

Empirical Study and Model of Personal Income

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- JAPAN

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 - Employment income earners who worked throughout a year
 - Results of Sample Survey for Self-assessed Income Tax
- Top Taxpayers (high frequency data)

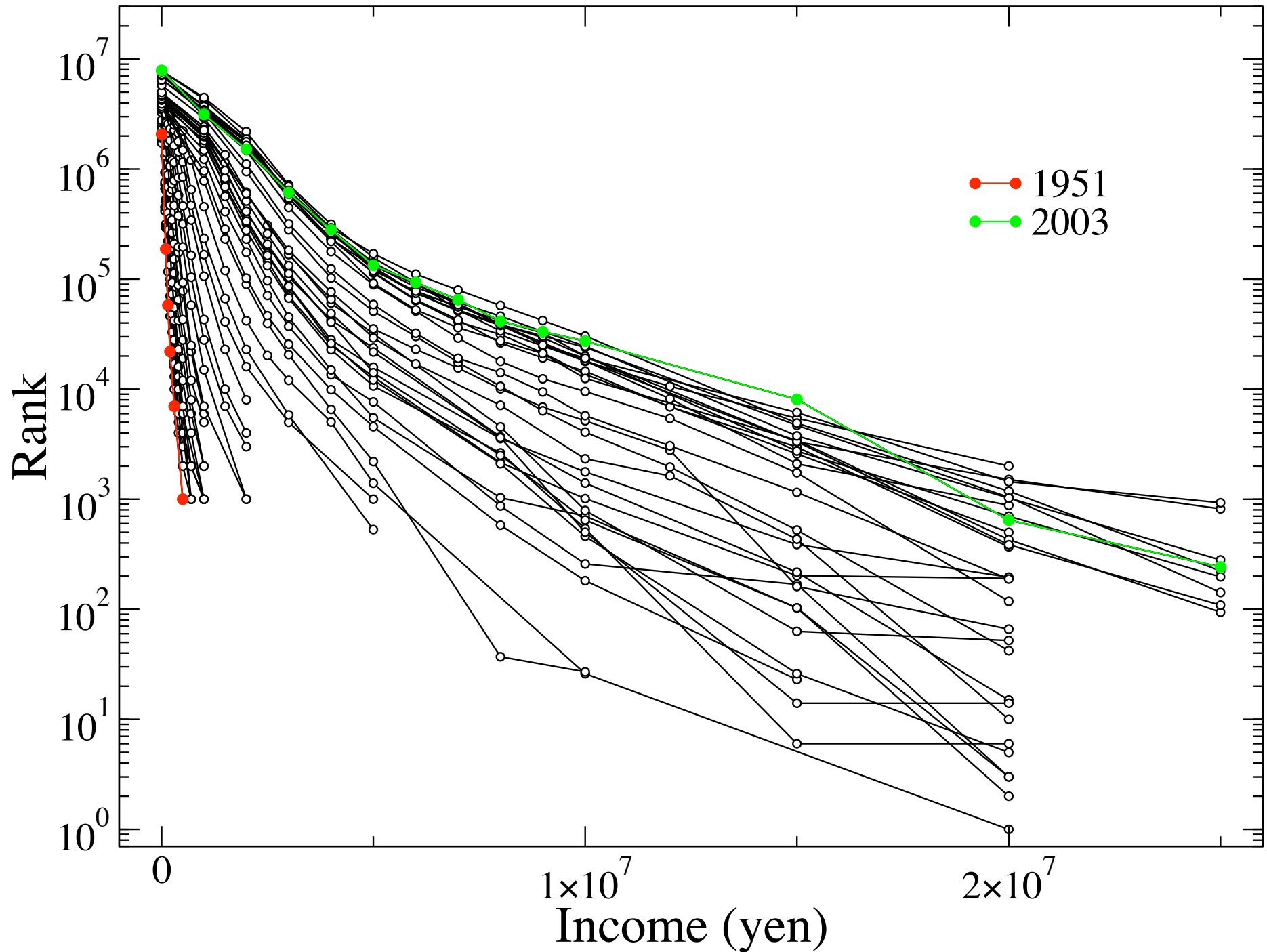
- US

- Statistics of Income Database of the Internal Revenue Service

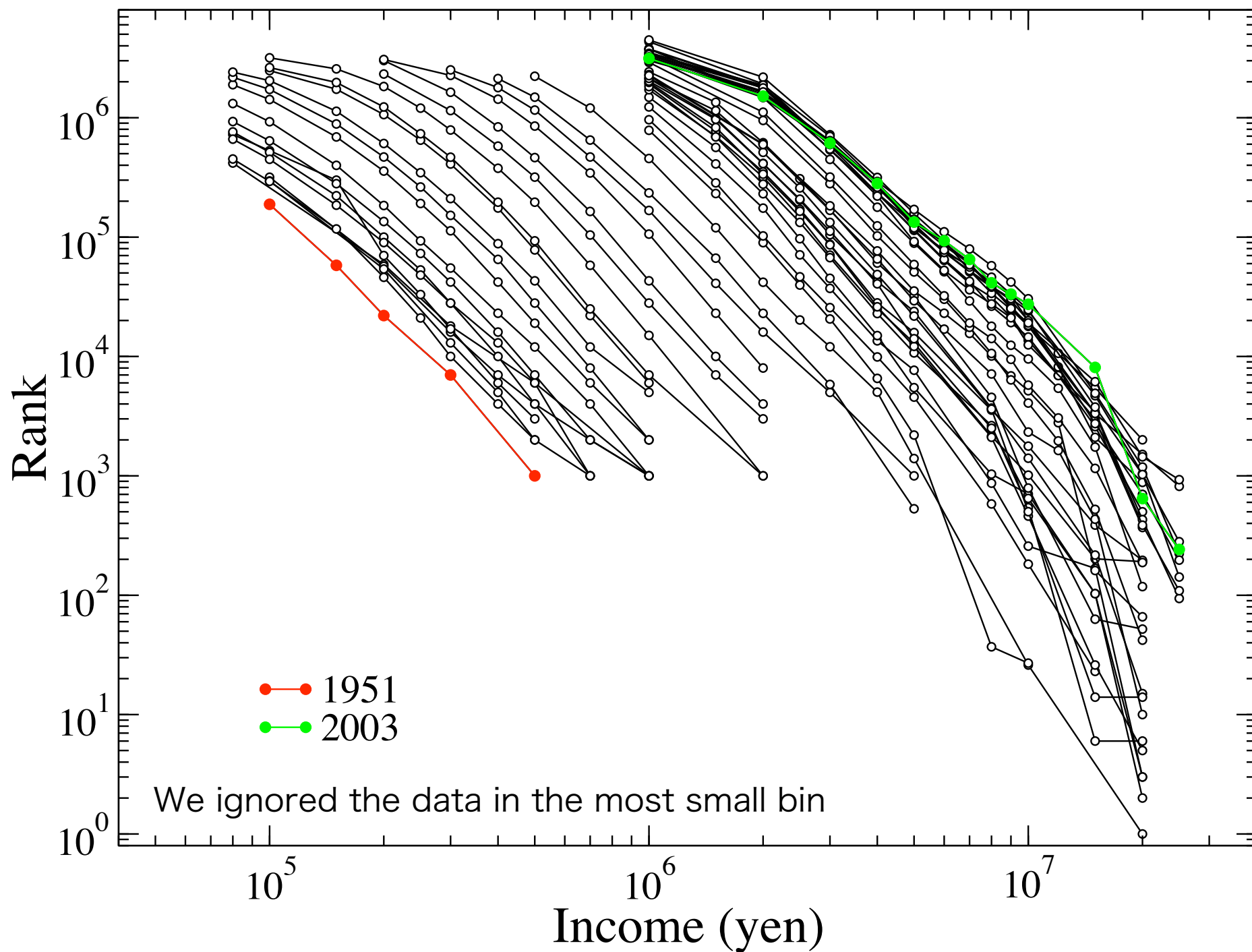
- Modeling

- $\text{Income} \approx \text{Employment income} + \text{Income from assets}$

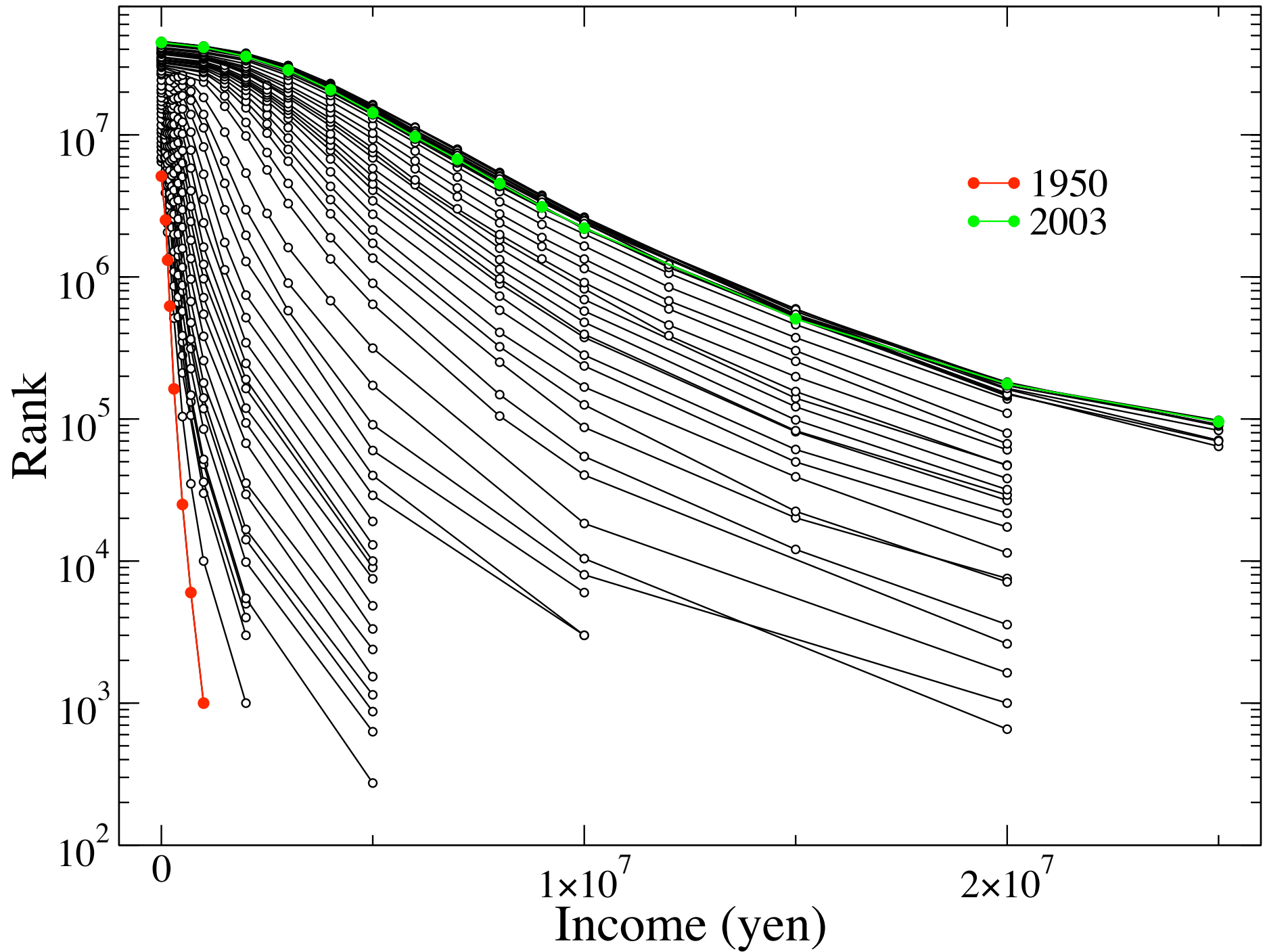
Employment income earners who worked for less than a year from 1951 to 2003



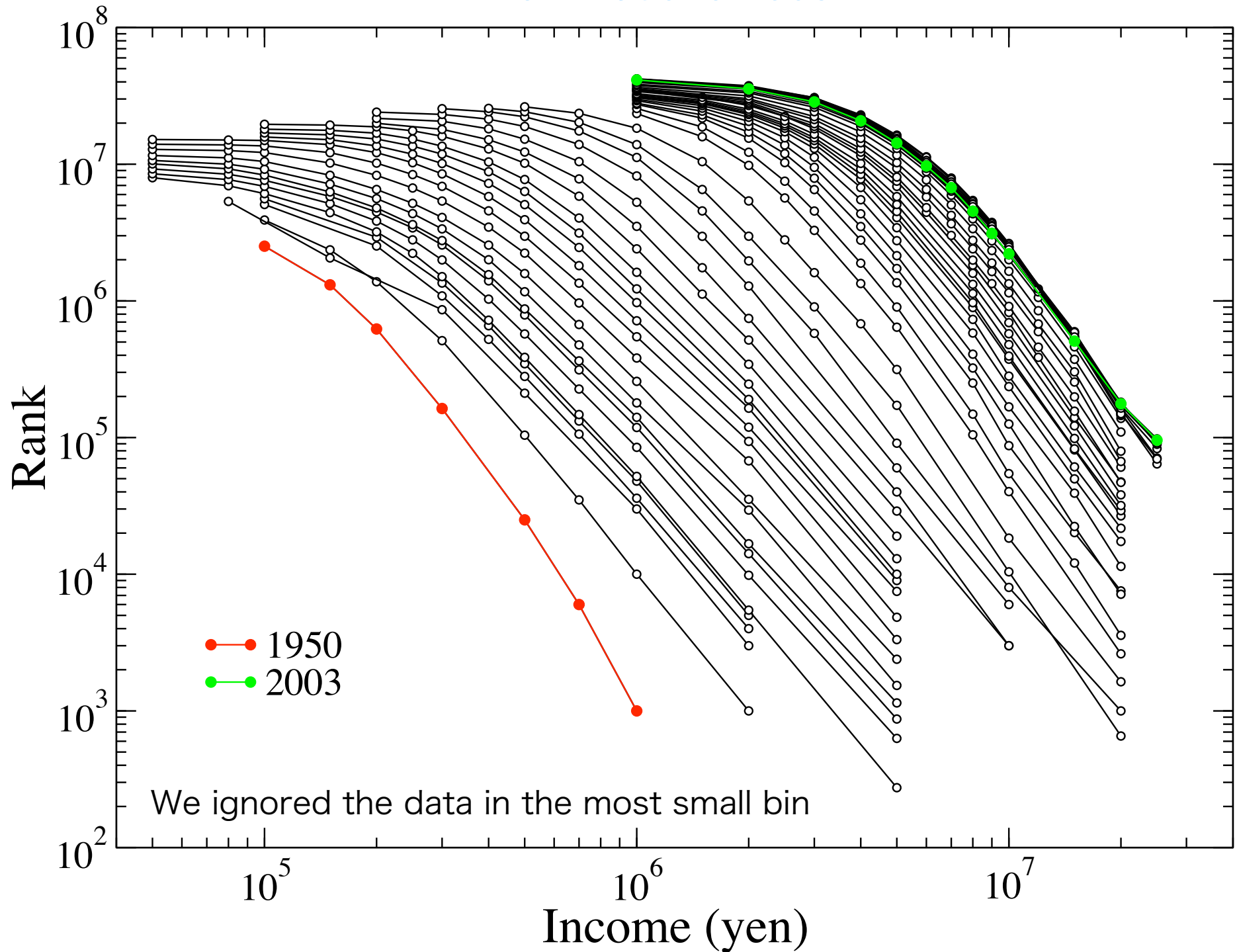
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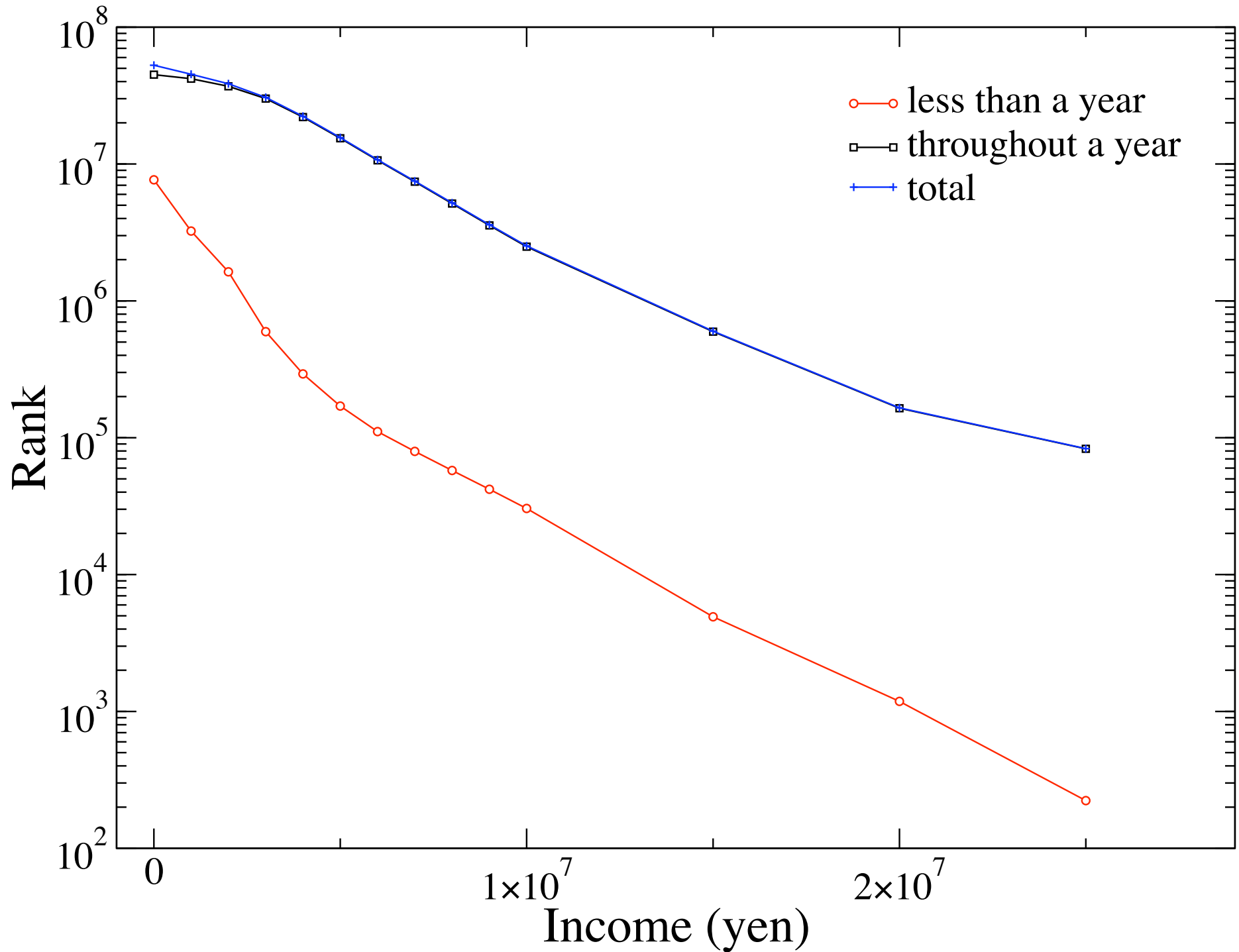
Employment income earners who worked throughout a year from 1950 to 2003



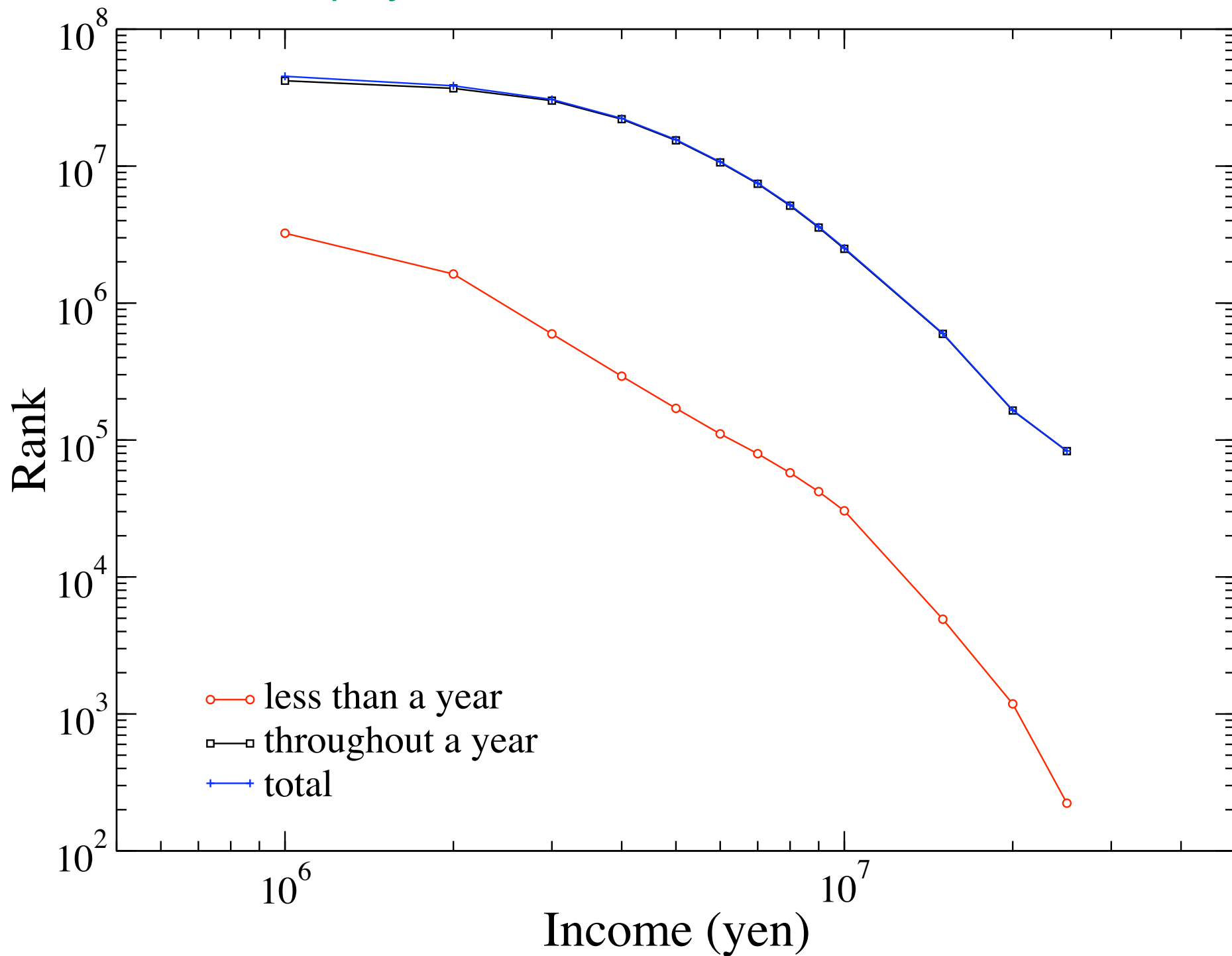
Employment income earners who worked throughout a year from 1950 to 2003



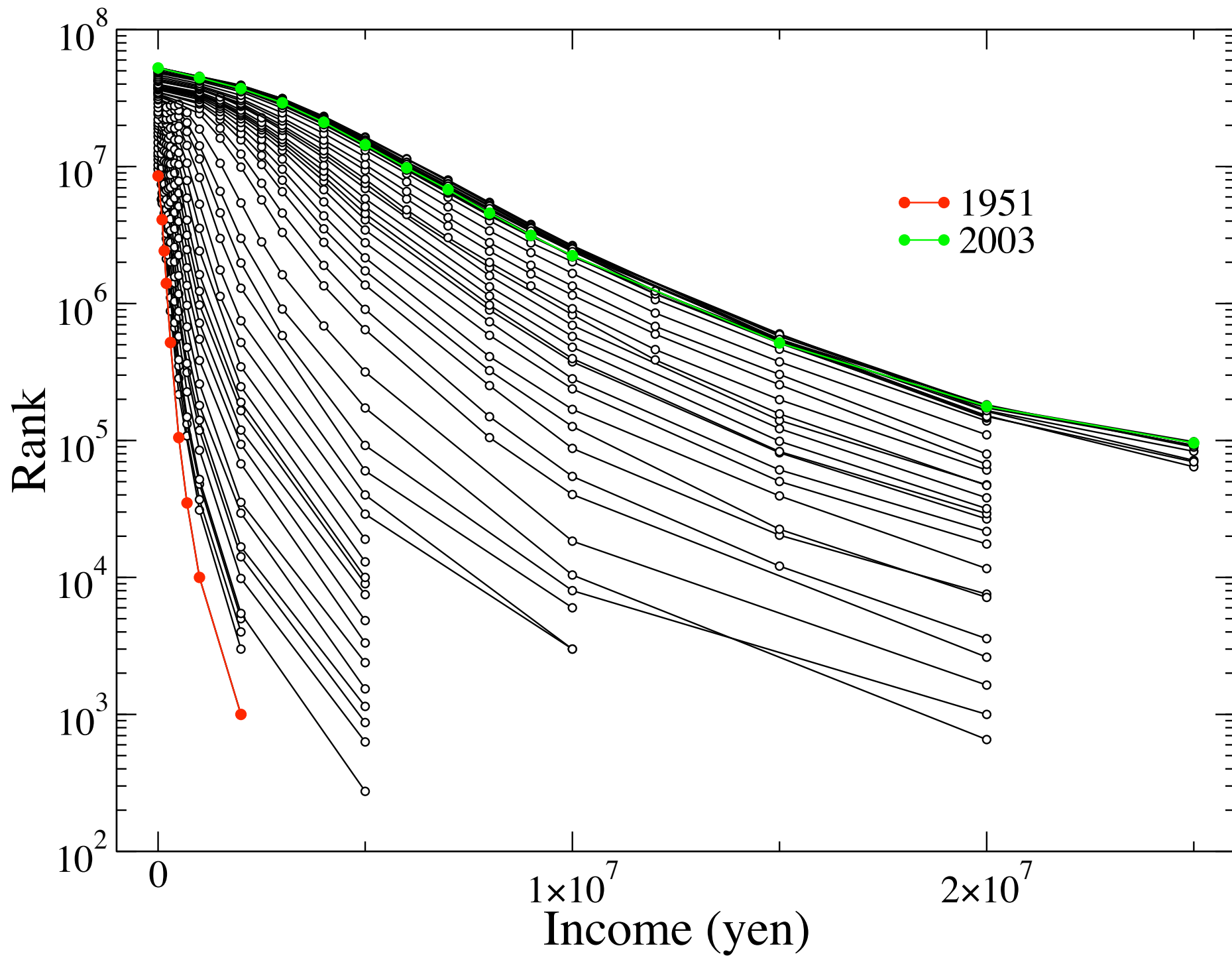
Employment Income Earners in 1999



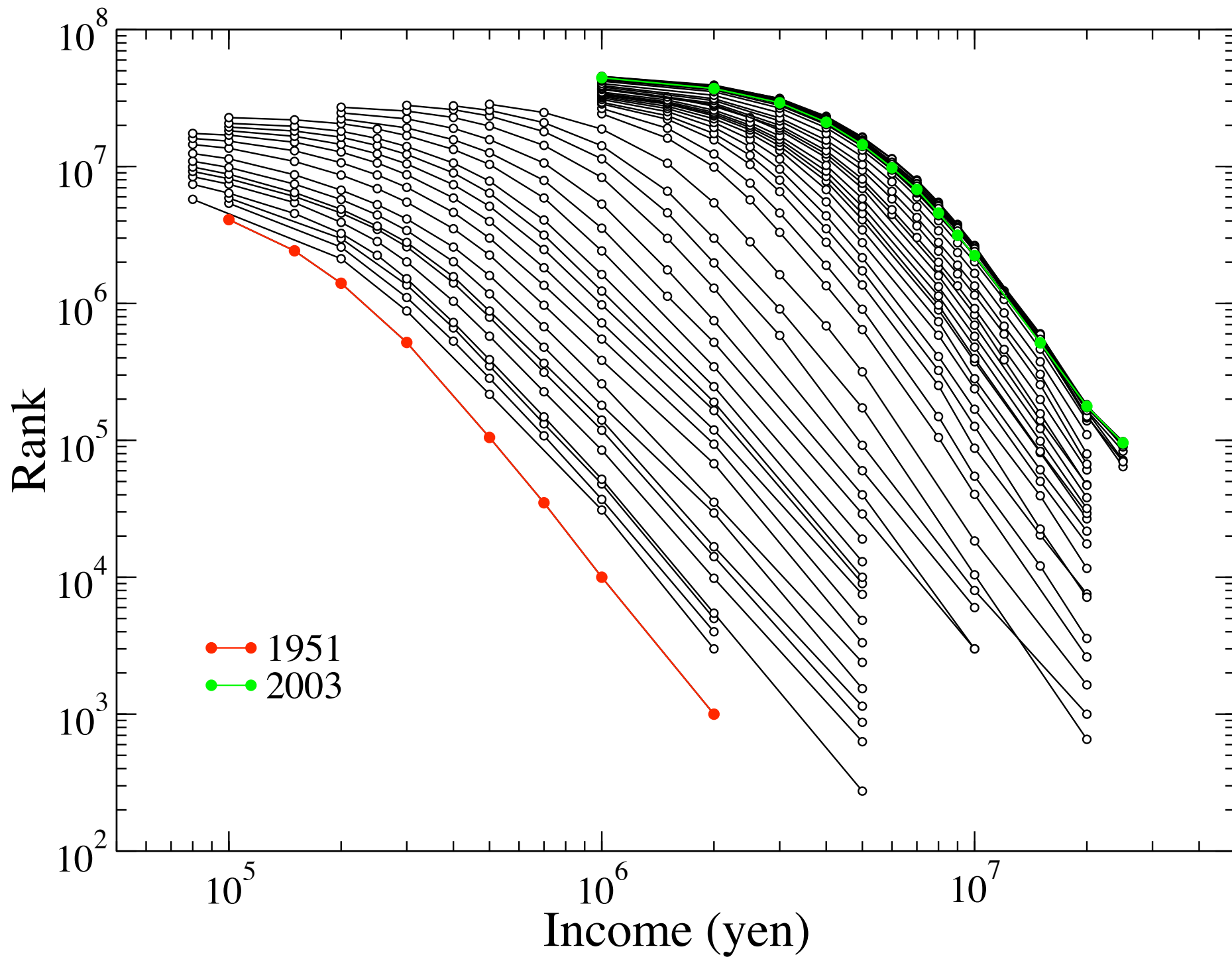
Employment Income Earners in 1999



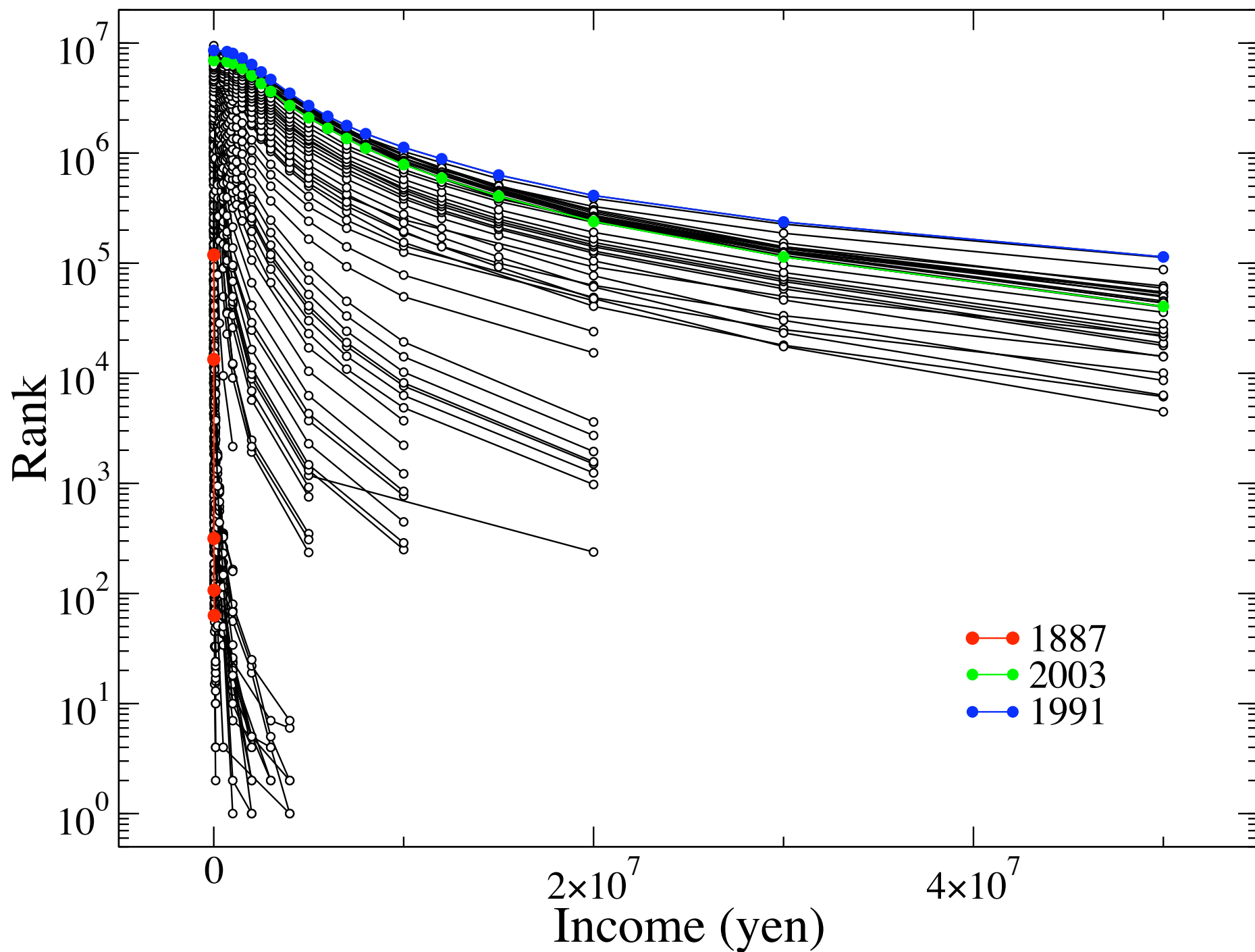
Employment Income from 1951 to 2003



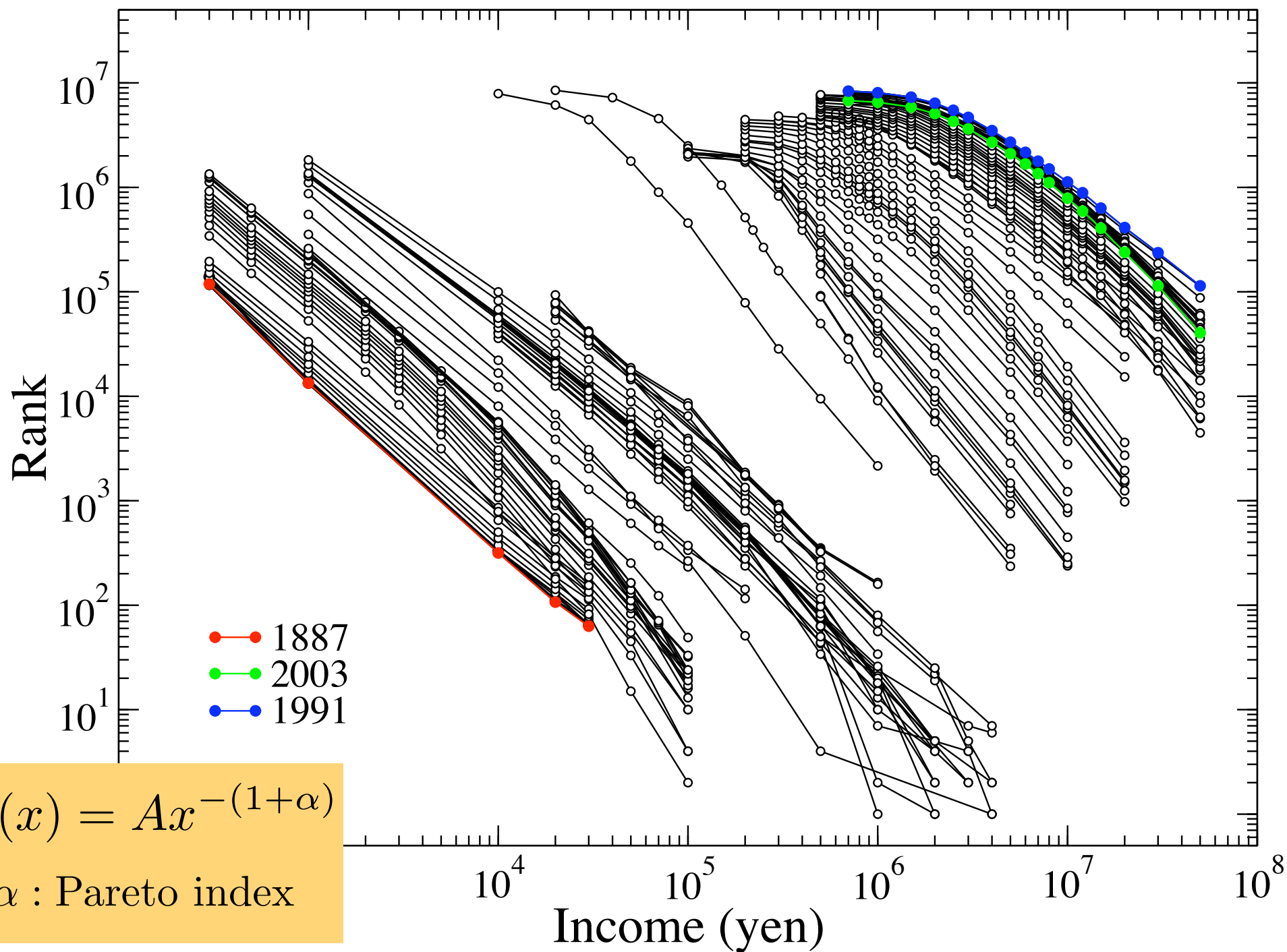
Employment Income from 1951 to 2003



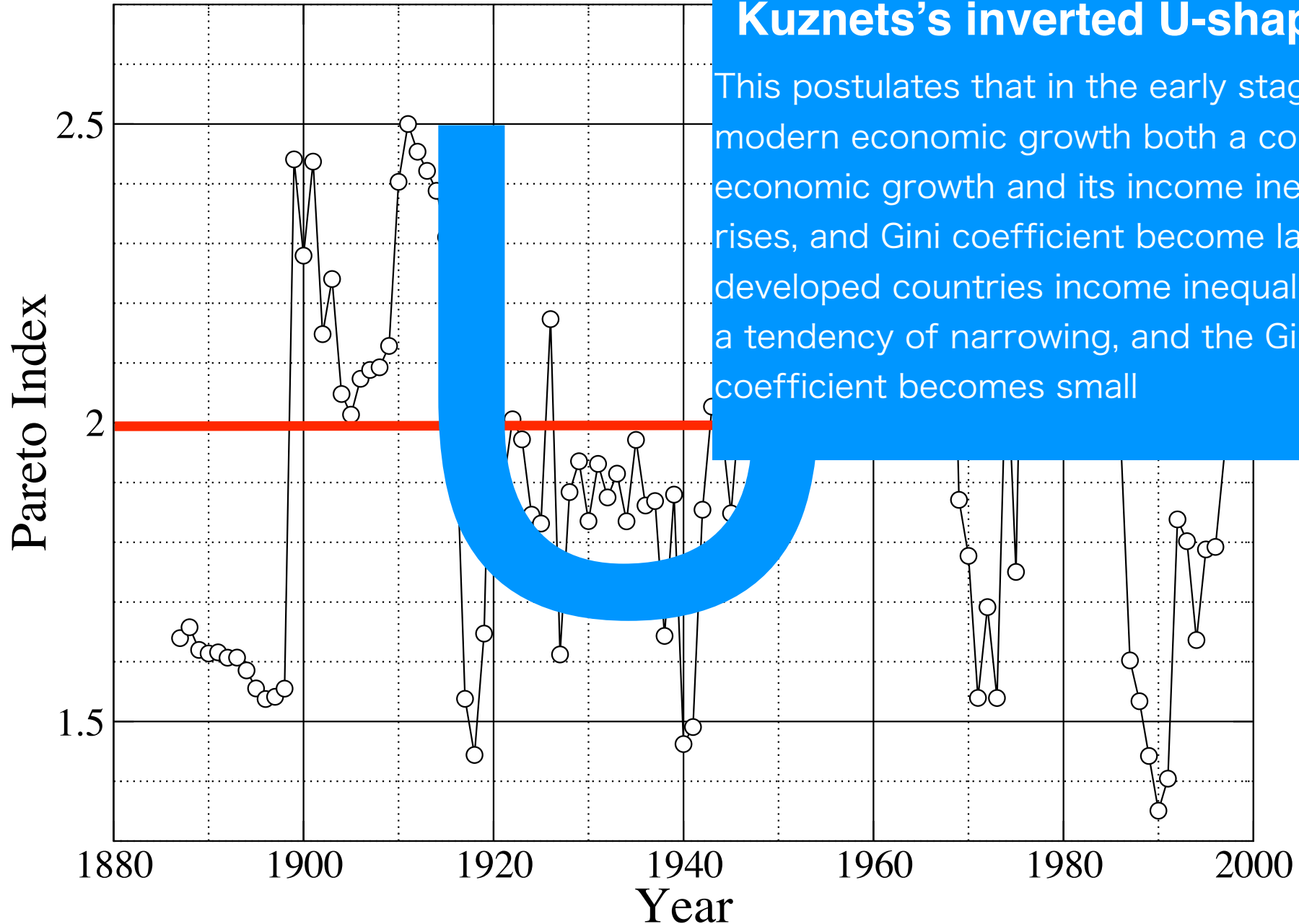
Results of Sample Survey for Self-assessed Income Tax from 1887 to 2003



Results of Sample Survey for Self-assessed Income Tax from 1887 to 2003



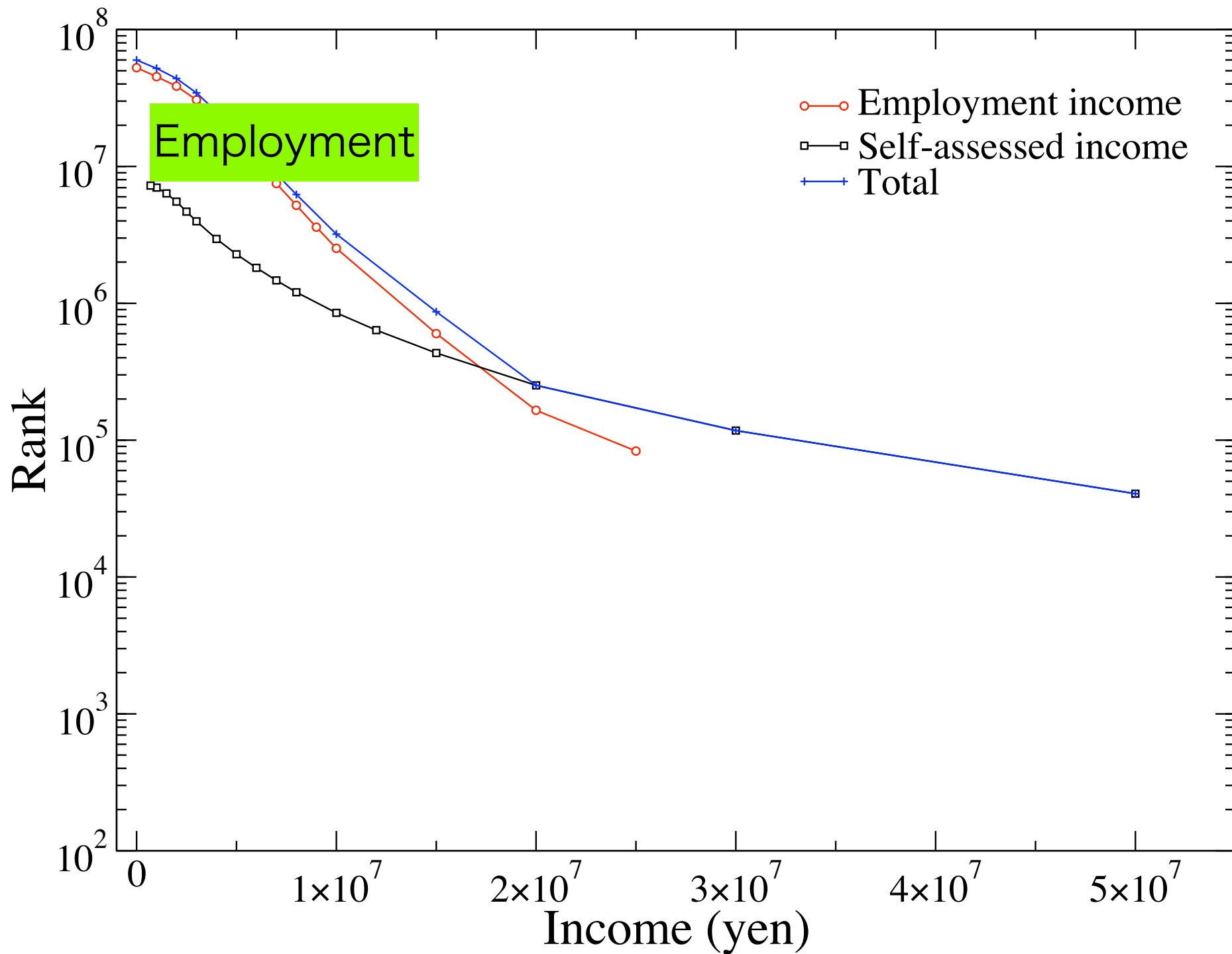
Pareto index



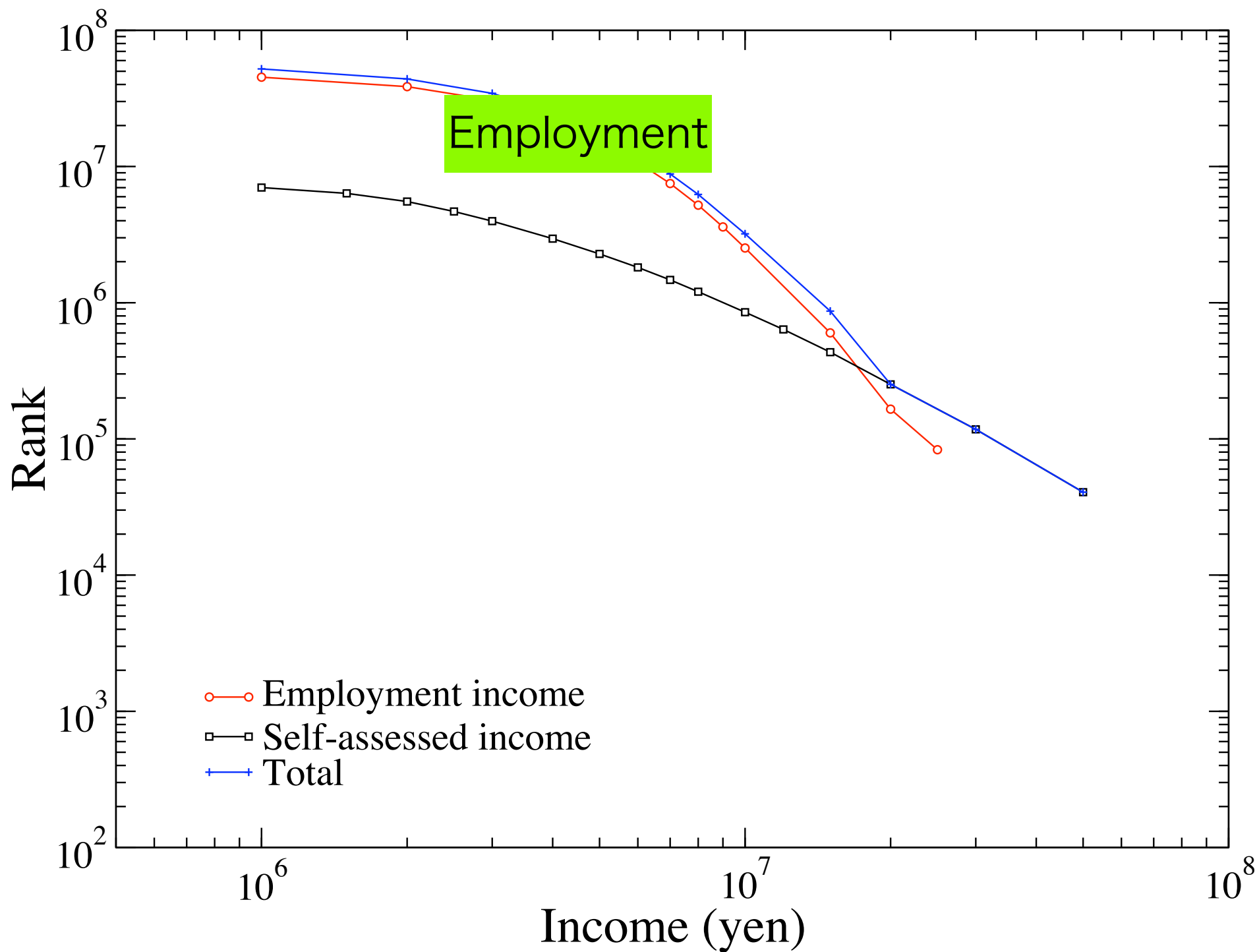
Kuznets's inverted U-shaped ?

This postulates that in the early stage of modern economic growth both a country's economic growth and its income inequality rises, and Gini coefficient become large. For developed countries income inequality shows a tendency of narrowing, and the Gini coefficient becomes small

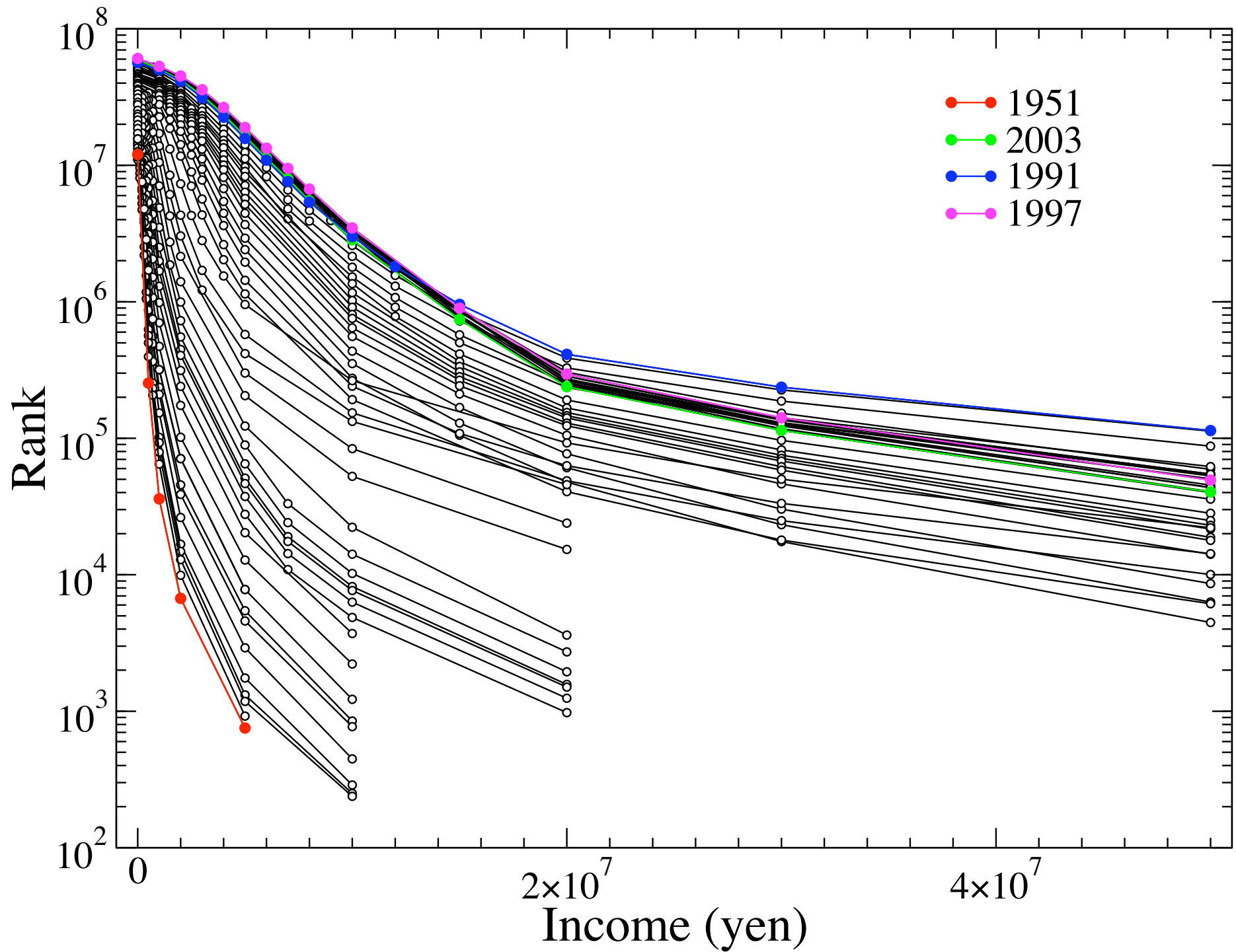
Total income in 1999



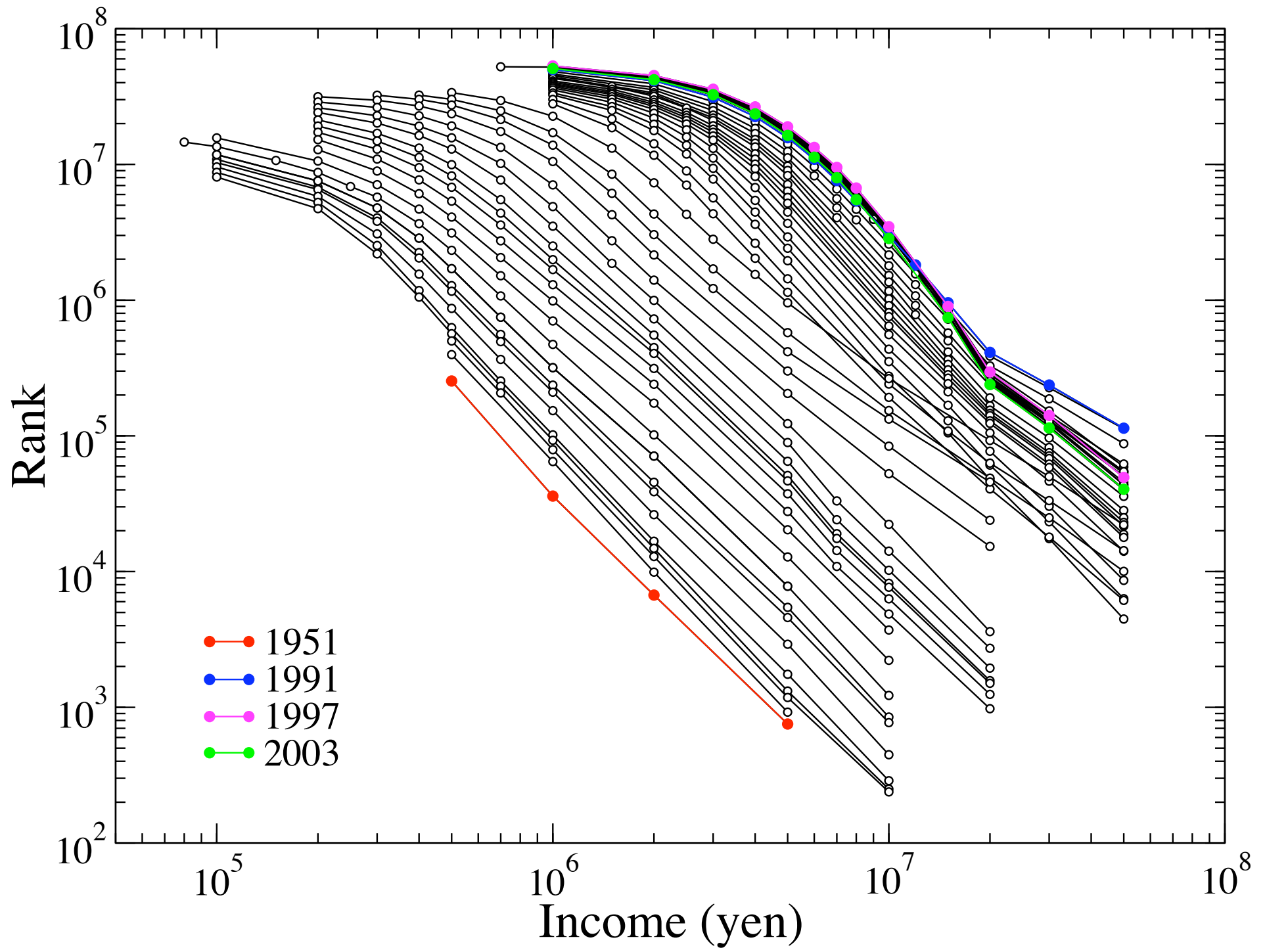
Total income in 1999



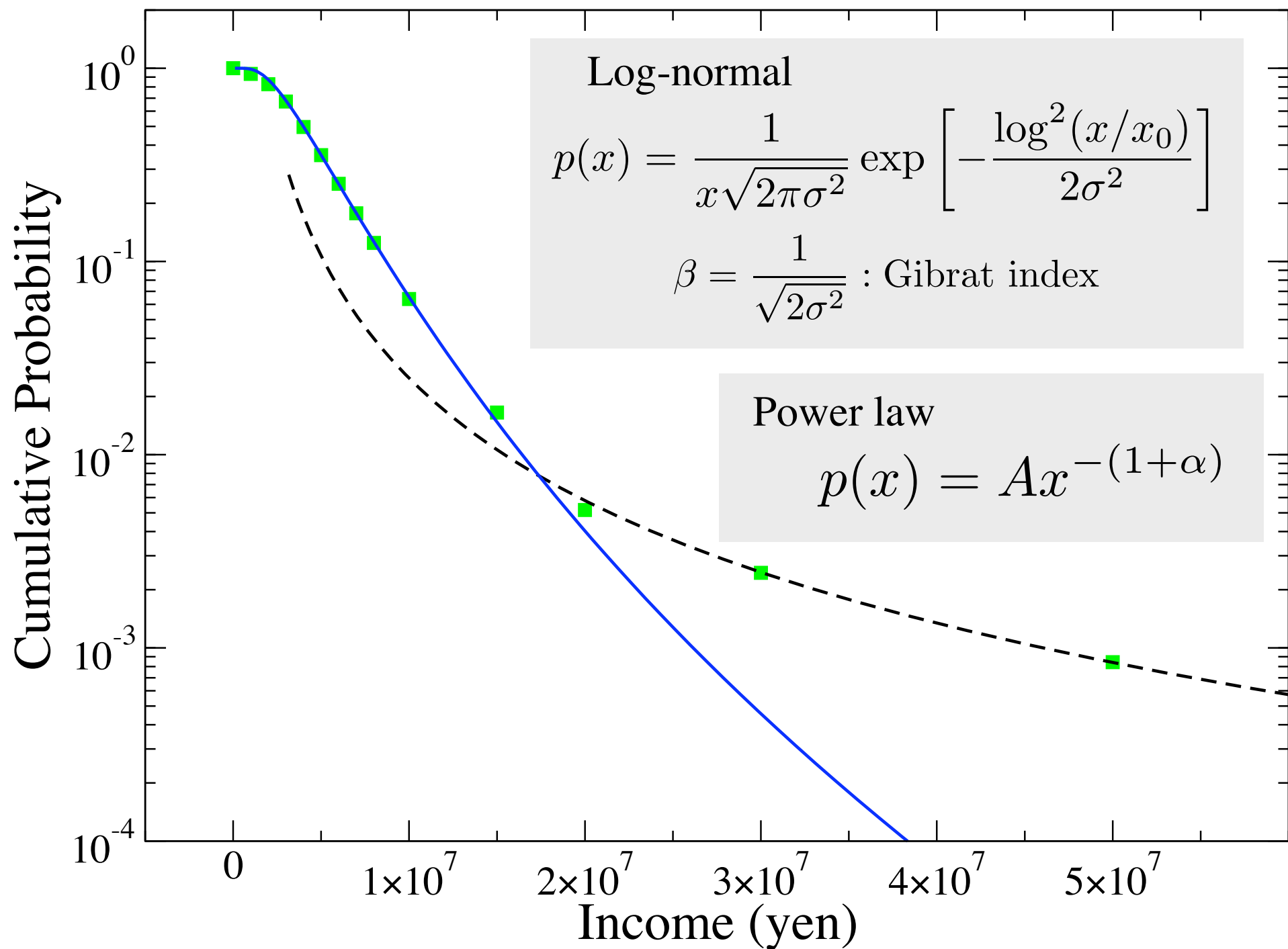
Total income from 1951 to 2003



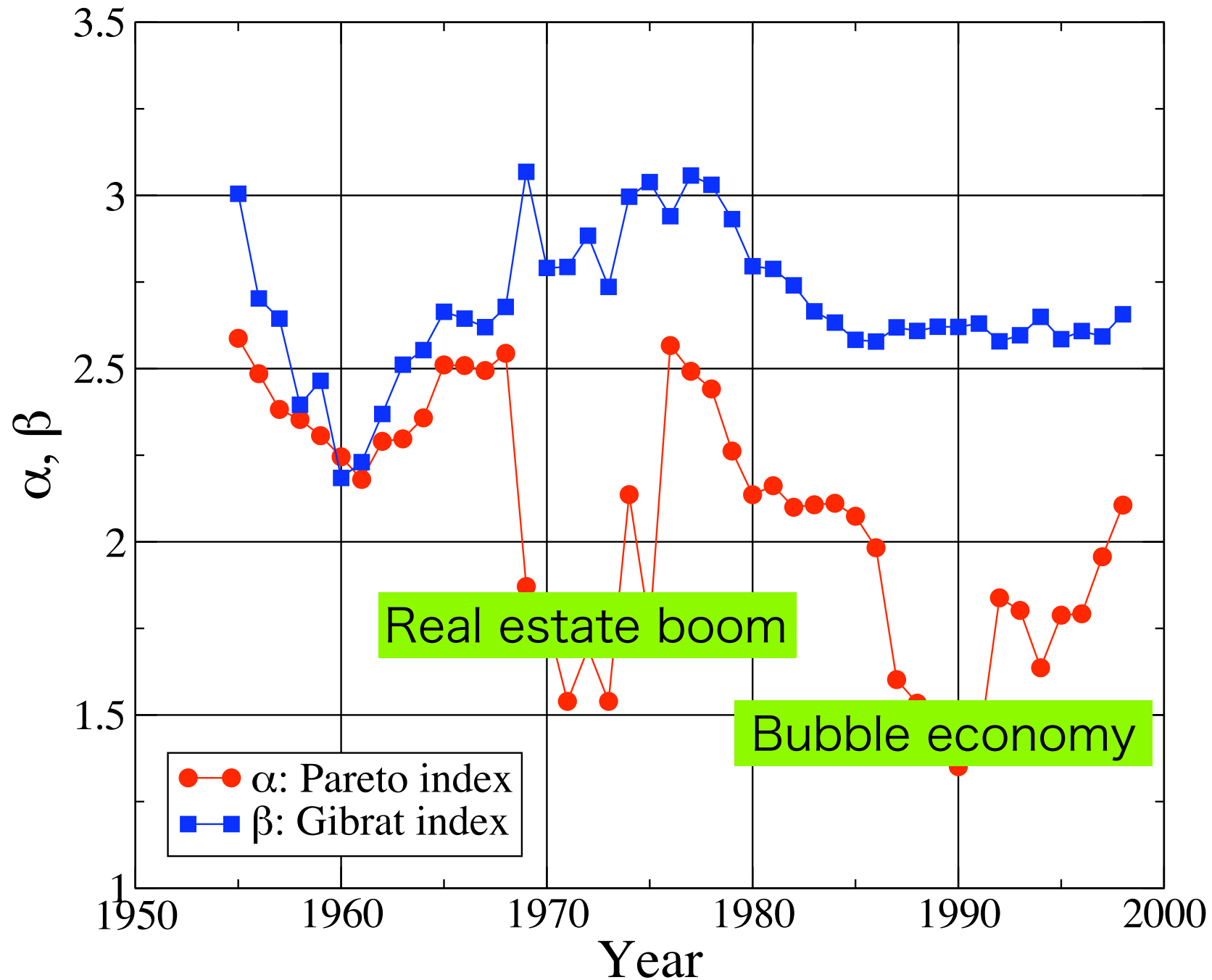
Total income from 1951 to 2003



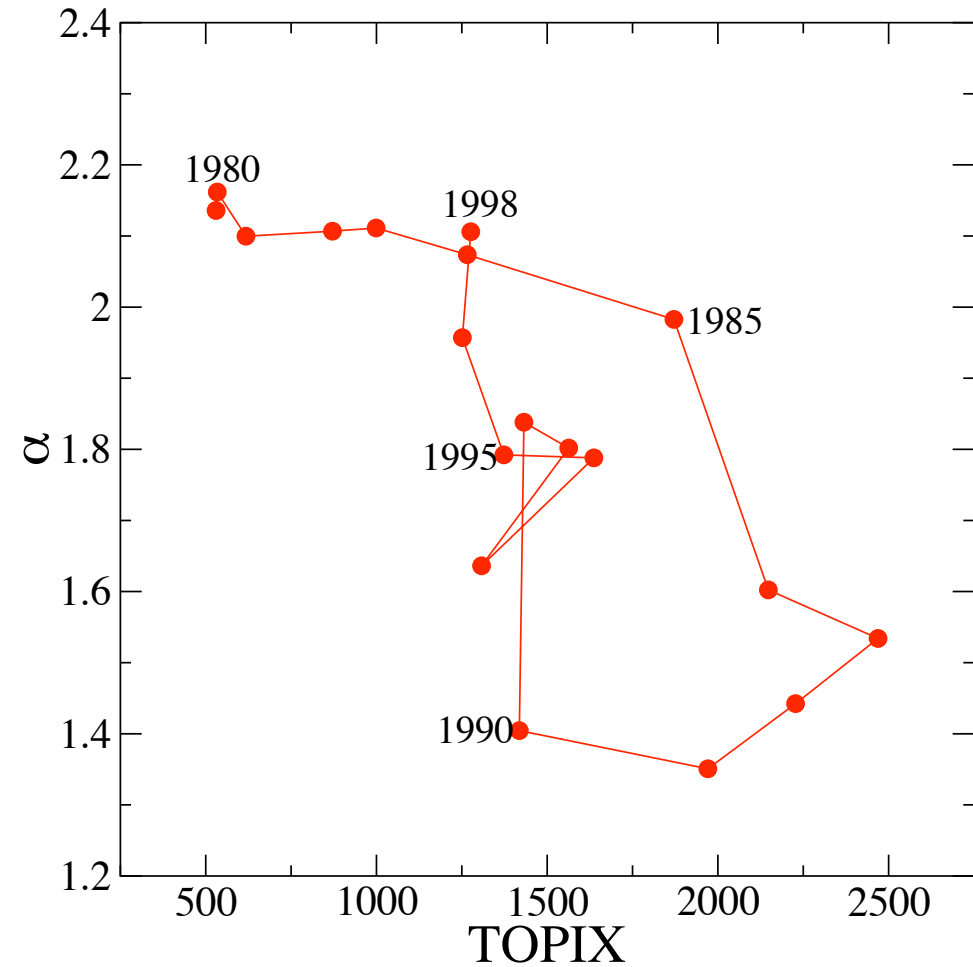
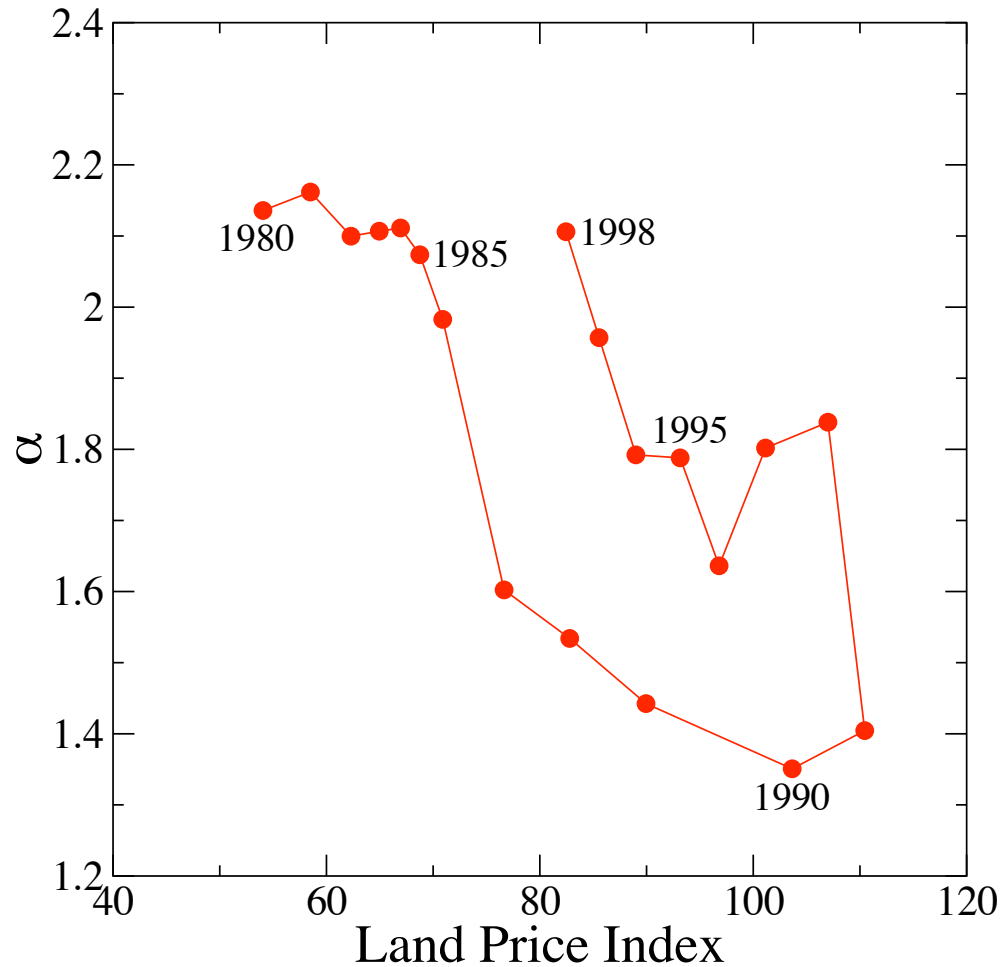
Total income in 1999



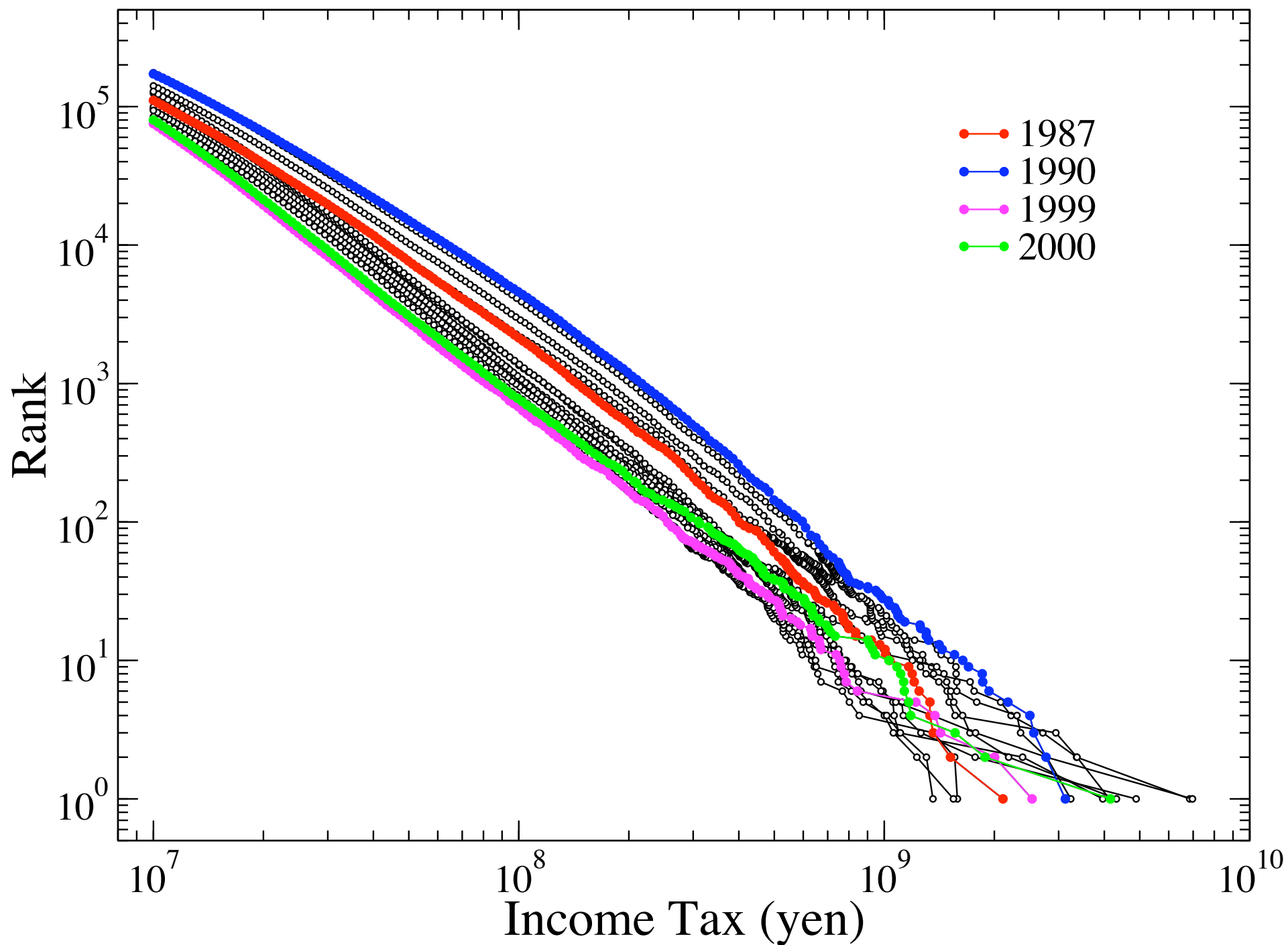
Pareto index and Gibrat index



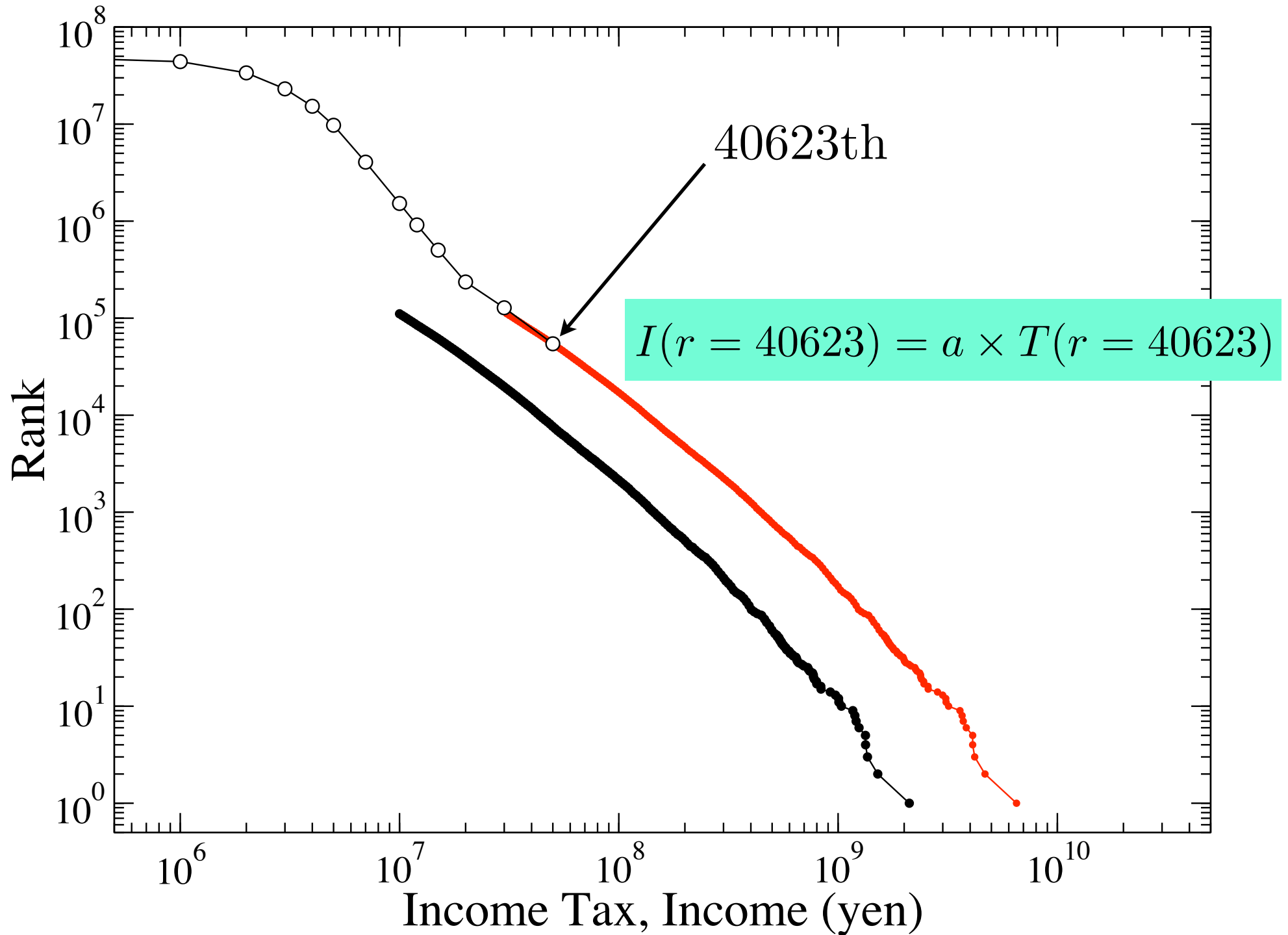
Bubble economy and Pareto index



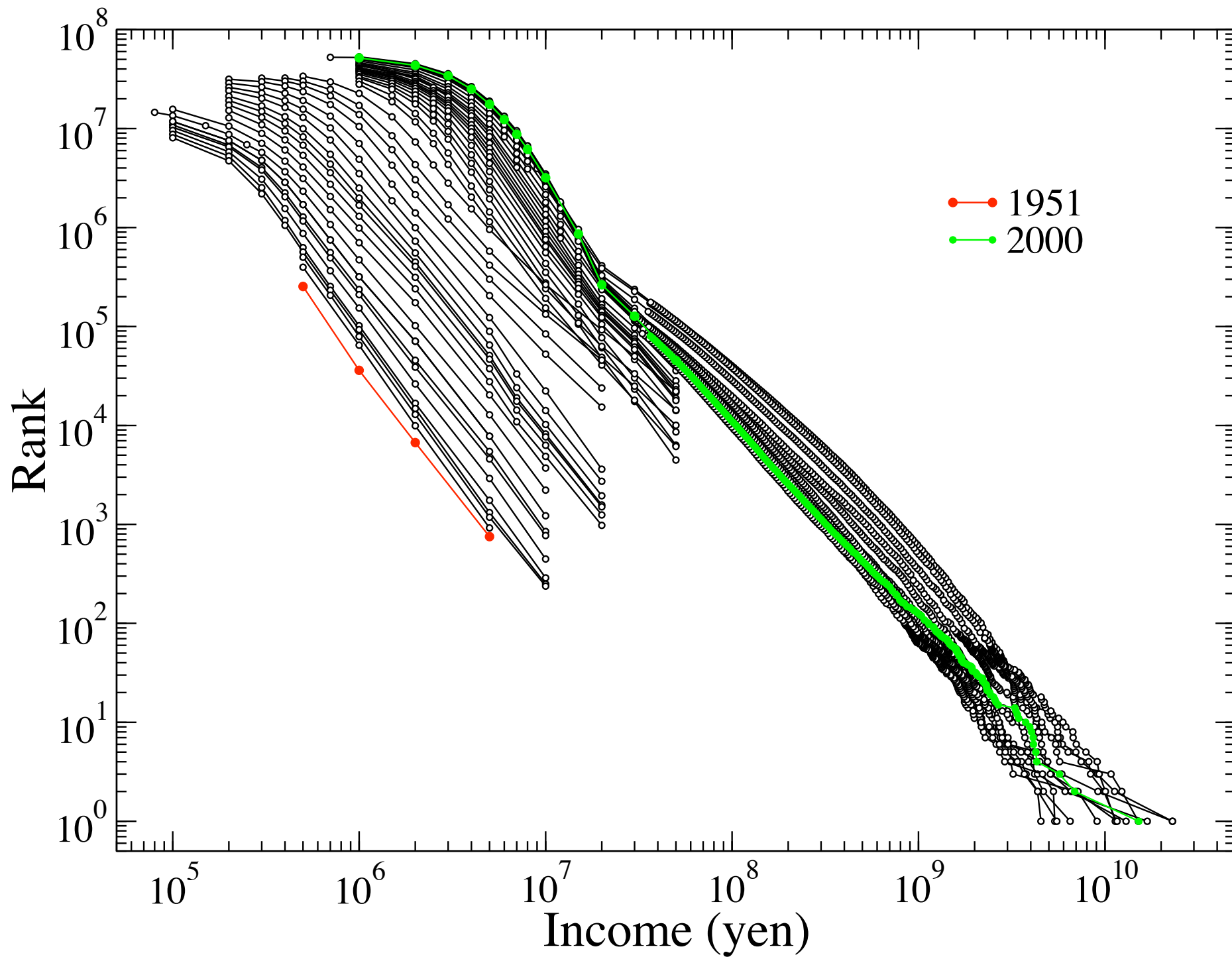
Top Taxpayers from 1987 to 2000



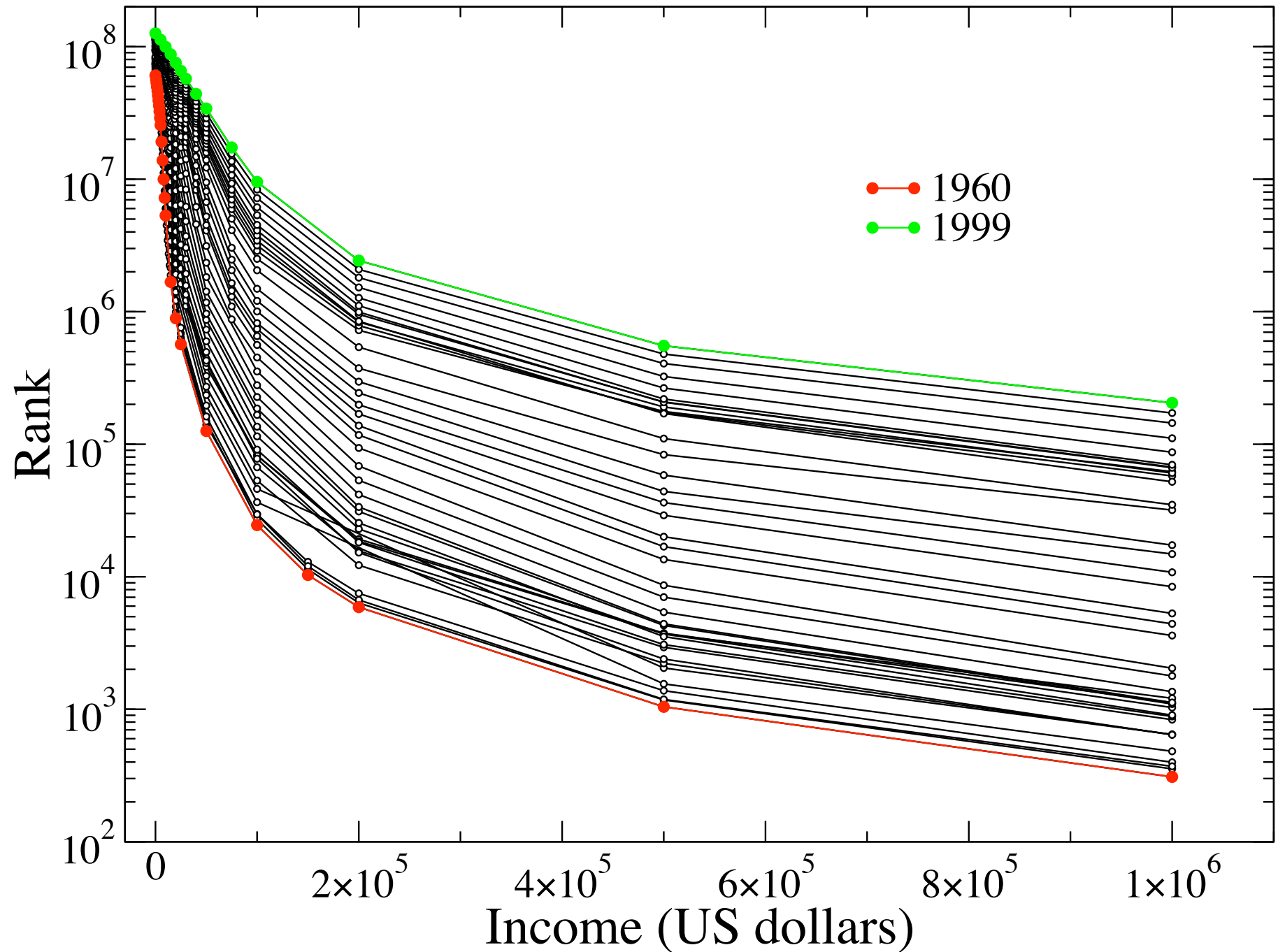
Total income in 1999 with converted income tax data



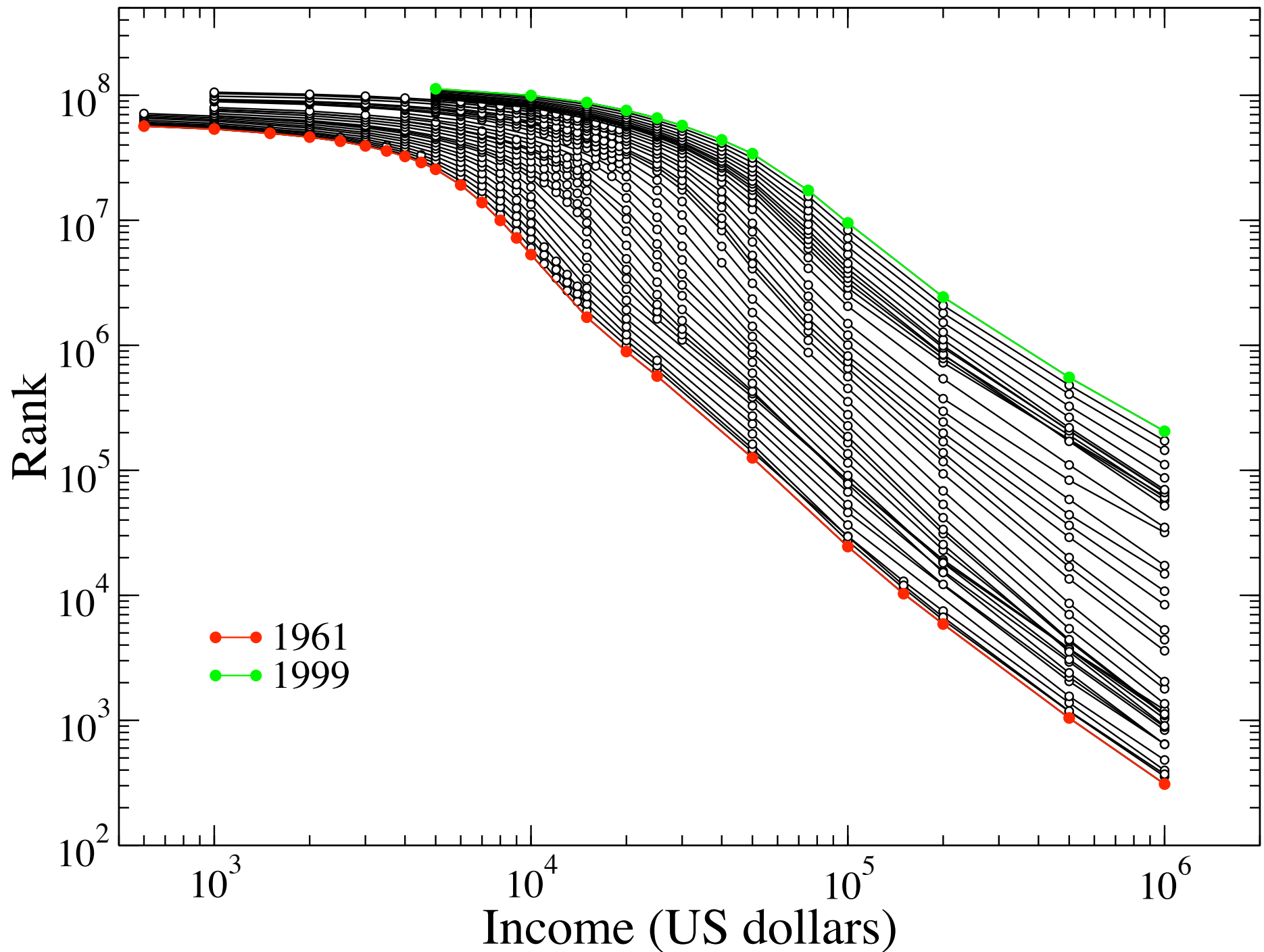
Total income from 1951 to 2000



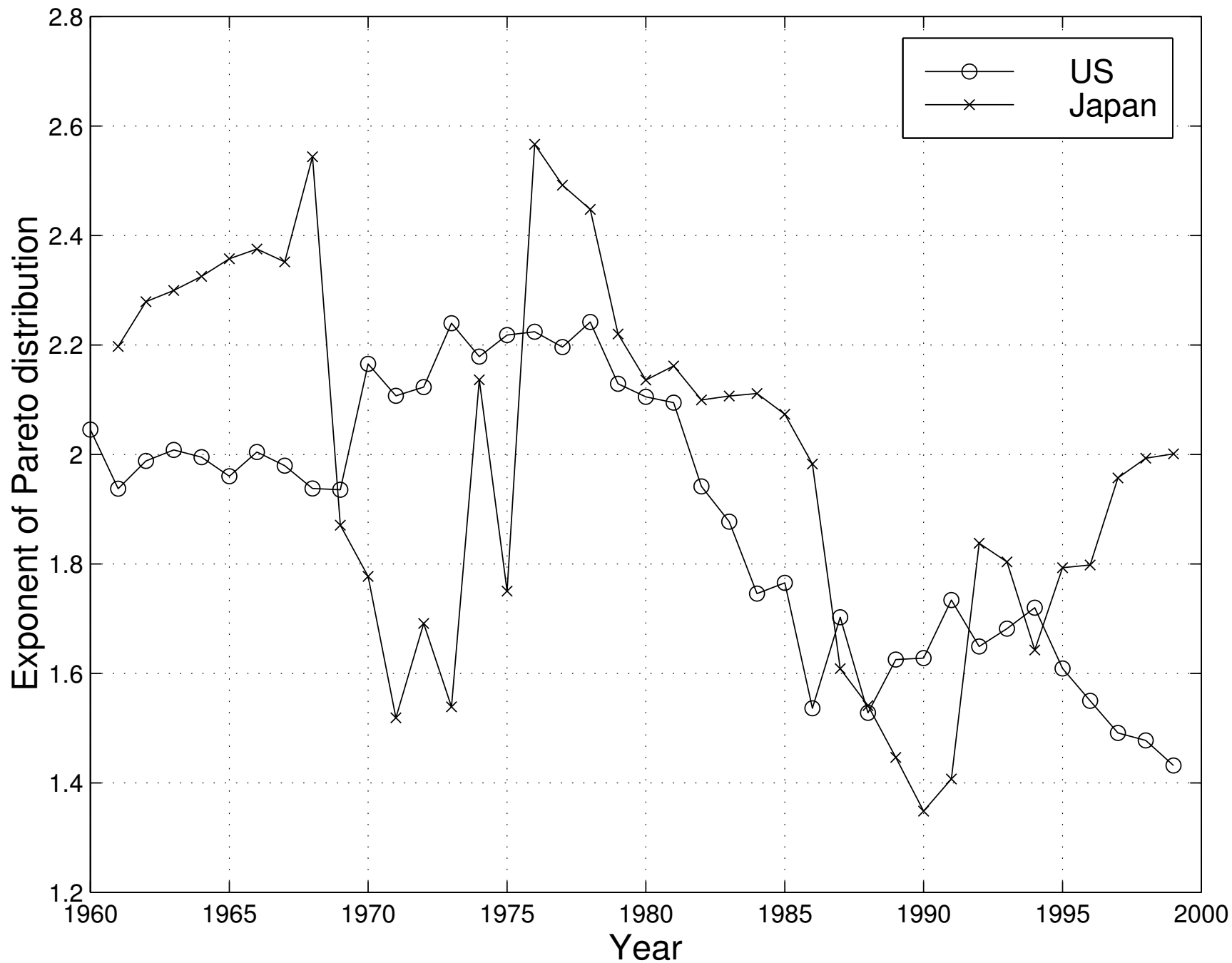
Statistics of Income database of the Internal Revenue Service from 1960 to 1999



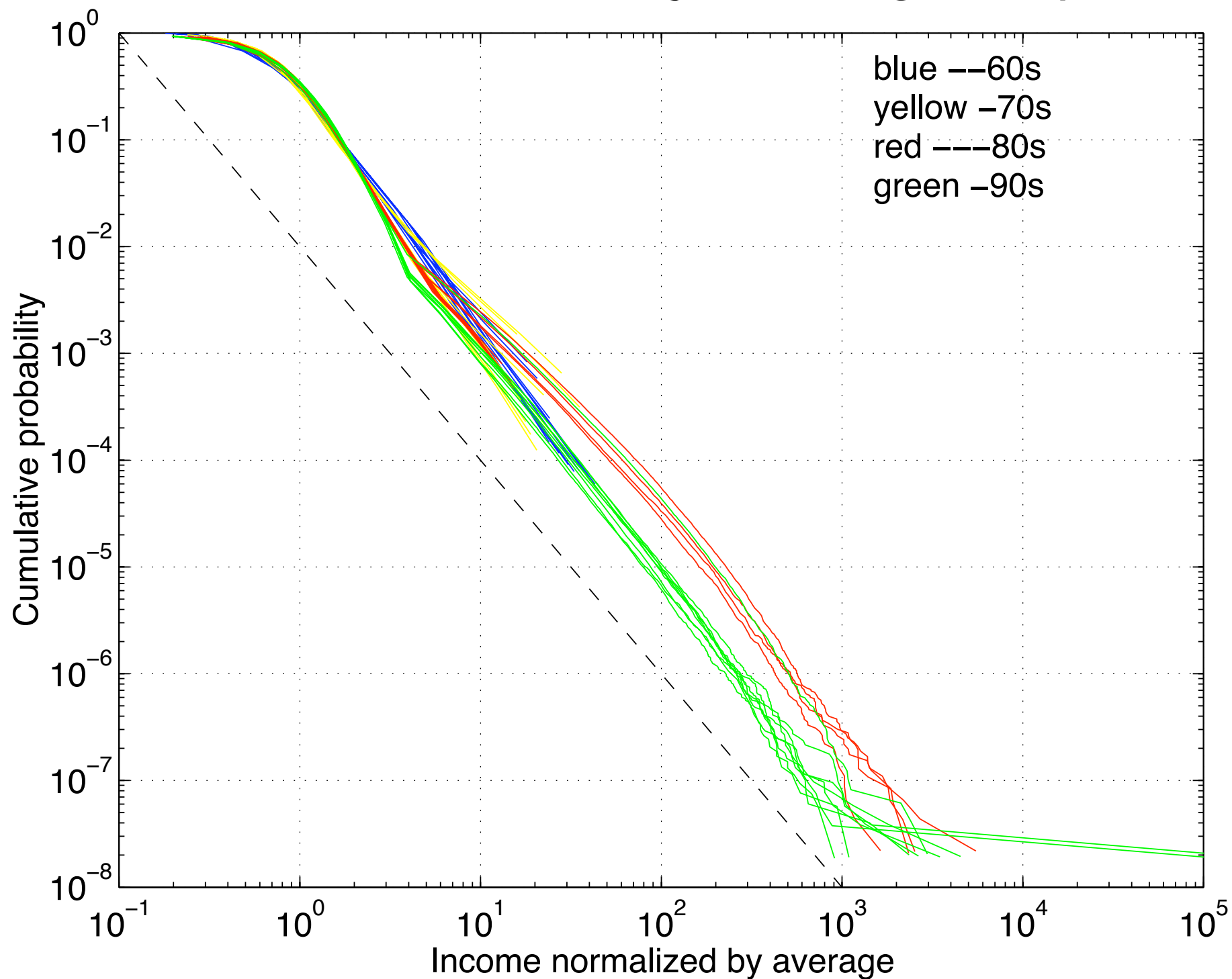
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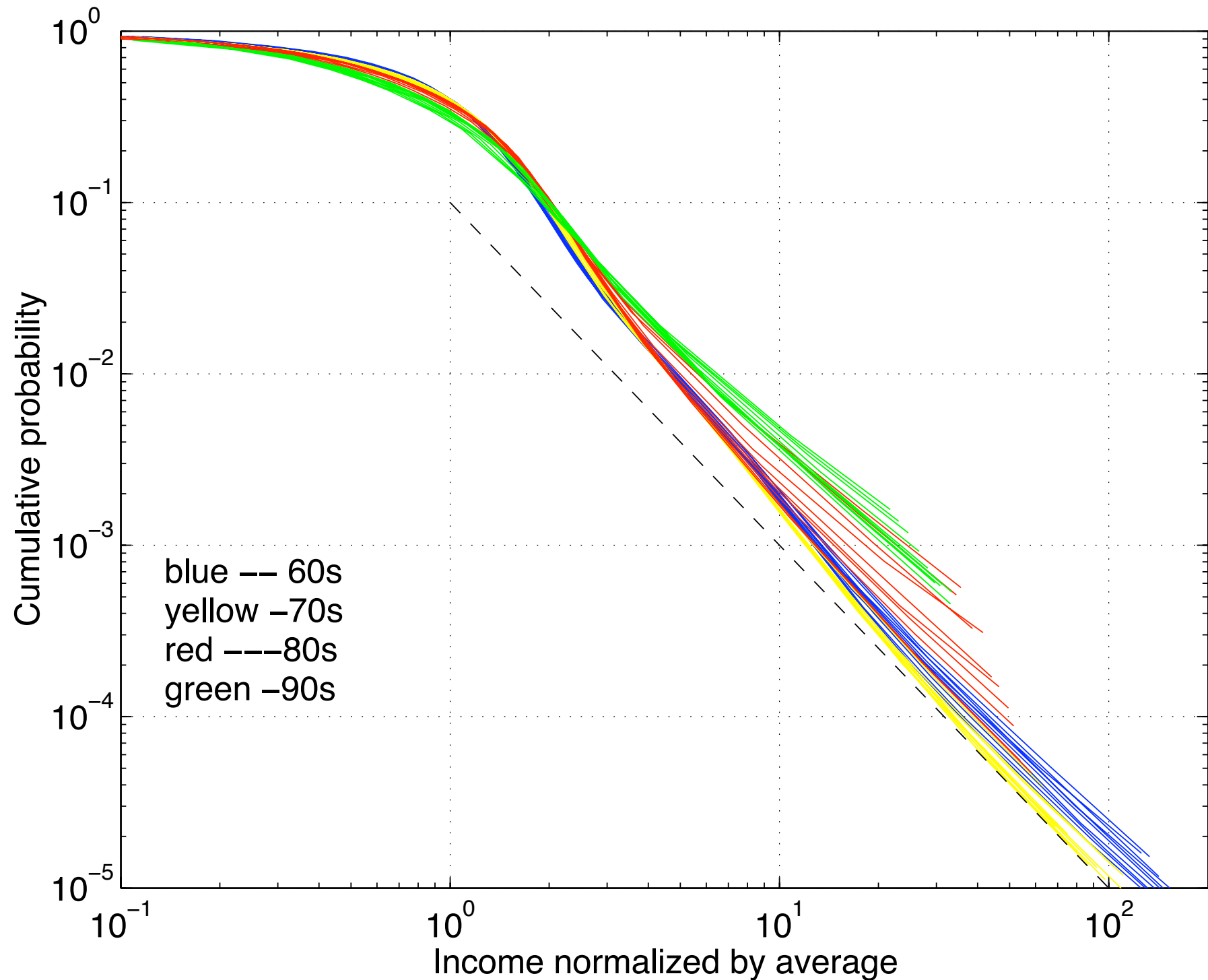
Pareto index



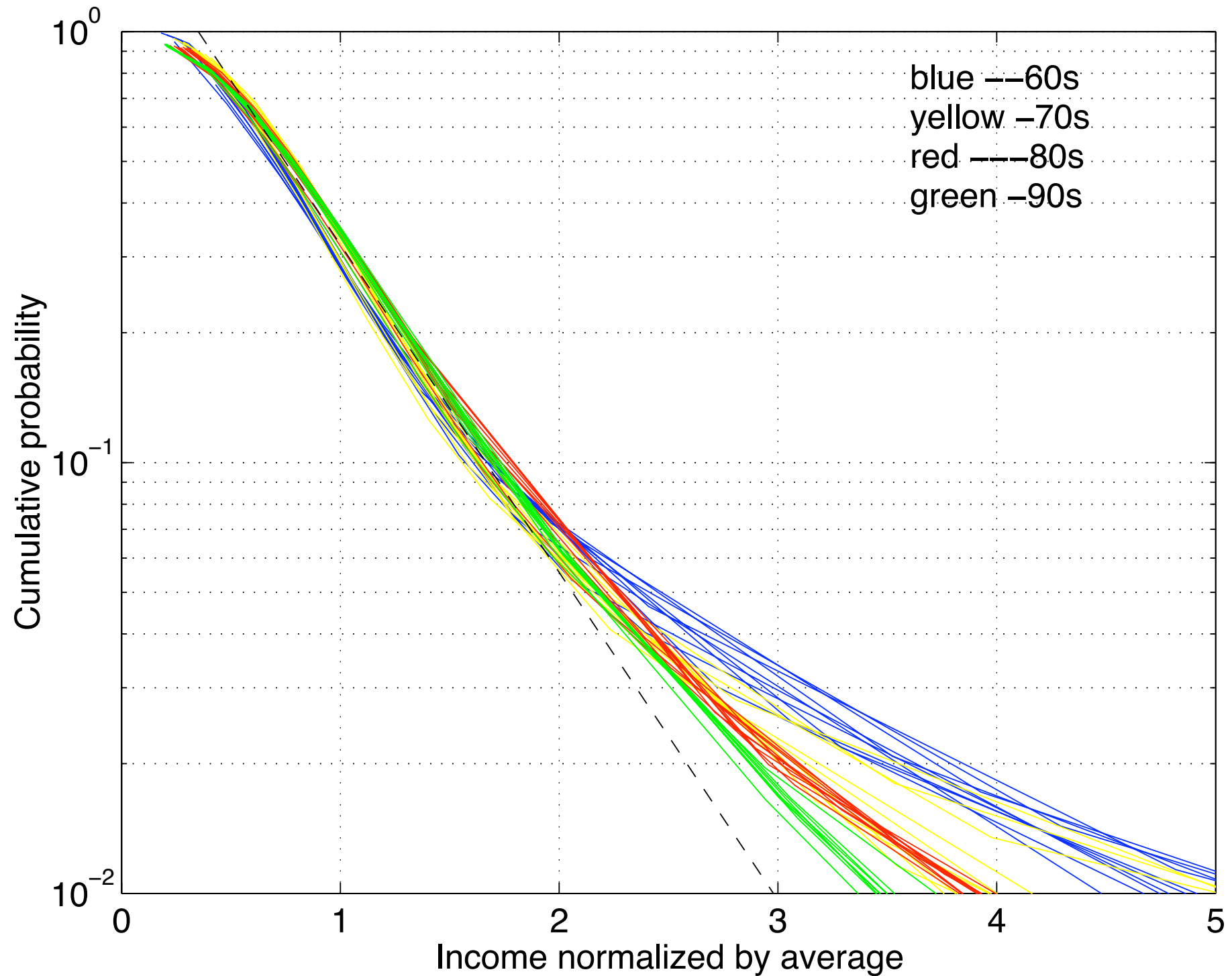
Income normalized by Average (Japan)



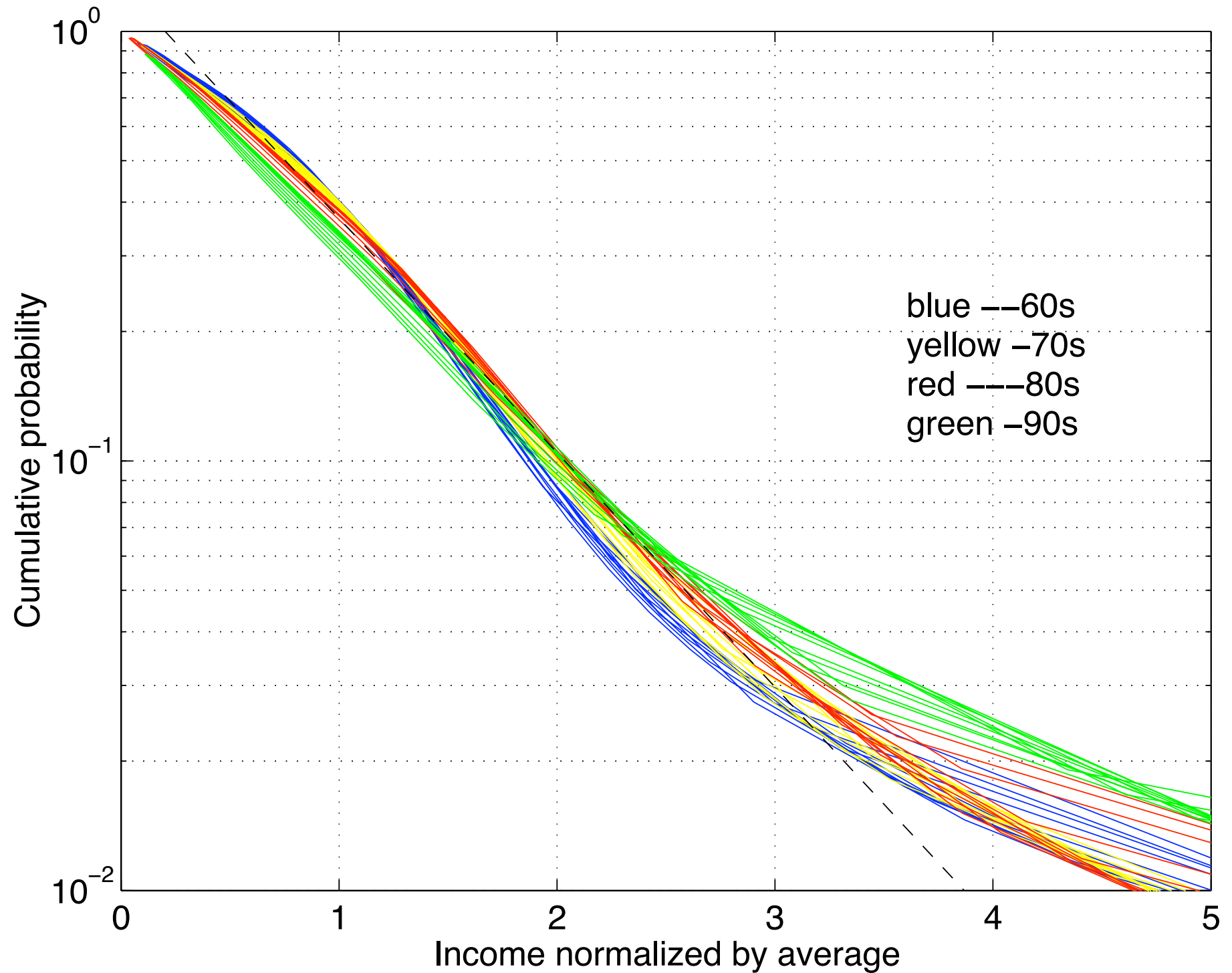
Income normalized by Average (U.S.)



Income normalized by Average (Japan)



Income normalized by Average (U.S.)



- Income normalized by average
 - Stationary distribution
- Middle income range
 - Boltzmann, exponential, log-normal
 - Income source: employment (labor) income
- High income range (top taxpayers)
 - Power law distribution
 - Income source: ?

Income source of top taxpayers

2003 2002 2001 2000

Capital gains of stocks, etc.
Long-term separate capital gains
Short-term separate capital gains

Assets

Retirement income

Forestry income

Income from public pension, etc.

Miscellaneous income

Sporadic income

Comprehensive capital gains

Employment

Wages & salaries

Assets

Rental income

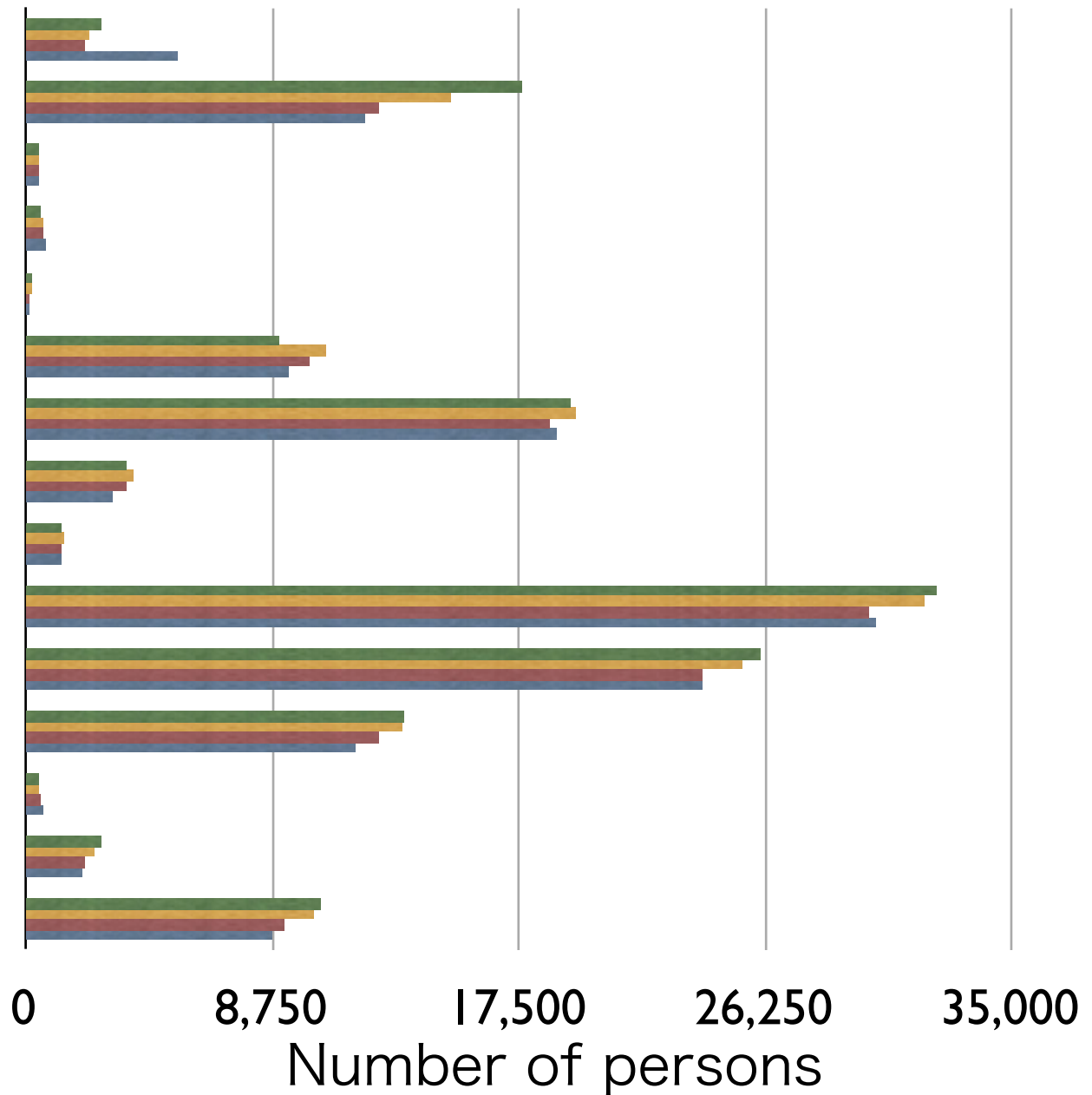
Dividends

Interest income

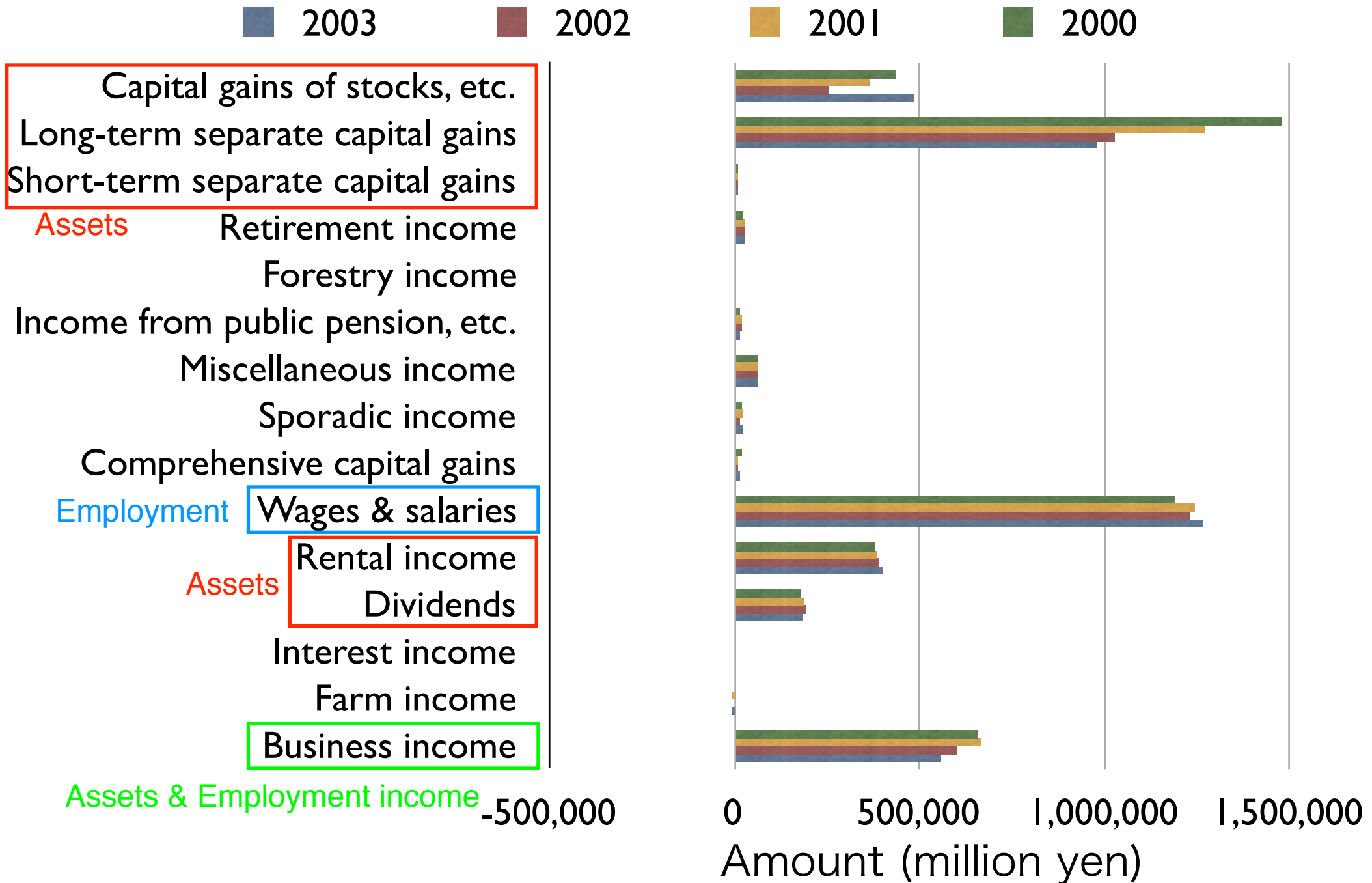
Farm income

Business income

Assets & Employment income



Income source of top taxpayers



- Income source in high income range
 - Employment income (Labor income)
 - Income from assets
- $\text{Income} = \text{Employment income} + \text{Income from assets}$

Model

- Income = Employment income + Income from assets
- Employment income process

Employment income

$$w_i(t+1) = uw_i(t) + s\epsilon_i\bar{w}(t) \quad i = 1 \sim N$$

The trend growth rate of employment income, u , reflects an automatic growth in nominal wage.

We use an average inflation rate for the period 1961--1999,

$$u = 1.0422$$

$$\epsilon_i = N(0, 1)$$

s determines the level of income for the middle class

We choose $s = 0.32$ to fit the middle part of the empirical distribution

Reflective lower bound:
subsistence level of income

$$w_i(t) > \bar{w}(t)$$

$$\bar{w}(t) = v\bar{w}(t-1)$$

$$v = 1.0673$$

Time average growth rate of the nominal income per capita

● Assets accumulation process

Assets

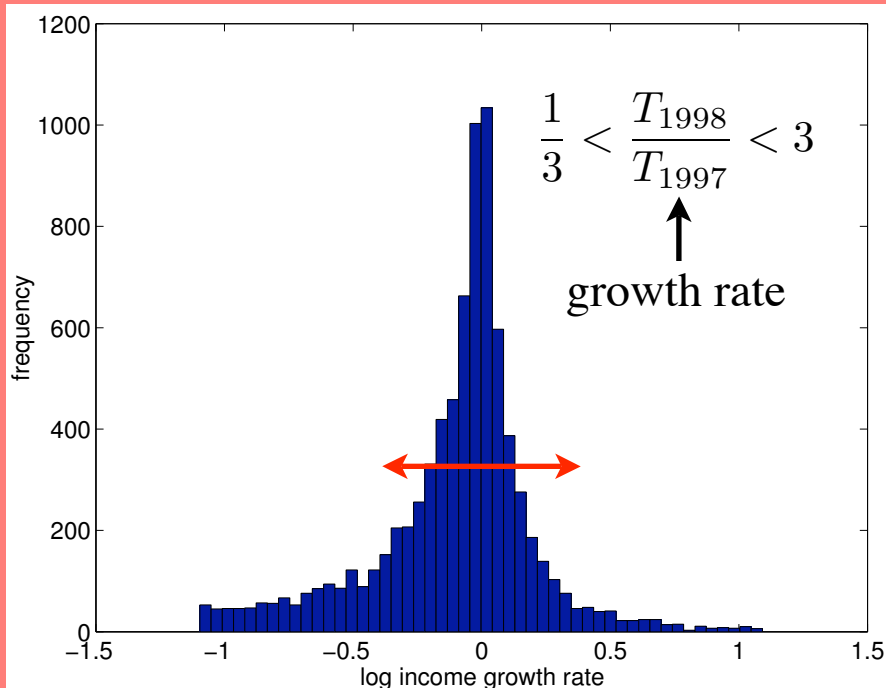
$$a_i(t+1) = \gamma_i(t)a_i(t) + w_i(t) - c_i(t)$$

We assume that the log return follows a normal distribution

$$\ln \gamma = N(y, x^2)$$

The log-variance of the asset return is estimated from top taxpayers data

$$x = 0.3122$$



Consumption

$$c_i(t) = \bar{w}(t) + b \left\{ a_i(t) + w_i(t) - \bar{w}(t) \right\}$$

Subsistence income determines the minimum level of consumption

$b = 0.059$ is chosen from the empirical range (0.05--0.1) estimated from Japanese micro data in 1990s by Hori et al. (2003)

The log-mean of the assets growth y cannot be estimated by the same growth rate data, since 1997 was not a typical year.

We estimate y by using a time-average growth rate of Nikkei average index, that is 0.0595 in average over 1961--1999.

$$y = 0.0595 - \frac{x^2}{2} \approx 0.01$$

- Income

$$\tilde{I}_i(t) = w_i(t) + E[\gamma_i(t) - 1]a_i(t)$$

- Normalized income

$$I_i(t) = \frac{\tilde{I}_i(t)}{E[\tilde{I}_i(t)]}$$

- Pareto index in the steady state

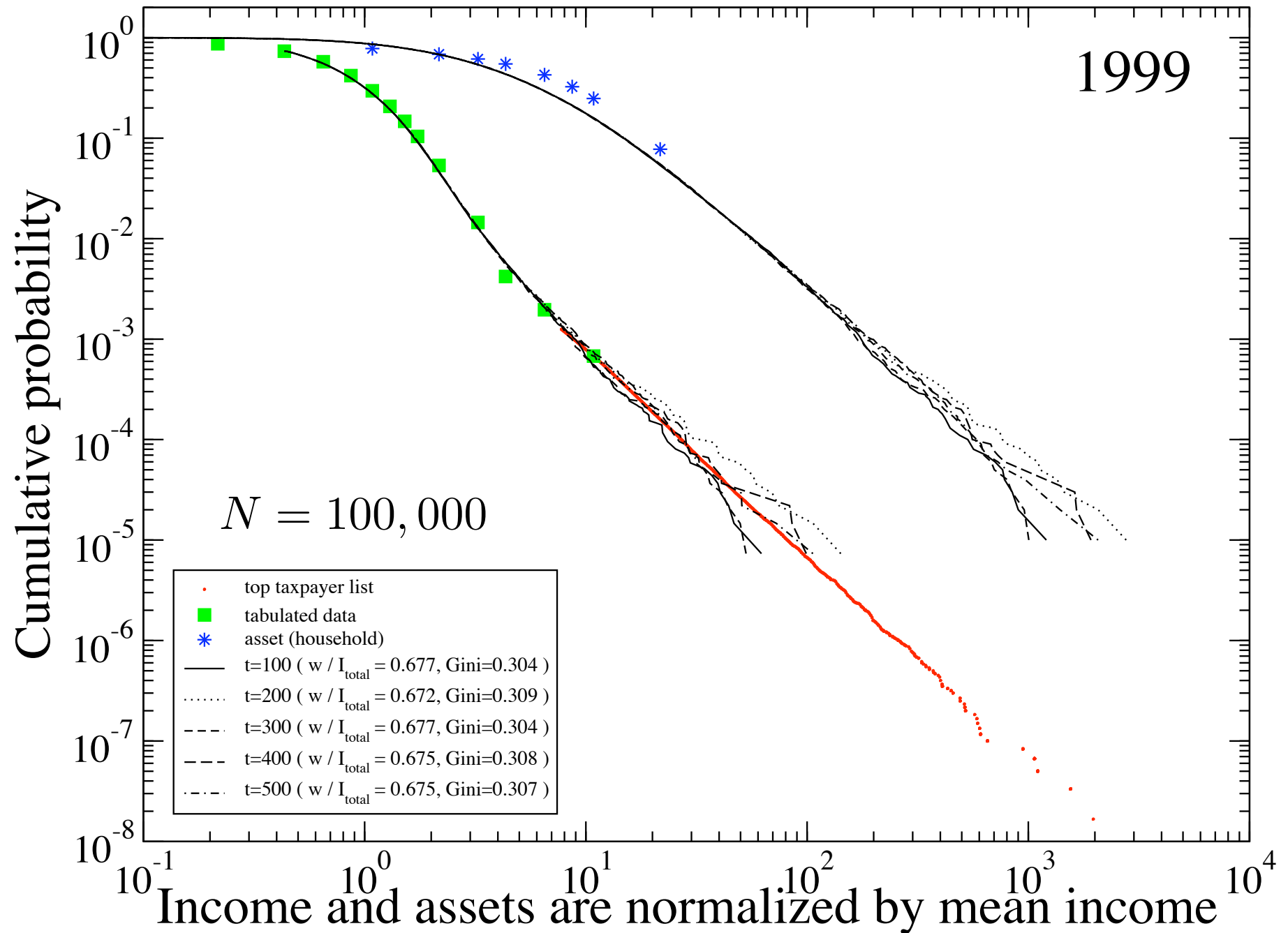
$$z \equiv \lim_{t \rightarrow \infty} \frac{w(t) - c(t)}{\langle a(t) \rangle} \quad \text{steady state value}$$

$$\alpha \approx 1 + \frac{2z}{gx^2} \quad \langle a(t) \rangle : \text{averaged assets}$$

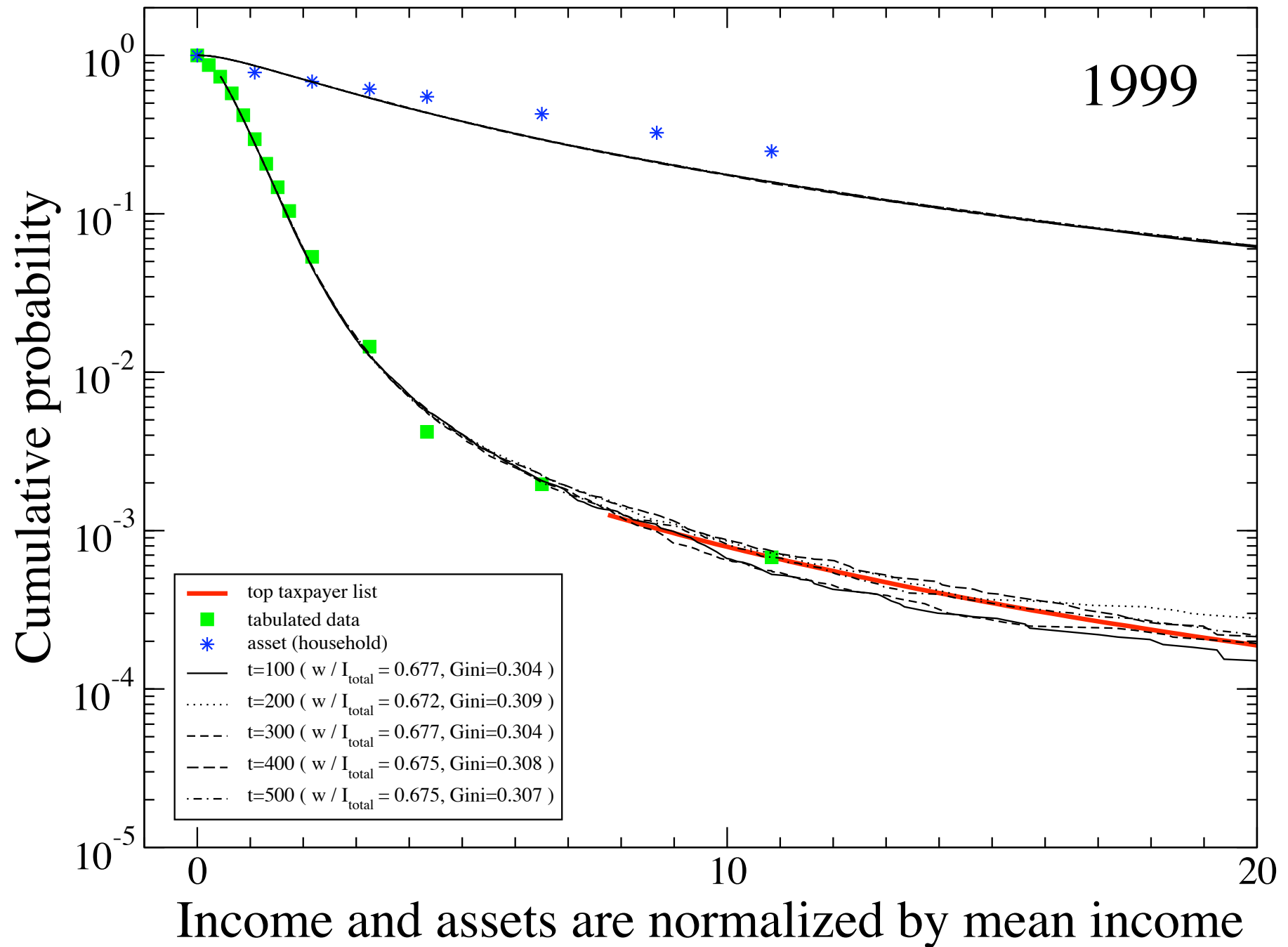
$$g \equiv \lim_{t \rightarrow \infty} g(t) \quad \text{steady state value}$$

$$gx^2 \sim 2z \quad g(t) : \text{growth rate of } \langle a(t) \rangle$$

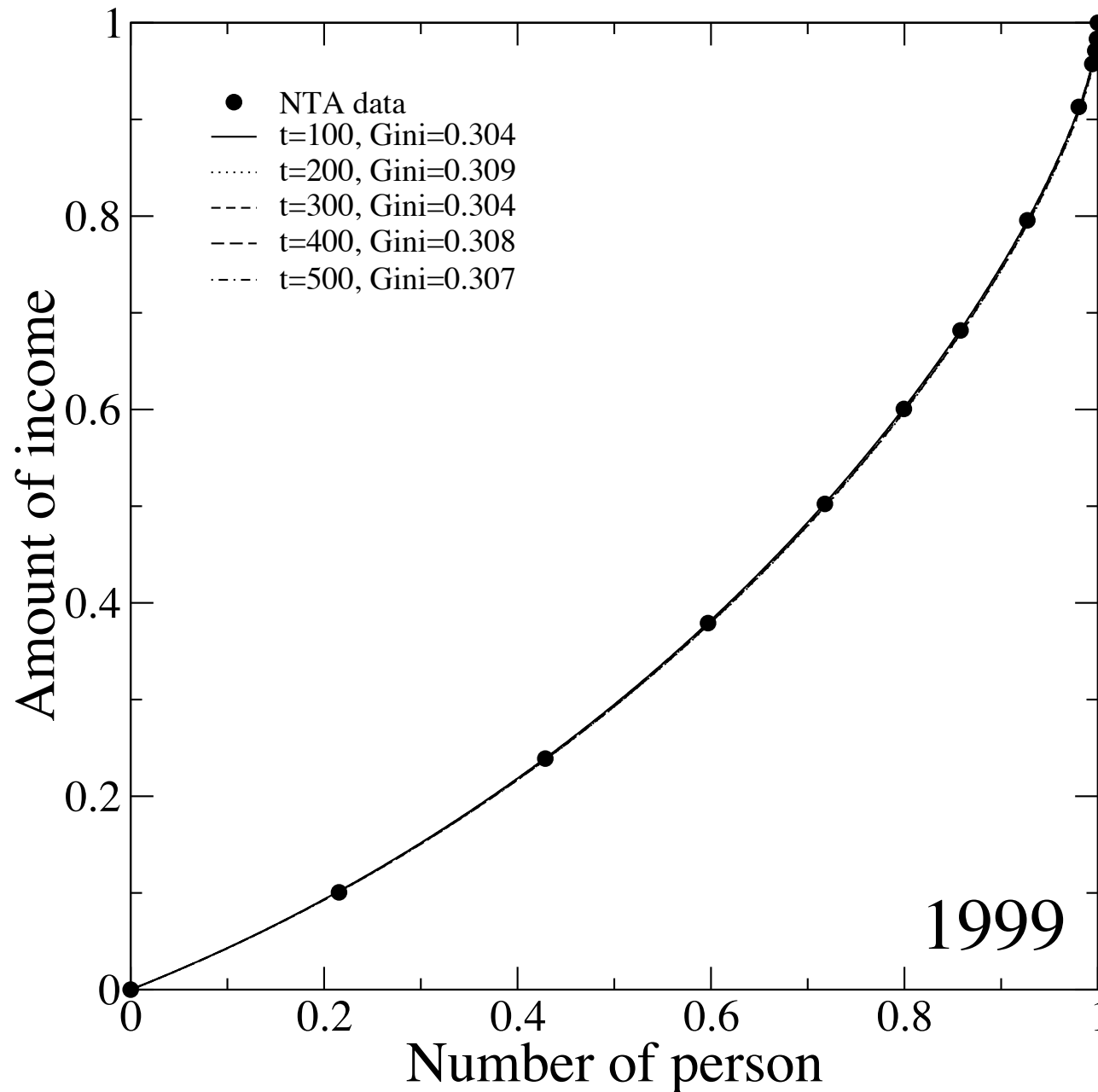
Simulation Result



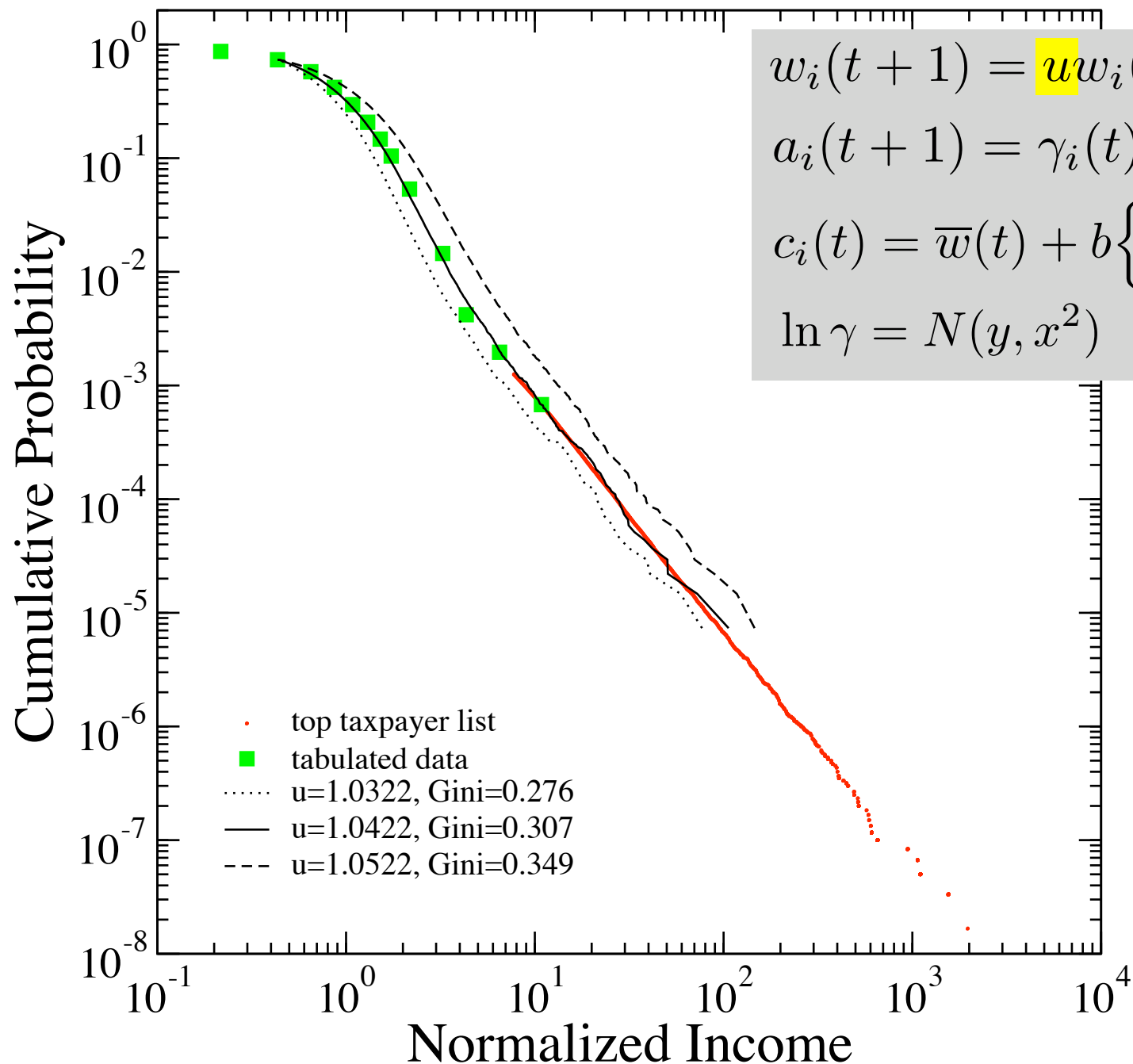
Simulation Result



Simulation Result



Parameter Sensitivity



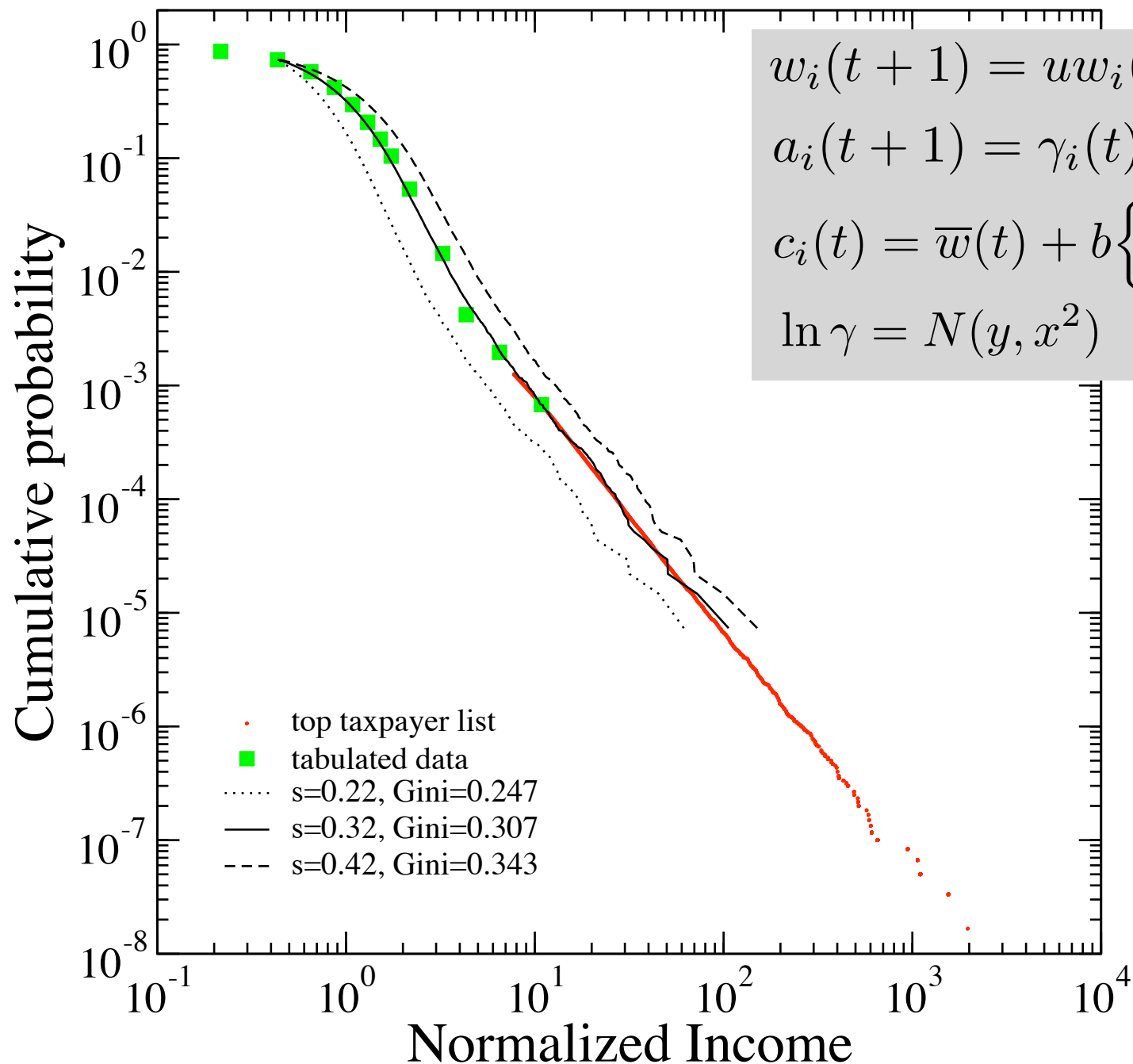
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$$a_i(t+1) = \gamma_i(t) a_i(t) + w_i(t) - c_i(t)$$

$$c_i(t) = \bar{w}(t) + b \left\{ a_i(t) + w_i(t) - \bar{w}(t) \right\}$$

$$\ln \gamma = N(y, x^2)$$

Parameter Sensitivity



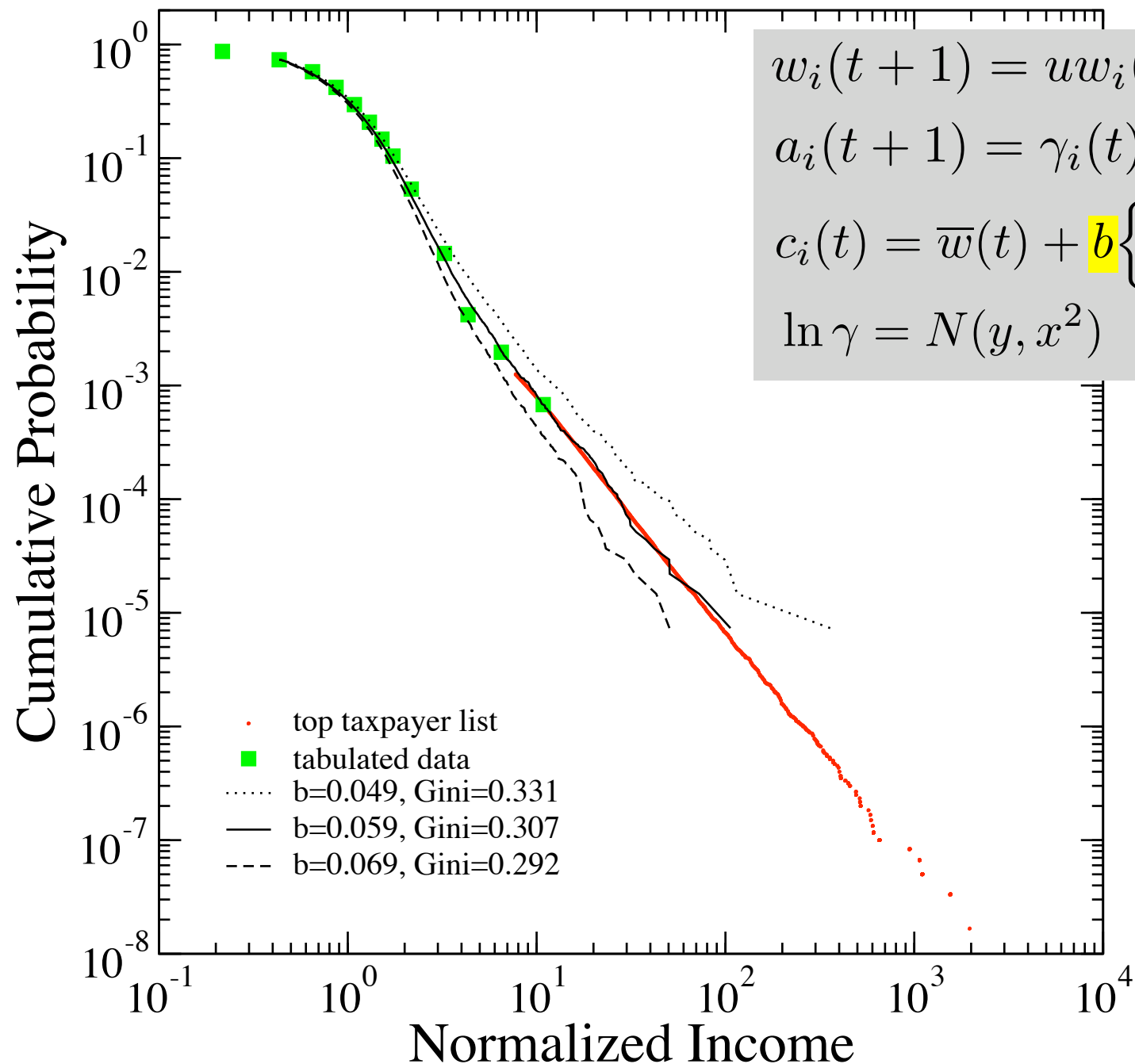
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Parameter Sensitivity



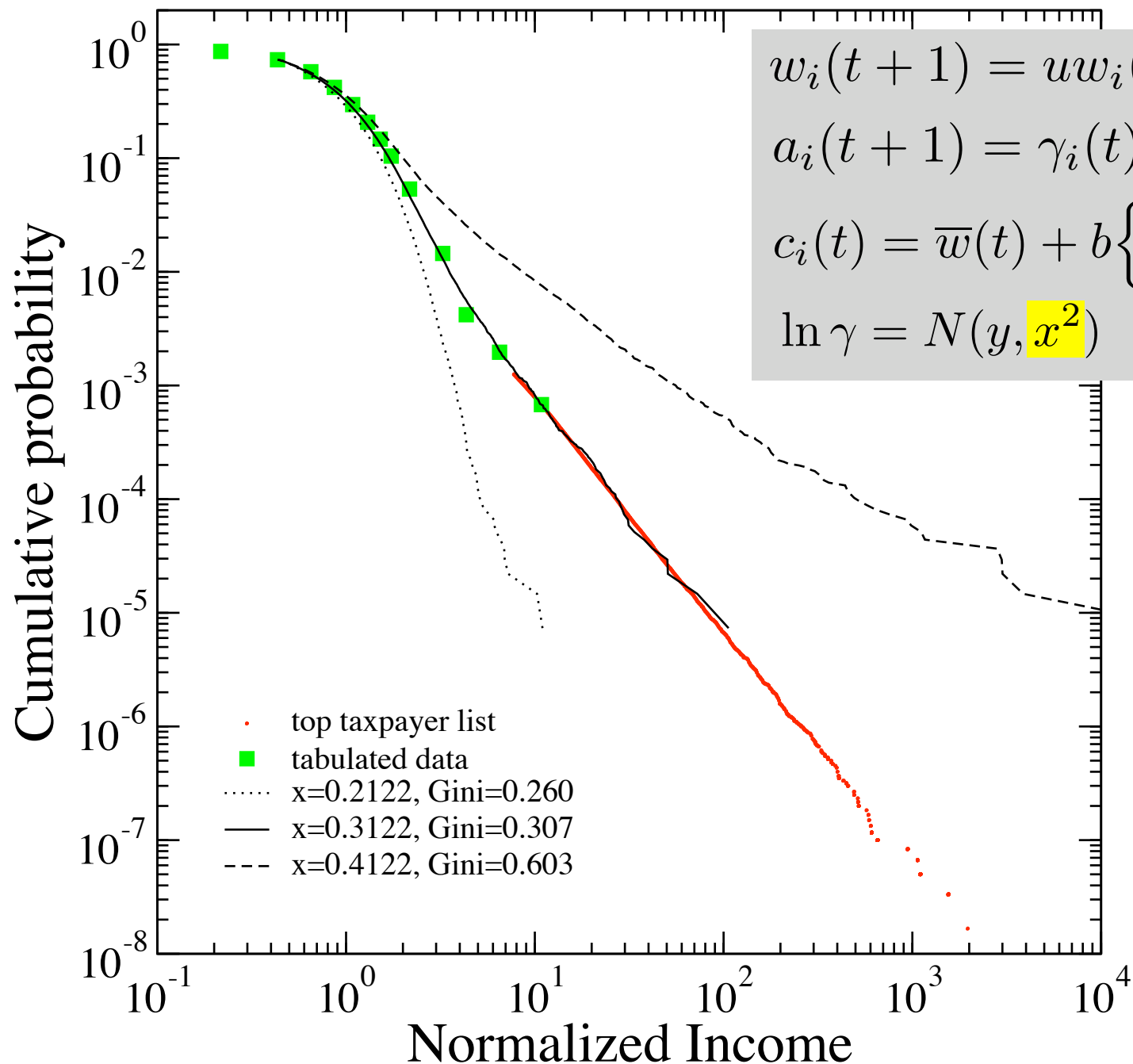
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Parameter Sensitivity



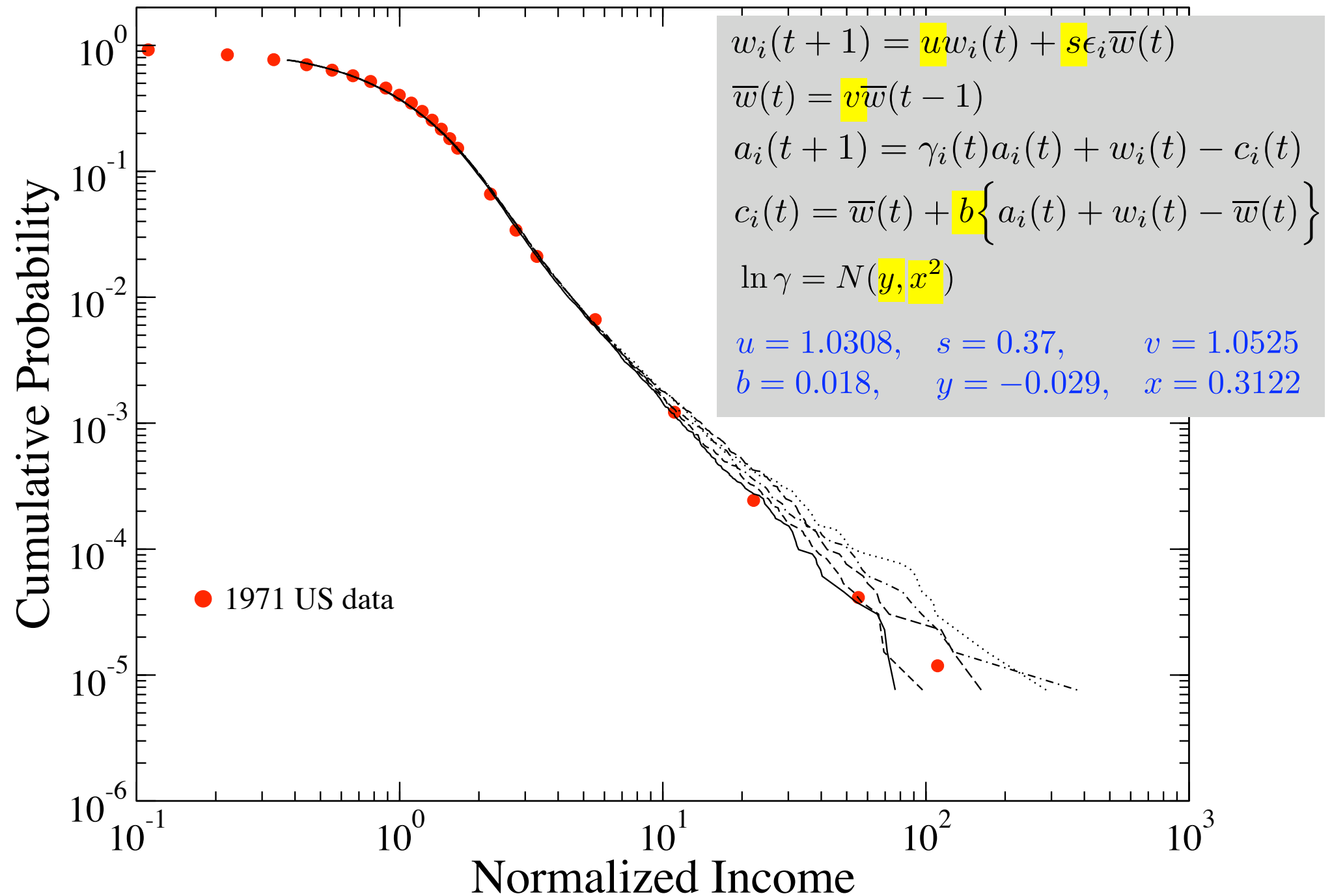
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$$\ln \gamma = N(y, x^2)$$

Simulation Result (U.S)



Questions

- Can we know the low income distribution?
- Can we obtain a model explaining the income growth distribution?
- Can we know characteristics of money flow networks?
- Can we understand economics as complex networks?

Flow: income
Stock: wealth

