

Despite agents being identical in all respects, one gets wealth stratification via the eventualities of the exchange process

Some Extensions

- allowing for pair-wise exchange rather than an aggregate market (makes no difference)
- introduction of agents with monopoly power
- introduction of agents with less volatile preferences

Monopolists

- we assume pair-wise exchange, but attribute stronger bargaining power to some agents
- while competitive agents would trade at a price equilibrating their demand and supply, monopolists would enforce a price (an exchange relation) that maximizes their utility
- note: though this can be viewed as *exploitation* of the competitive agents, it is not *expropriation* (as in Angle etc.). A trade only happens if it is still advantageous even for the 'exploited'.

The monopolist's price

- Monopolist maximizes:

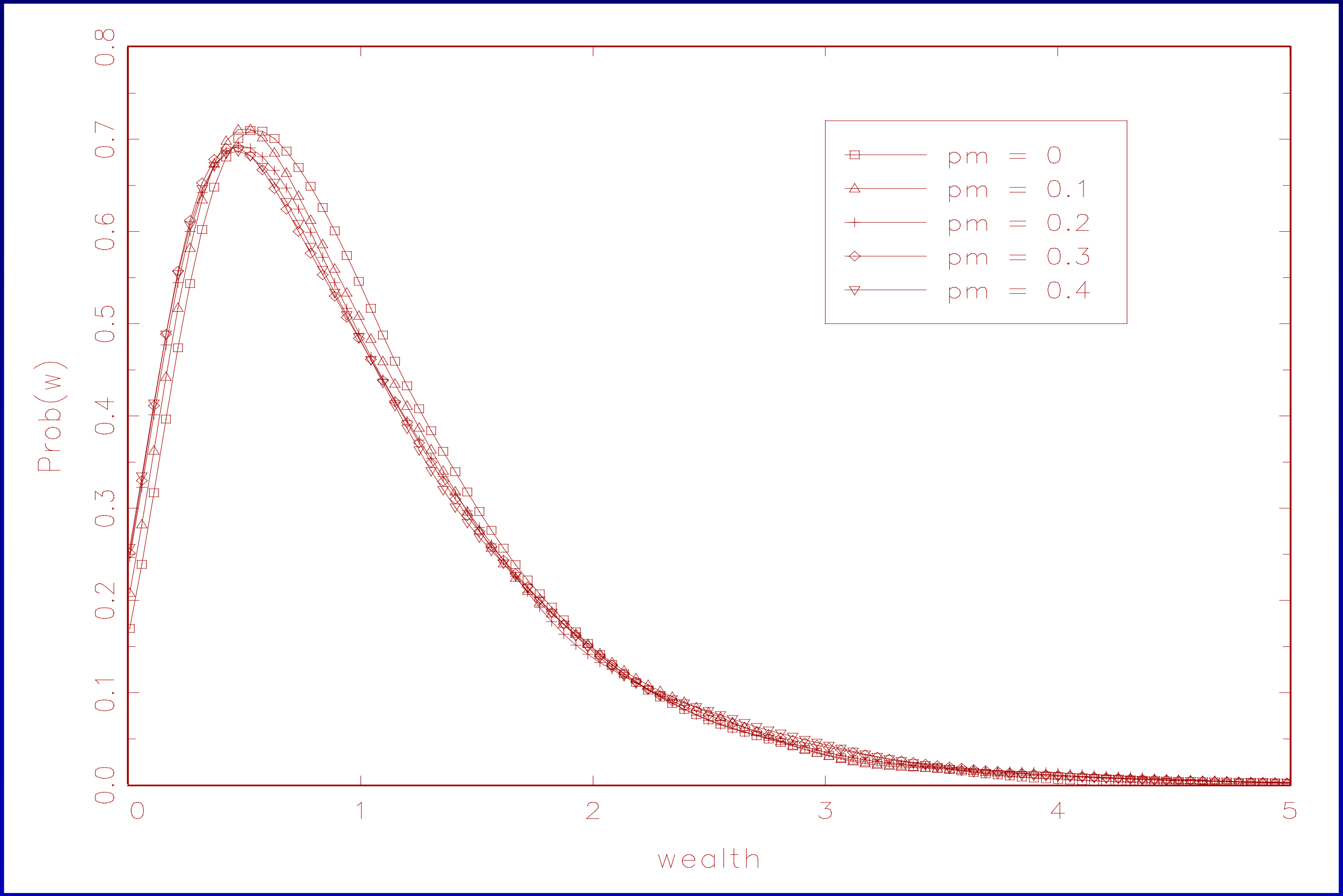
$$U_{i,t} = x_{i,t}^{f_{i,t}} y_{i,t}^{1-f_{i,t}}$$
$$= (x_{i,t-1} + x_{j,t-1} - x_{j,t})^{f_{i,t}} (y_{i,t-1} + y_{j,t-1} - y_{j,t})^{1-f_{i,t}}$$

subject to the demand/supply functions of his trading partner j.

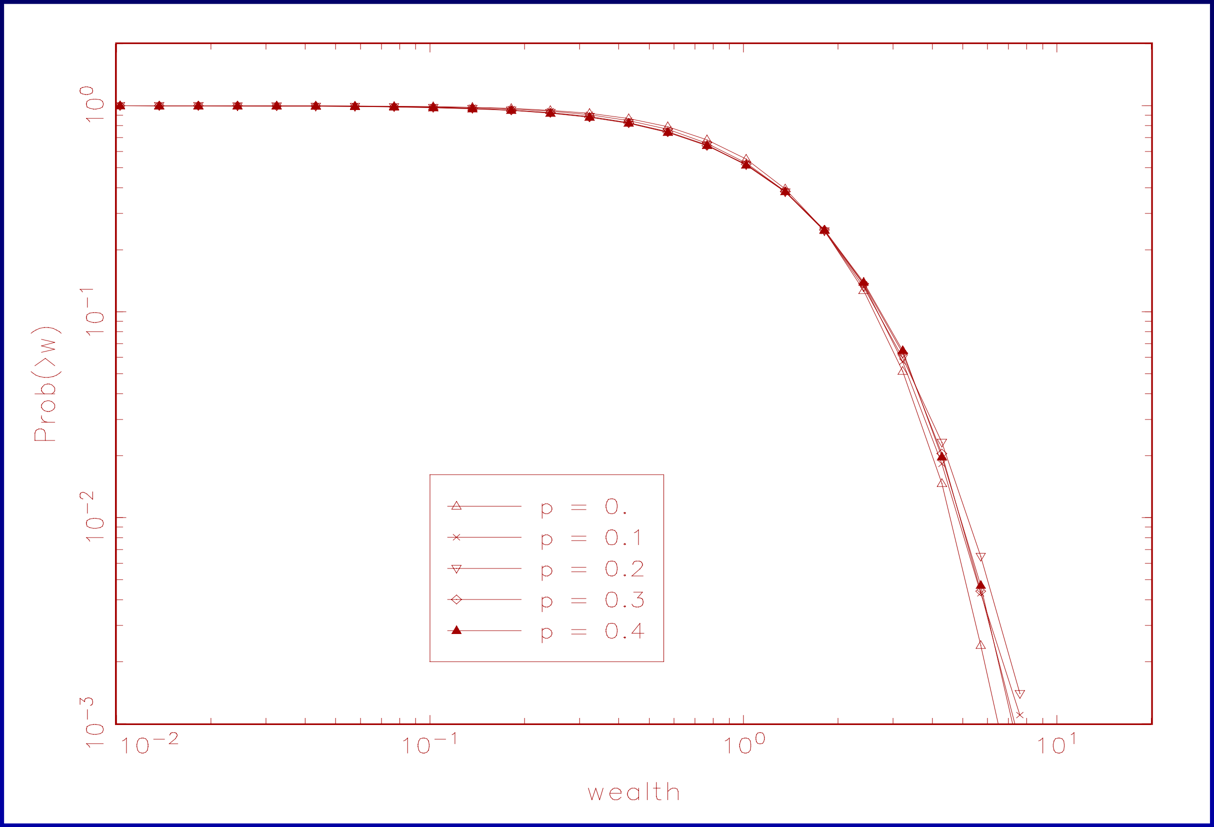
- the monopolist's price is the positive solution of:

$$-f_{i,t} f_{j,t} y_{j,t-1} (f_{j,t} y_{j,t-1} + y_{i,t-1}) p^2 + (2f_{i,t} - 1) f_{j,t} (1 - f_{j,t}) y_{j,t-1} x_{j,t-1} p$$
$$+ (1 - f_{i,t})(1 - f_{j,t}) x_{j,t-1} (x_{i,t-1} + (1 - f_{j,t}) x_{j,t-1}) = 0$$

Monopoly agents: small effect on wealth distribution



Result: slight change of shape, no Pareto tails



Estimated Gamma Parameters

p_{mon}	0	0.1	0.2	0.3	0.4
λ	2.01	1.89	1.73	1.72	1.67
σ	0.50	0.53	0.58	0.58	0.60
w(mon./ w(non-m.)	-	1.89	1.85	1.78	1.75

Parenthetically: we could allow any degree of bargaining power between the extreme cases of monopoly and perfect competition via the standard bargaining ansatz:

$$\max W = (\Delta U_{i,t})^{\alpha} (\Delta U_{j,t})^{1-\alpha}$$

with: α: bargaining strength of agent i

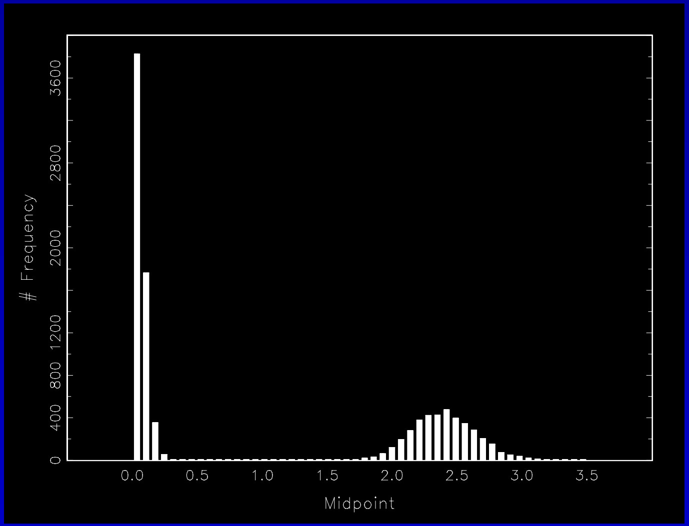
Natural Differences among Agents: Steady against More Volatile Agents

Some agents have more restricted interval of variation of their preferences:

fraction p with $f_i(t) \sim U[0.4,0.6]$

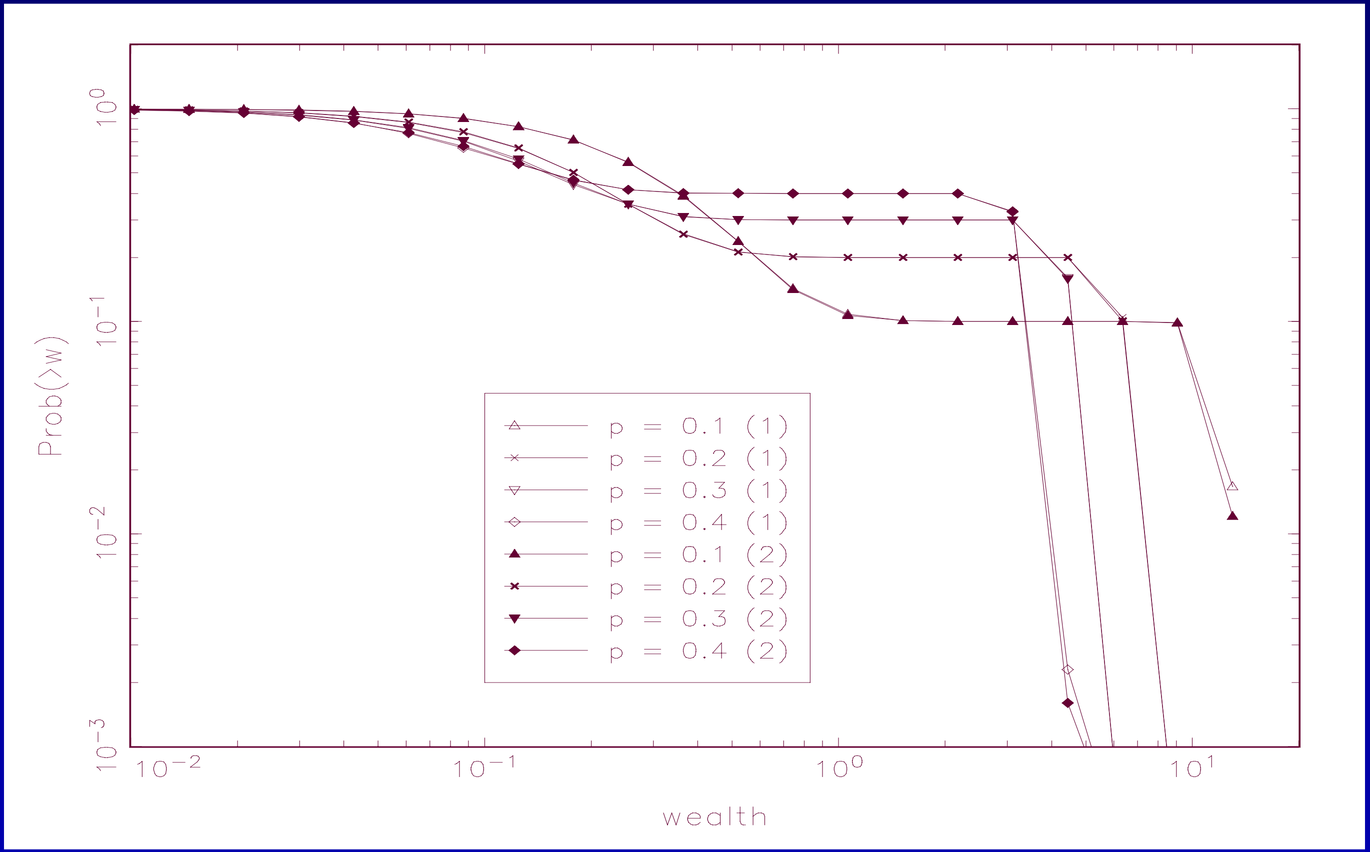
fraction 1- p with $f_i(t) \sim U[0,1]$

-> advantage to the more steady agents who have to rely less on appropriate trading partners to meet their needs



Result: bi-modal *stationary* distribution,

Example: p = 0.4, *no* monopolists



Inverse of Cumulative Distribution for Various Fractions of More "Steady" Agents:

(1) After 100,000 rounds, (2) after 200,000 rounds

Summary

- agent-based financial market models do ***not*** always exhibit a strong correlation between the distribution of wealth and that of asset returns

- 97% of the empirical wealth distribution can be explained by different degrees of luck in an otherwise unbiased exchange process

- the gas model (aka inequality process) can be reformulated in an way that avoids the paradoxes of the theft and fraud economy

- economic power *per se* does ***not*** necessarily lead to Pareto tails

Further Research: How to Add the Missing 3%?

- further reinforcement of wealth stratification, e.g., give monopoly power to those in the highest wealth class?

  ✏ mimics a law of proportionate effects

- introduction of growth, investment, savings (non-conservative system!)

  ✏ historically, emergence of inequality seems to be connected with transition from hunter/gatherer economies to more differentiated economies, development of inequality shows characteristic tendencies during industrialization