Pareto-Zipf law, Gibrat's law, detailed-balance and their breakdown

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collaboration with

- personal income and firms

Hideaki Aoyama (Kyoto University)

Wataru Souma (ATR)

- firm size

Mauro Gallegati, Corrado Di Guilmi (Ancona)

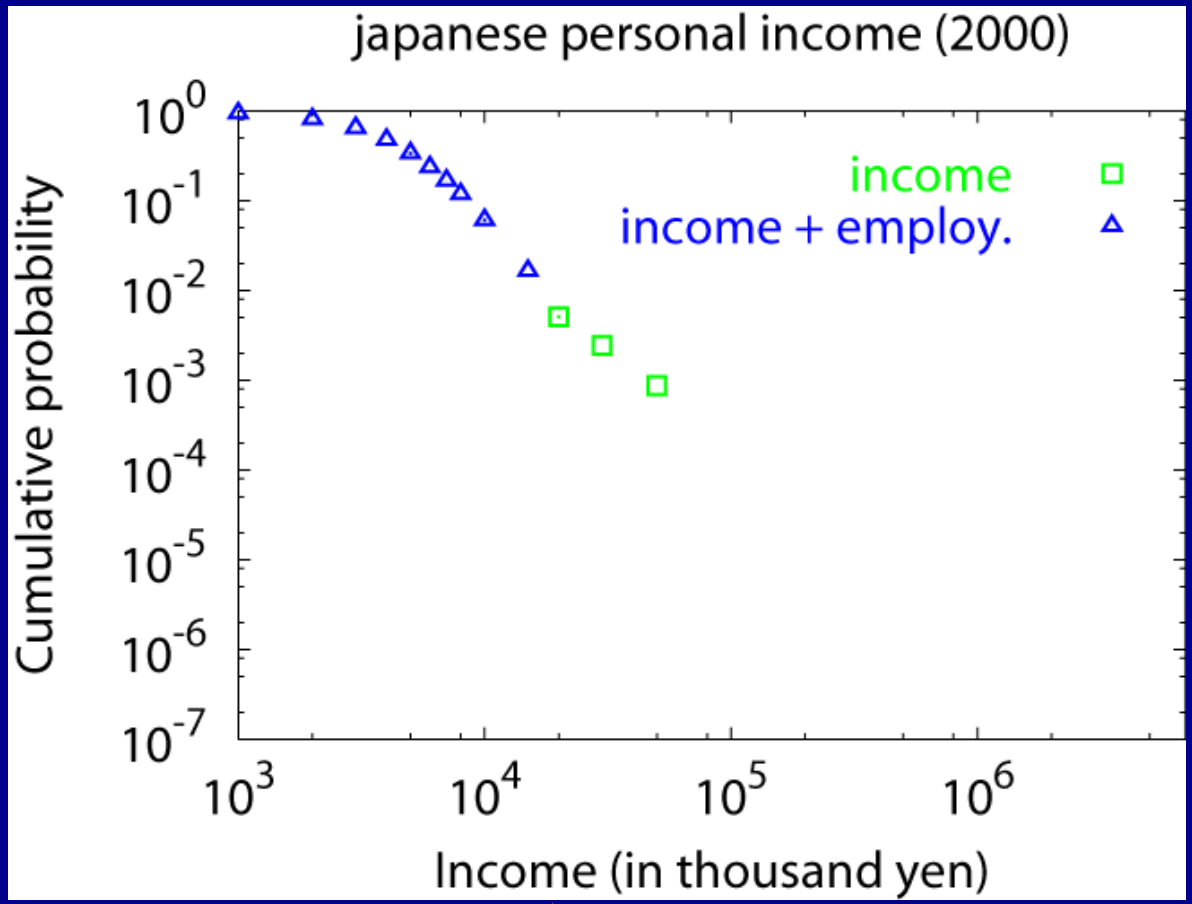
japanese personal income

def. of income

wages/salaries, income from self-employment,
capital gains (lands/stocks etc.),
miscellaneous (public pension, transfer, etc.)
N.B. inheritance not included

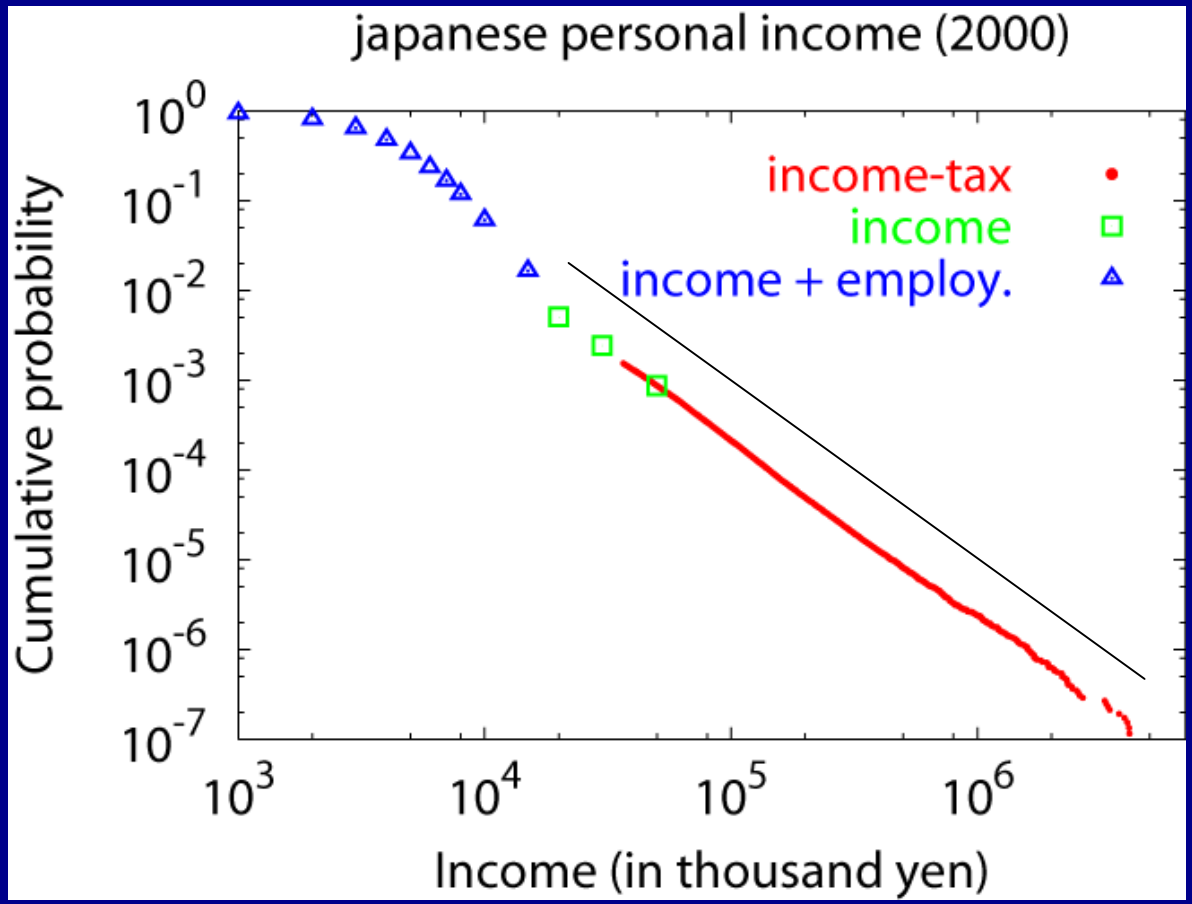
data sources

1. Employment income
 - sample survey for salaries workers in companies
 - sampled out of 45 million people
2. Income data
 - compiled and tabulated
 - 7 million people who filed tax returns
3. Income-tax data
 - complete list of high-income taxpayers
 - 80,000 people w. names and addresses



200,000 Euros/USD

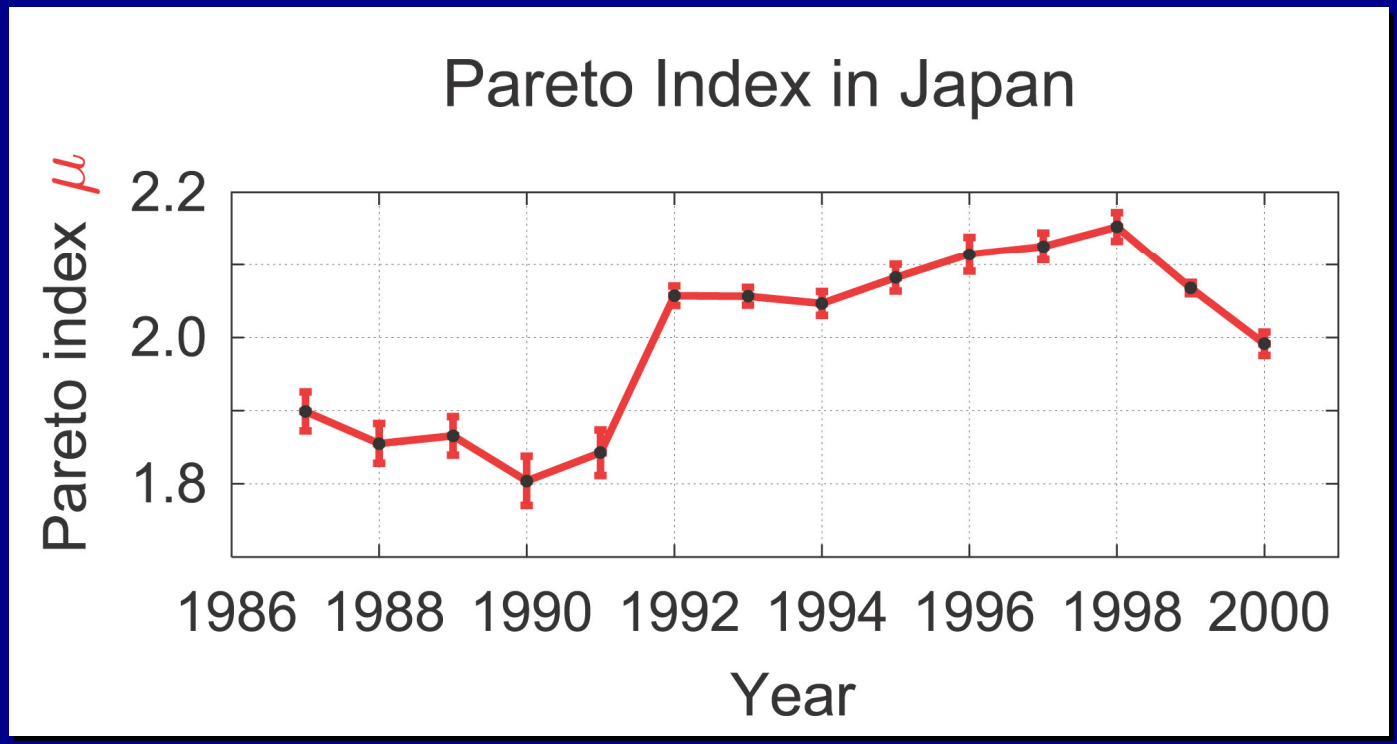
100 Yen=1 Euros/USD



$\mu \simeq 2$

$$P_{>}(x) \propto x^{-\mu}$$

Annual change of Pareto index 1987-2000



Problem addressed

- Pareto's law: snapshot of distribution
- many scenarios for power-law

- Champernowne (1953)
- Mandelbrot (1961)
- Ijiri & Simon (1977)
- Montroll & Shlesinger (1983)
- Levy & Solomon (1996)
- Bouchaud & Mezard (2000)
- Solomon & Richmond (2001)
- etc.

Empirical study of dynamics is lacking

Contents

1. growth and fluctuation: Gibrat's law
2. detailed-balance, Pareto and Gibrat
   Pareto $\longleftrightarrow$ Gibrat
3. temporal breakdown (non-staionarity)
   and risky assets
4. Firm size
   scaling in non-Pareto region (firms)
5. Conclusion

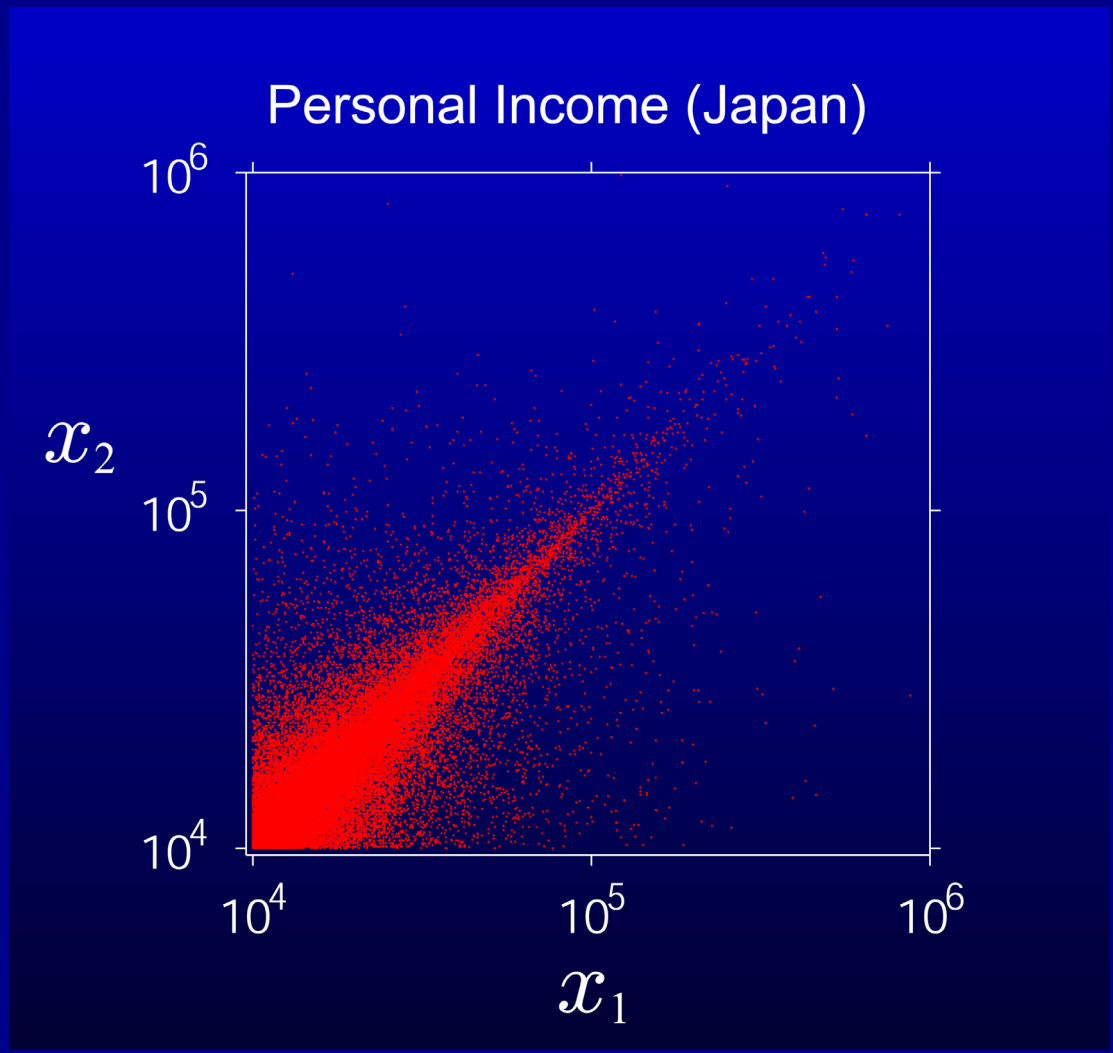
Growth and fluctuations

variable: x (income, firm size; flow and stock)

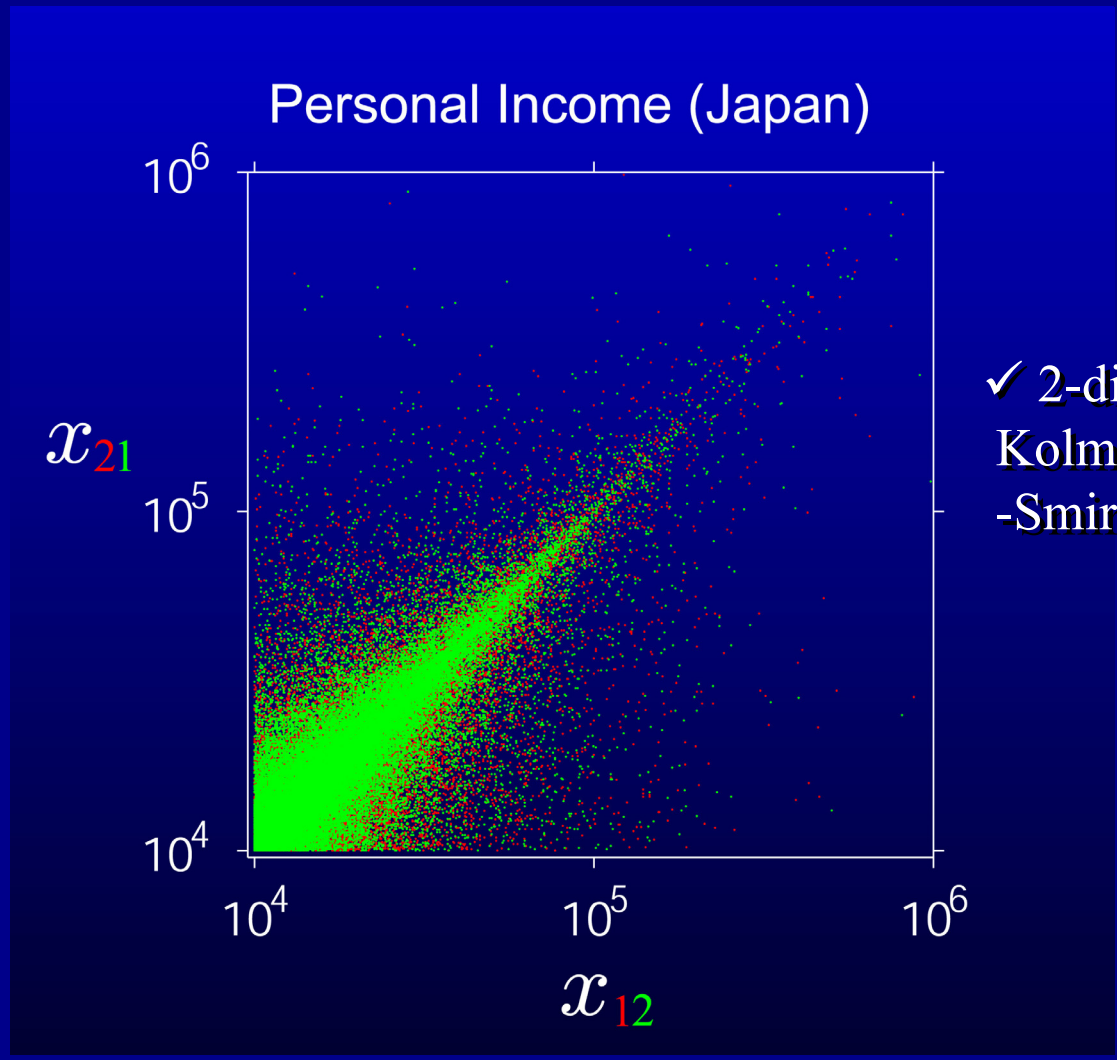
How does x change in time?

x_1 : at initial year
x_2 : at next year

$P_{12}(x_1, x_2)$


Personal Income (Japan)
10^6
10^5
10^4
x_2
10^4
10^5
10^6
x_1

$$P_{12}(x_1, x_2)$$



✓ 2-dim Kolmogorov -Smirnov test

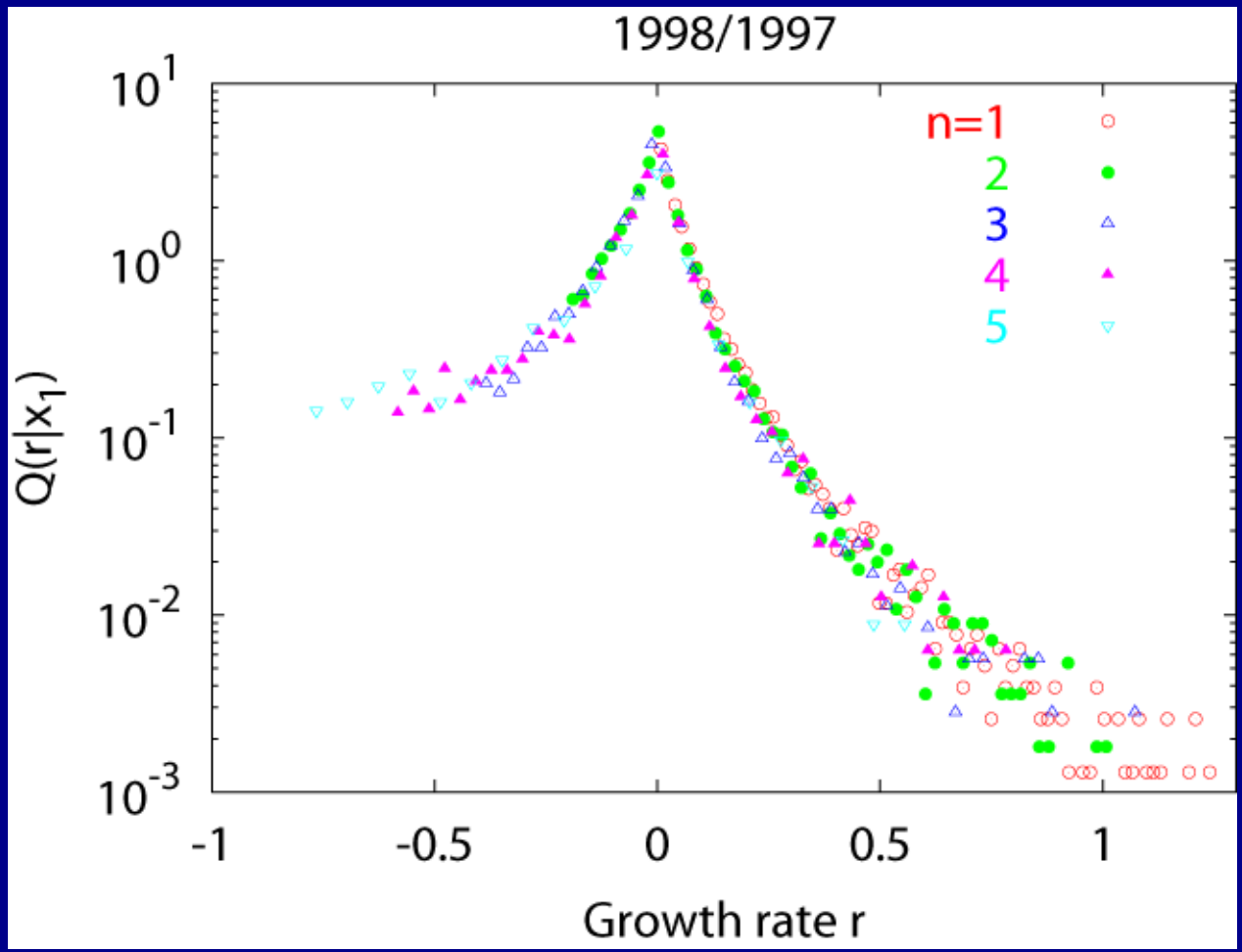
$$P_{12}(x_1, x_2) = P_{12}(x_2, x_1)$$

growth rate: $R = x_2/x_1$ $\longrightarrow$ $P_{1R}(x_1, R)$

$P_1(x)$: prob. distribution of x
$Q(R|x_1)$: conditional prob. distribution

$$P_{1R}(x_1, R) = P_1(x_1)Q(R\,|\,x_1)$$

$r \equiv \log_{10} R$: log growth rate



Gibrat's law

The growth rate R is independent from the first year's value x_1.

$$Q(R \mid x_1) = Q(R)$$

or,

$$P_{1R}(x_1, R) = P_1(x_1)Q(R) \quad (x_1 \gg x_0)$$

Robert Pierre Louis Gibrat (1932)

Pareto's law

Cumulative PDF $P_{>}(x) \propto x^{-\mu}$

PDF $P(x) \propto x^{-\mu-1}$

$(x_1 \gg x_0)$

μ : Pareto index

Vilfredo Pareto
(1897)

Relation of detailed-balance, Pareto and Gibrat

1. Detailed-balance

$$P_{12}(x_1, x_2) = P_{12}(x_2, x_1)$$

2. Gibrat's law

$$Q(R \,|\, x_1) = Q(R)$$

3. Pareto's law

$$P_1(x) \propto x^{-\mu-1}$$

How 1,2,3 are related with each other?

Under detailed-balance (1)
Gibrat's law (2)

Pareto's law (3)

plus

- "reflection" relation

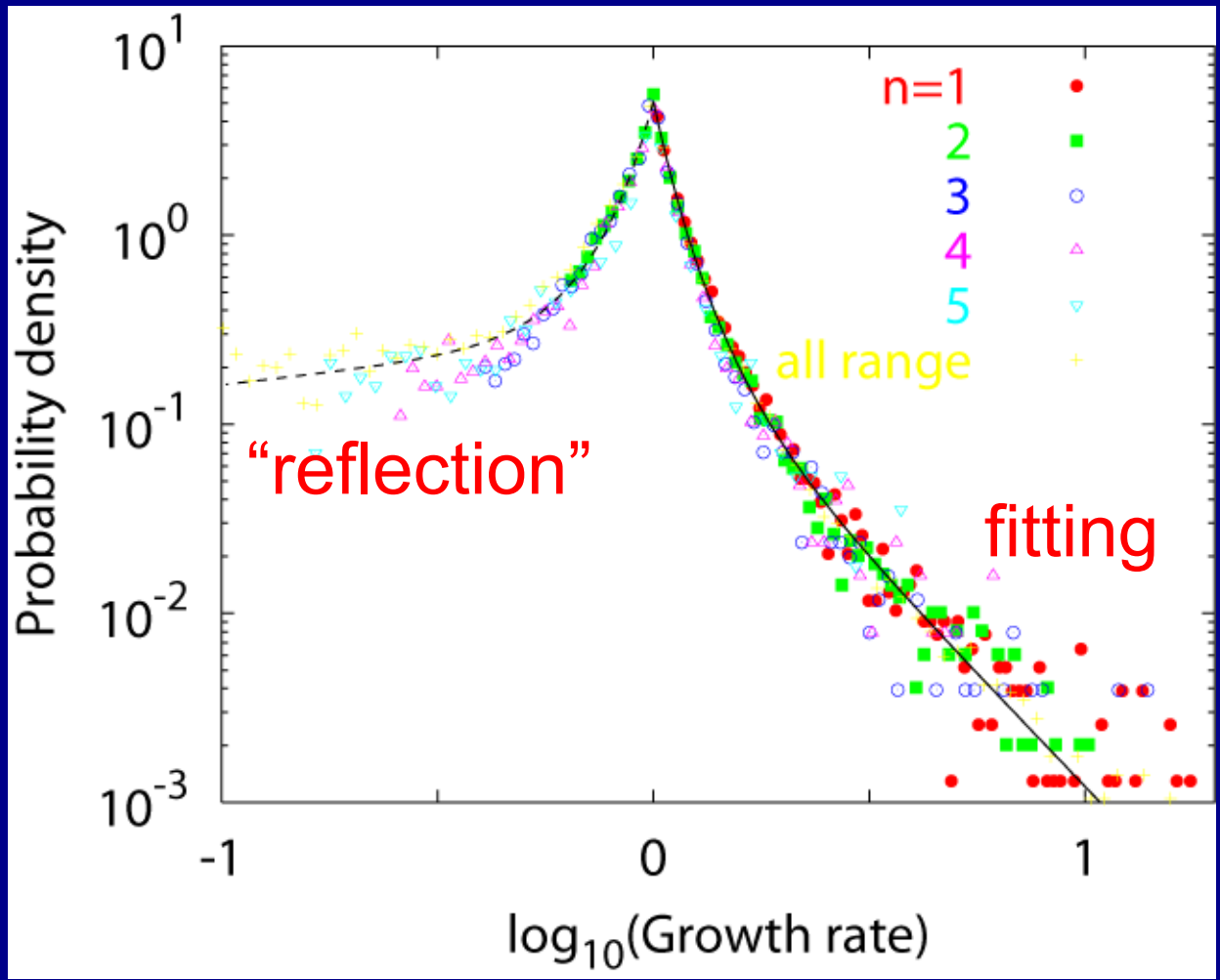
$$Q(R) = R^{-\mu-2} Q(R^{-1})$$

skewness

- shape at $R = 1$

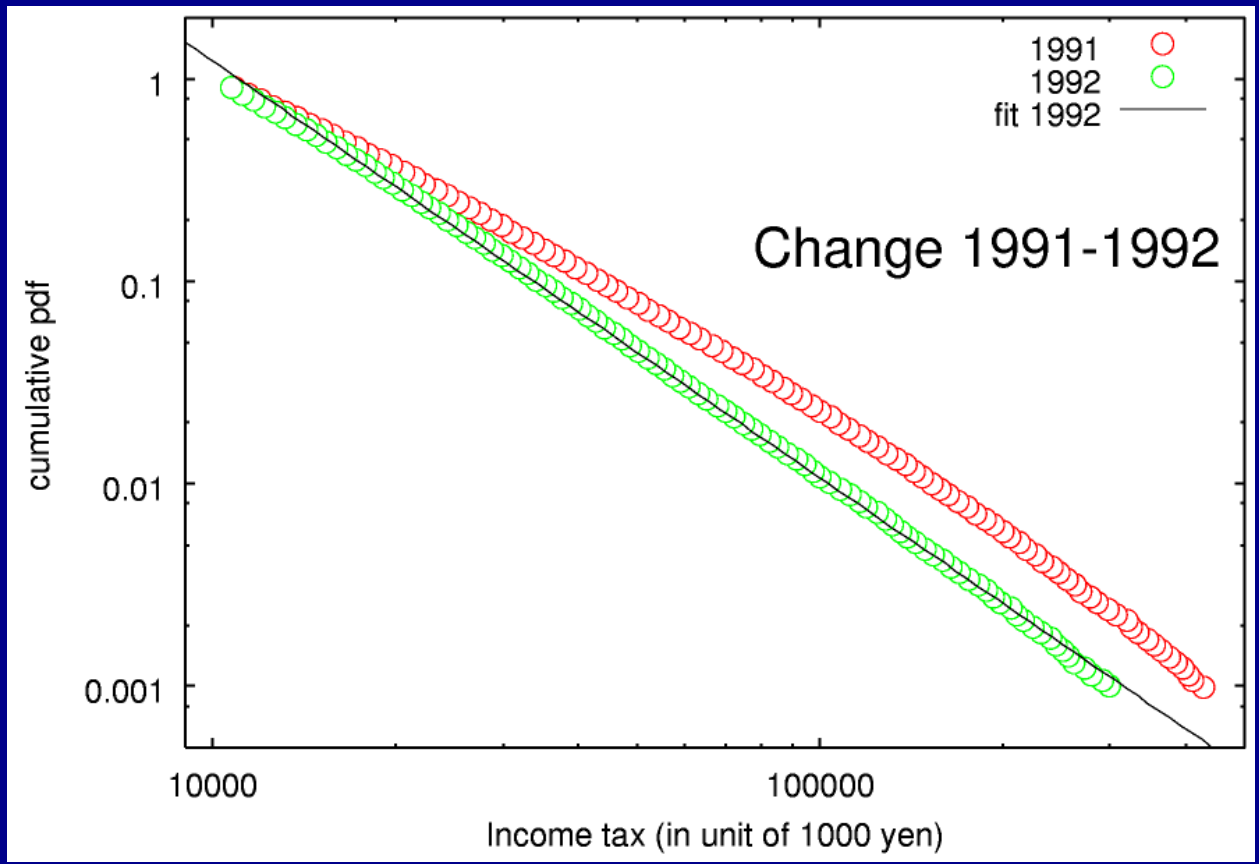
$$\frac{Q^{+\prime}(1)+Q^{-\prime}(1)}{Q(1)} = -\mu - 2$$

cusp


Probability density
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
-1
0
1
$\log_{10}$(Growth rate)
n=1
2
3
4
5
all range
"reflection"
fitting

Breakdown of all laws in non-staionary state

power-law breaks down under economic change



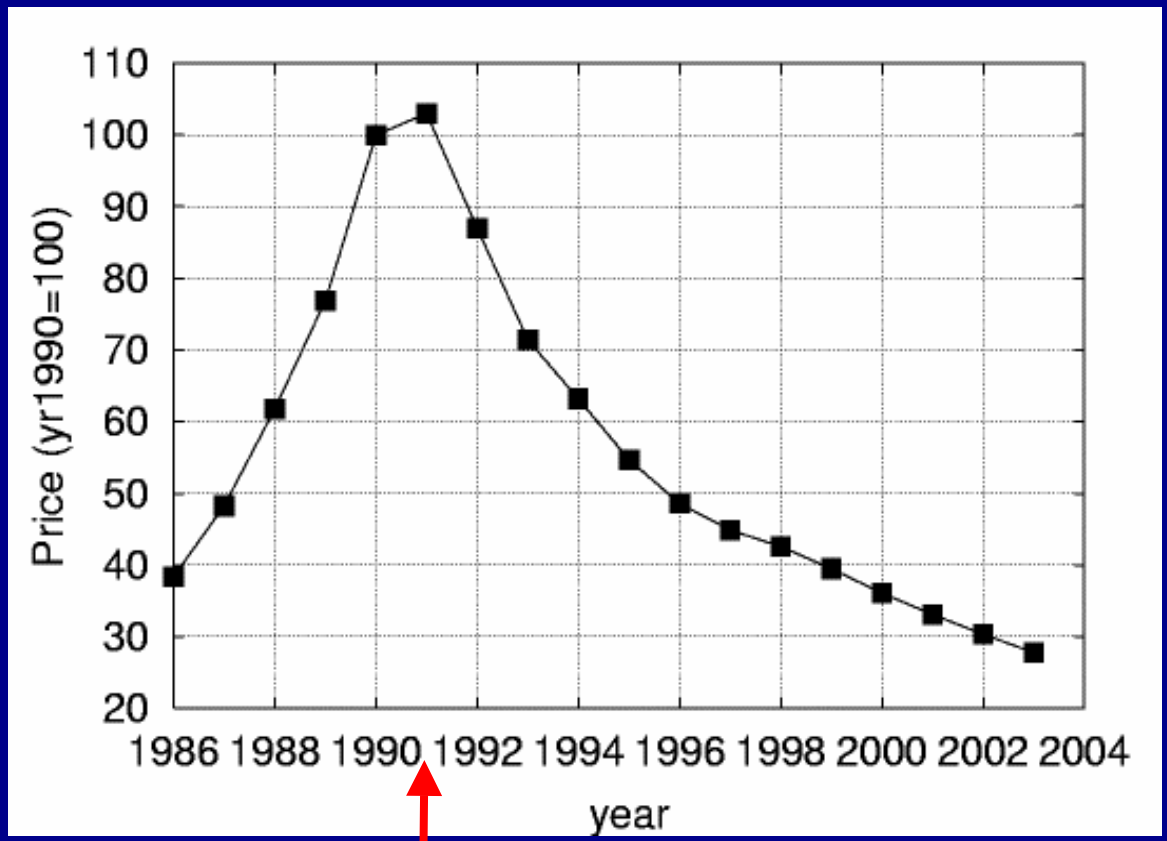
Q. Gibrat's law?

detailed-balance?

Q. What was economic

change?

speculative land-price rise and drop



1991

6 largest urban areas
avg. for residential/industrial/commercial

banks, firms and households

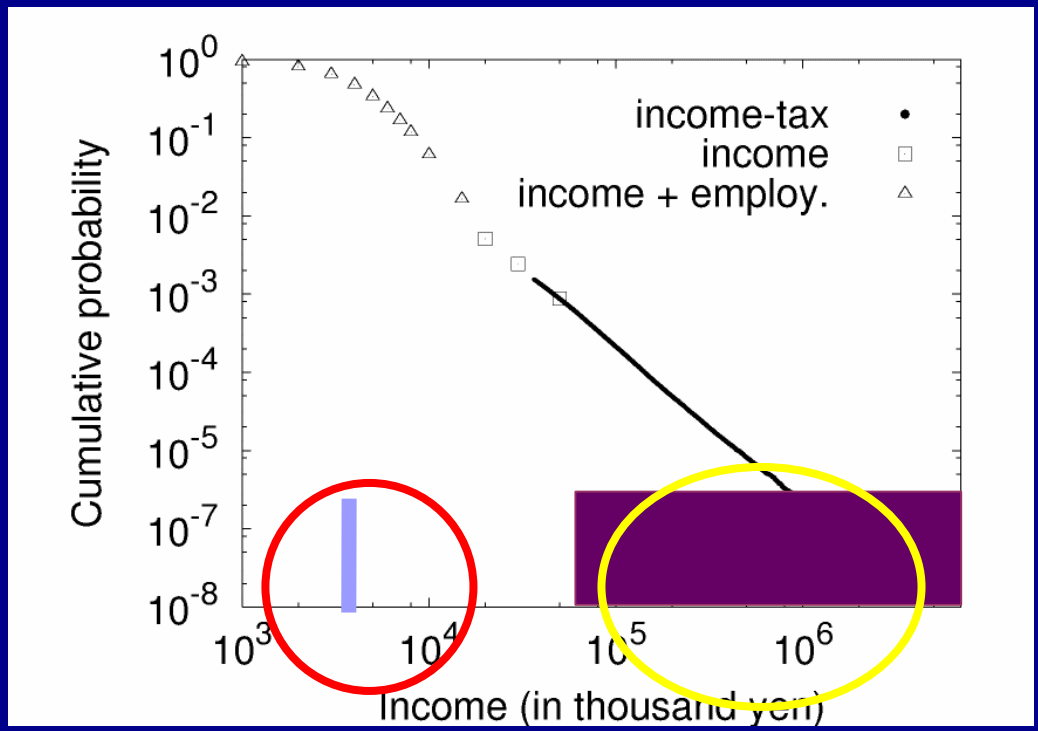
Banks = sources of finance
Firms = borrowers

- 1980's: large firms finance more directly from markets (stocks/corporate bonds etc.)
- Banks shift loans to small-business firms and individuals
- Speculation about larger demand for lands
- Small firms invest concentratedly on real estates

  (esp. Non-manufacturing sectors)
- Big effect to individuals

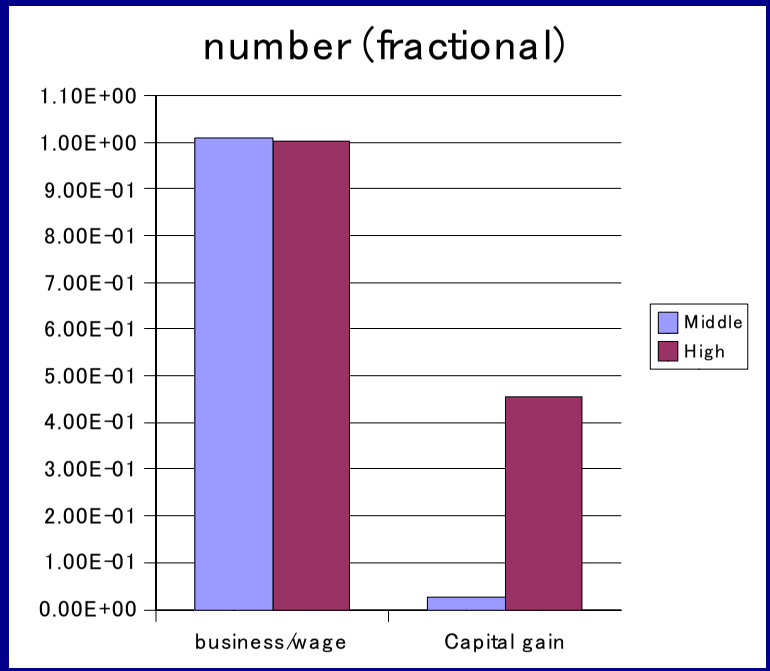
  e.g. balance-sheet change in firms and households

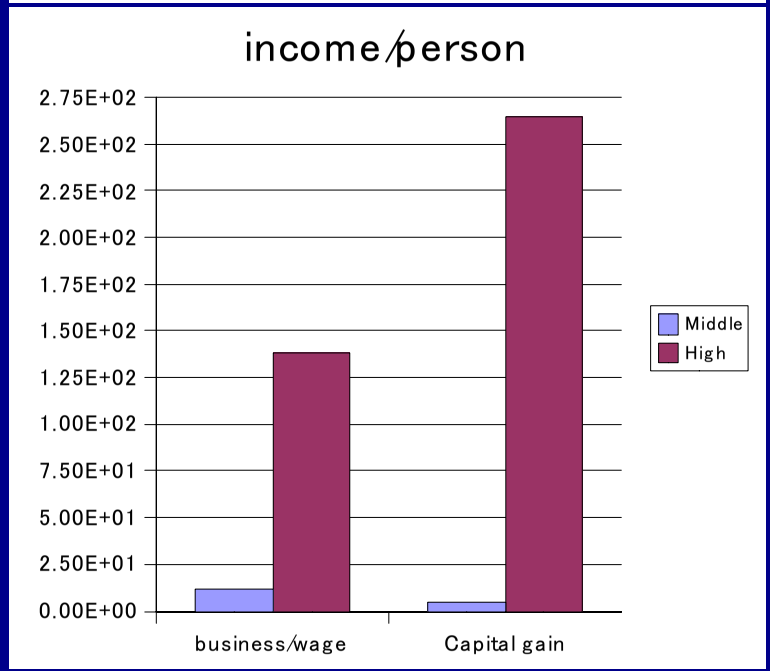
income sources



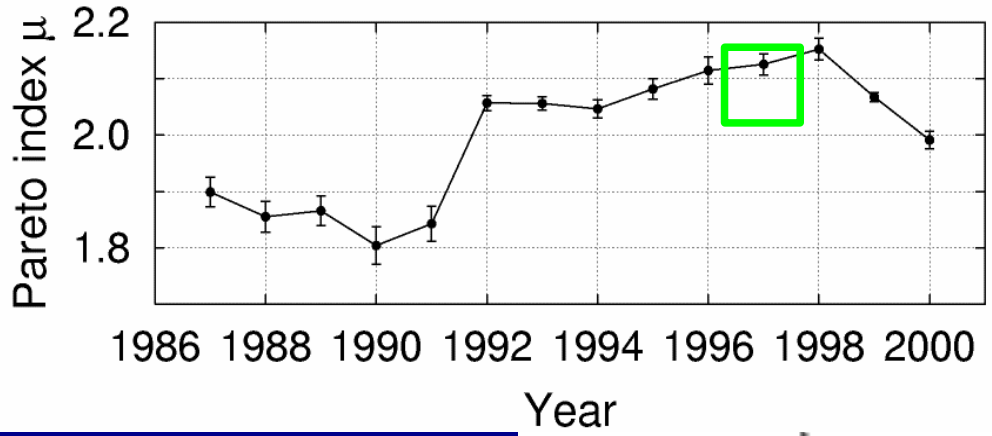
salary/wages capital gain

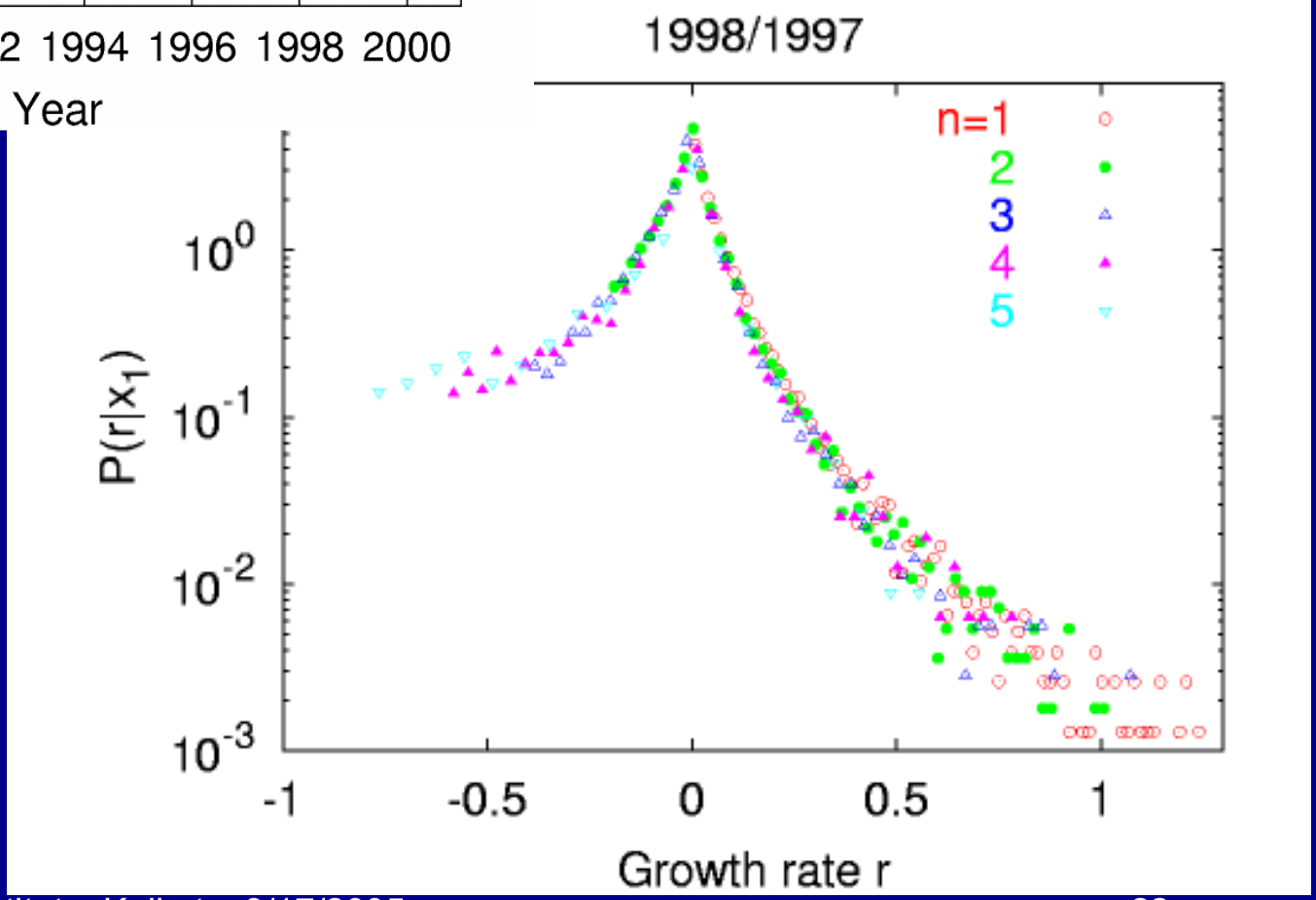
High-income from capital gain (esp. real estate)

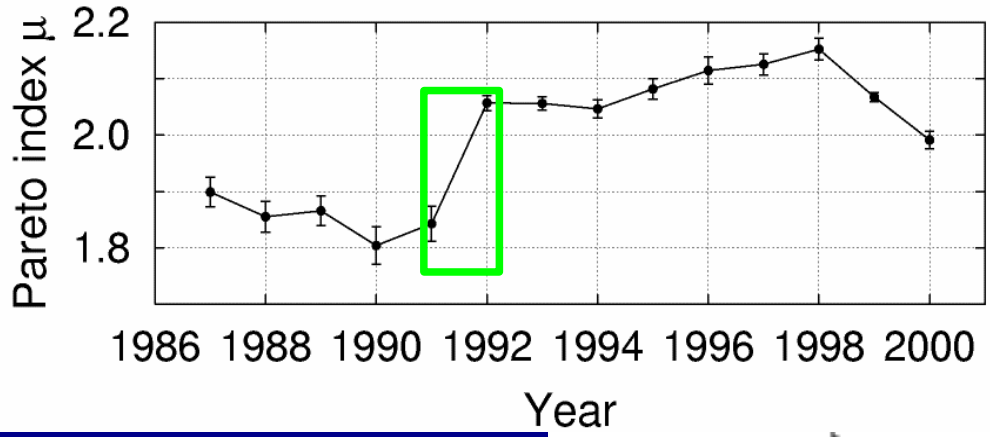




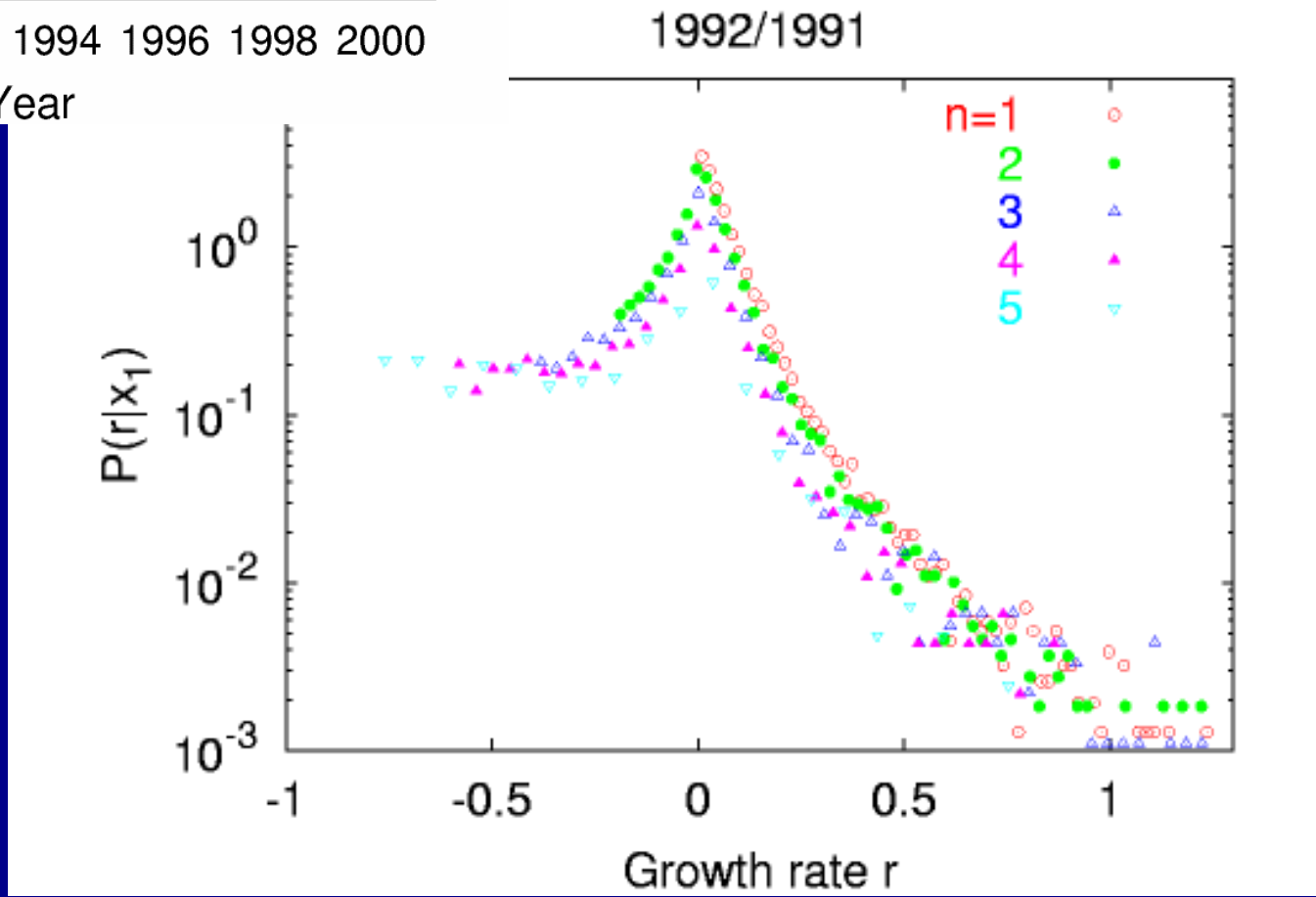
Gibrat's law

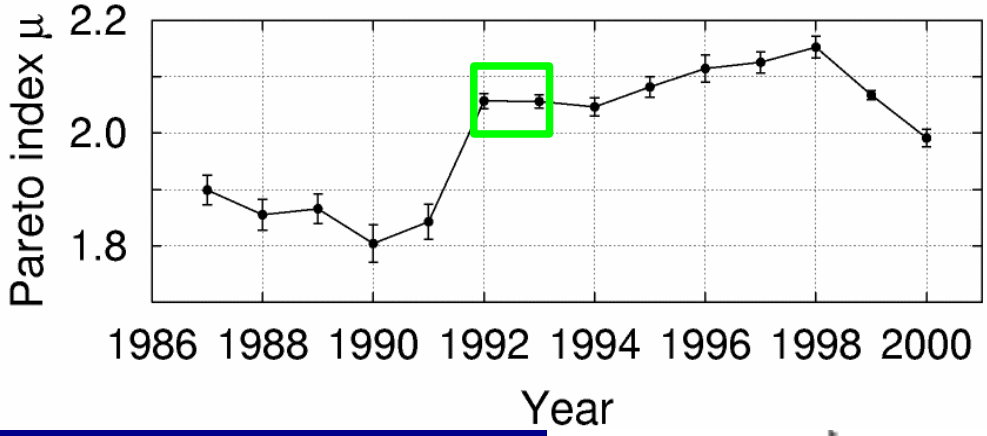

Pareto index μ
2.2
2.0
1.8
1986 1988 1990 1992 1994 1996 1998 2000
Year

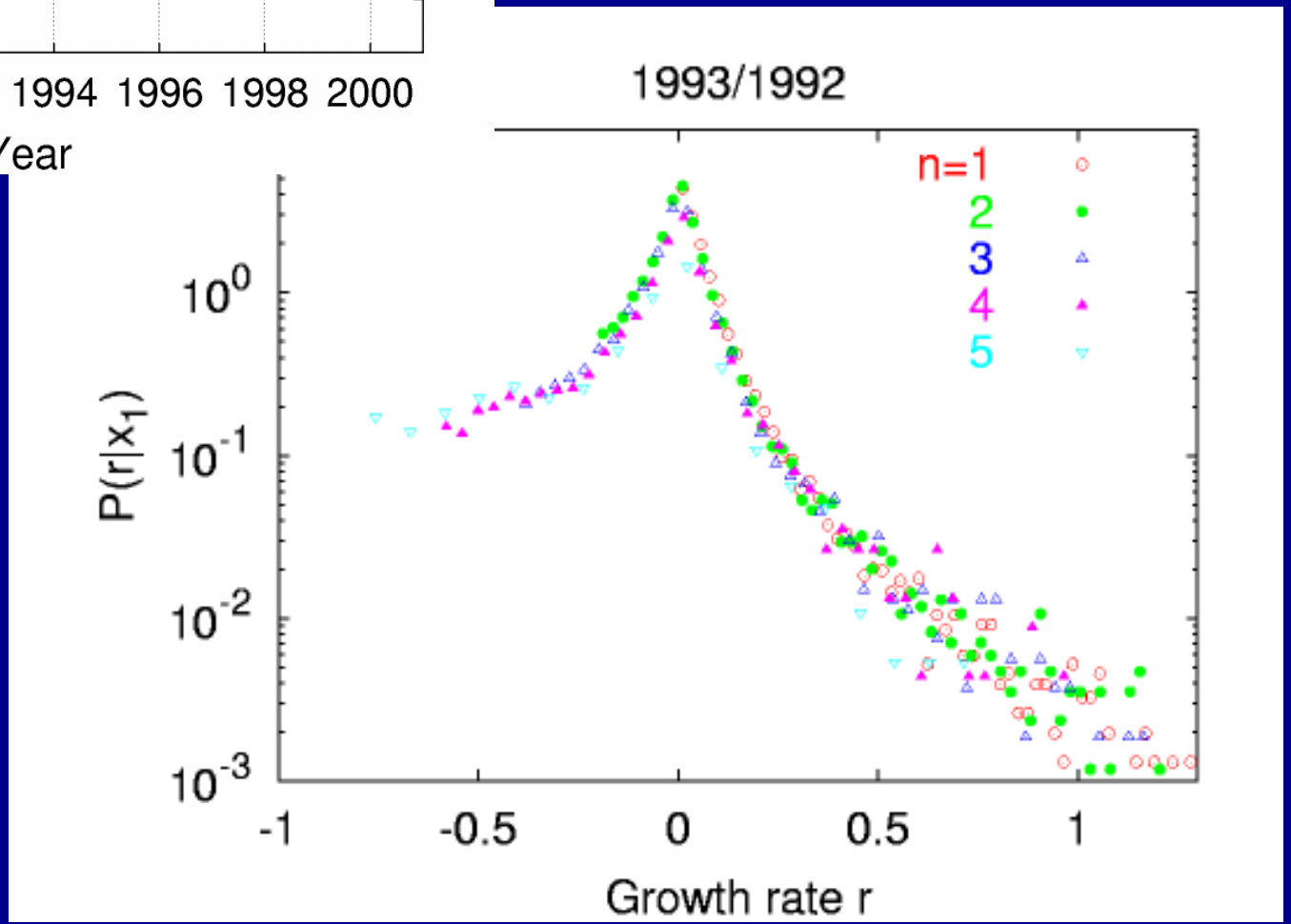

1998/1997
n=1
2
3
4
5
10^0
10^{-1}
10^{-2}
10^{-3}
$P(r|x_1)$
-1
-0.5
0
0.5
1
Growth rate r



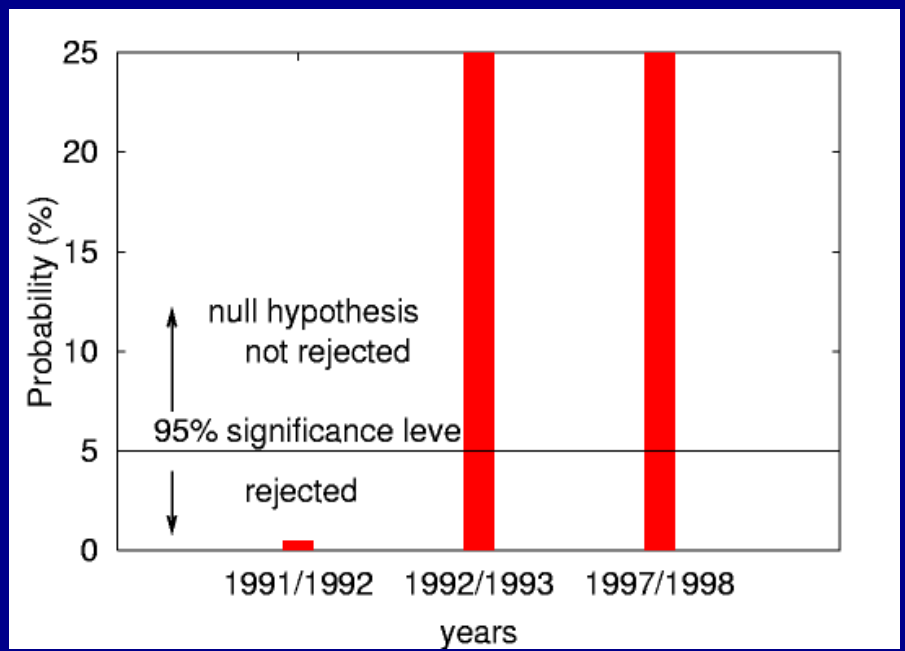
Gibrat breaks down




Pareto index μ
2.2
2.0
1.8
1986 1988 1990 1992 1994 1996 1998 2000
Year


1993/1992
n=1
2
3
4
5
10^0
10^{-1}
10^{-2}
10^{-3}
$P(r|x_1)$
-1
-0.5
0
0.5
1
Growth rate r

breakdown of detailed balance



null hypothesis

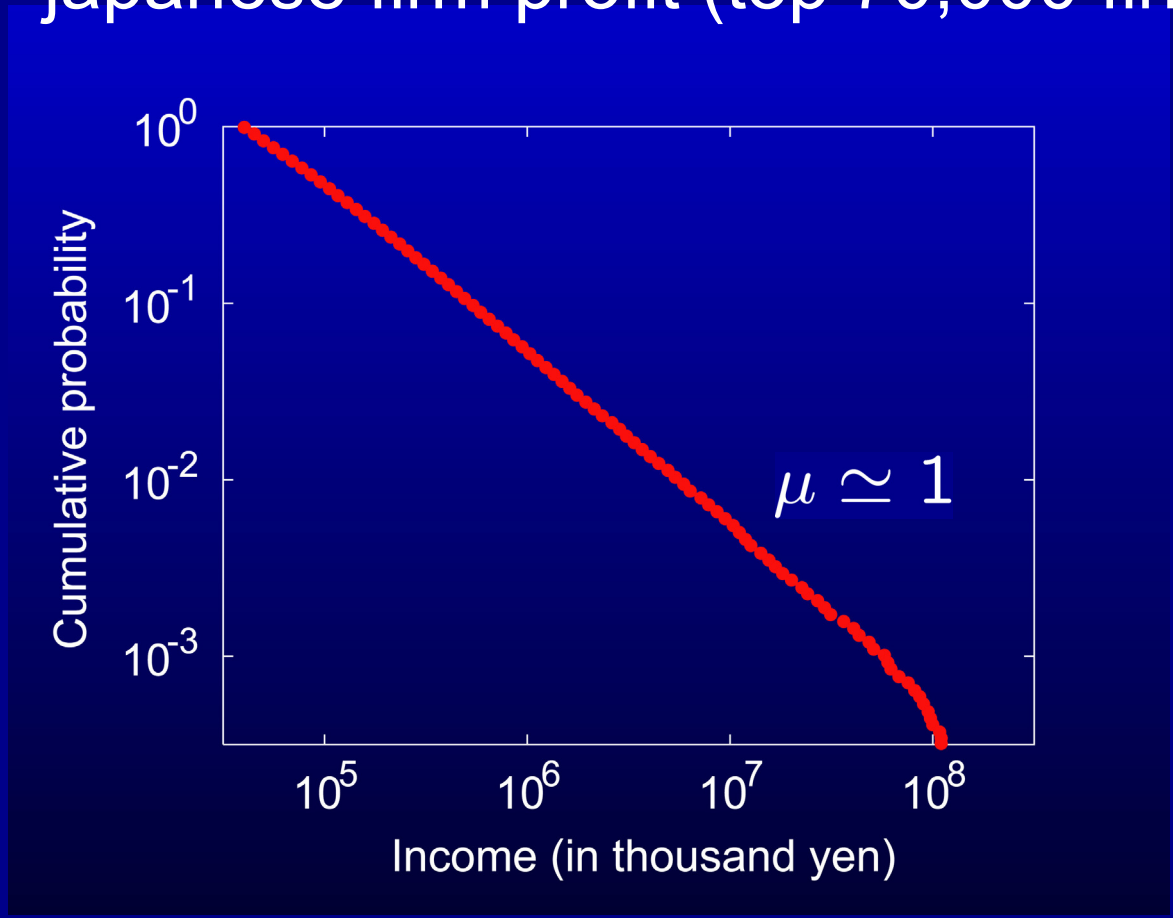
$$P_{12}(x_1, x_2) = P_{12}(x_2, x_1)$$

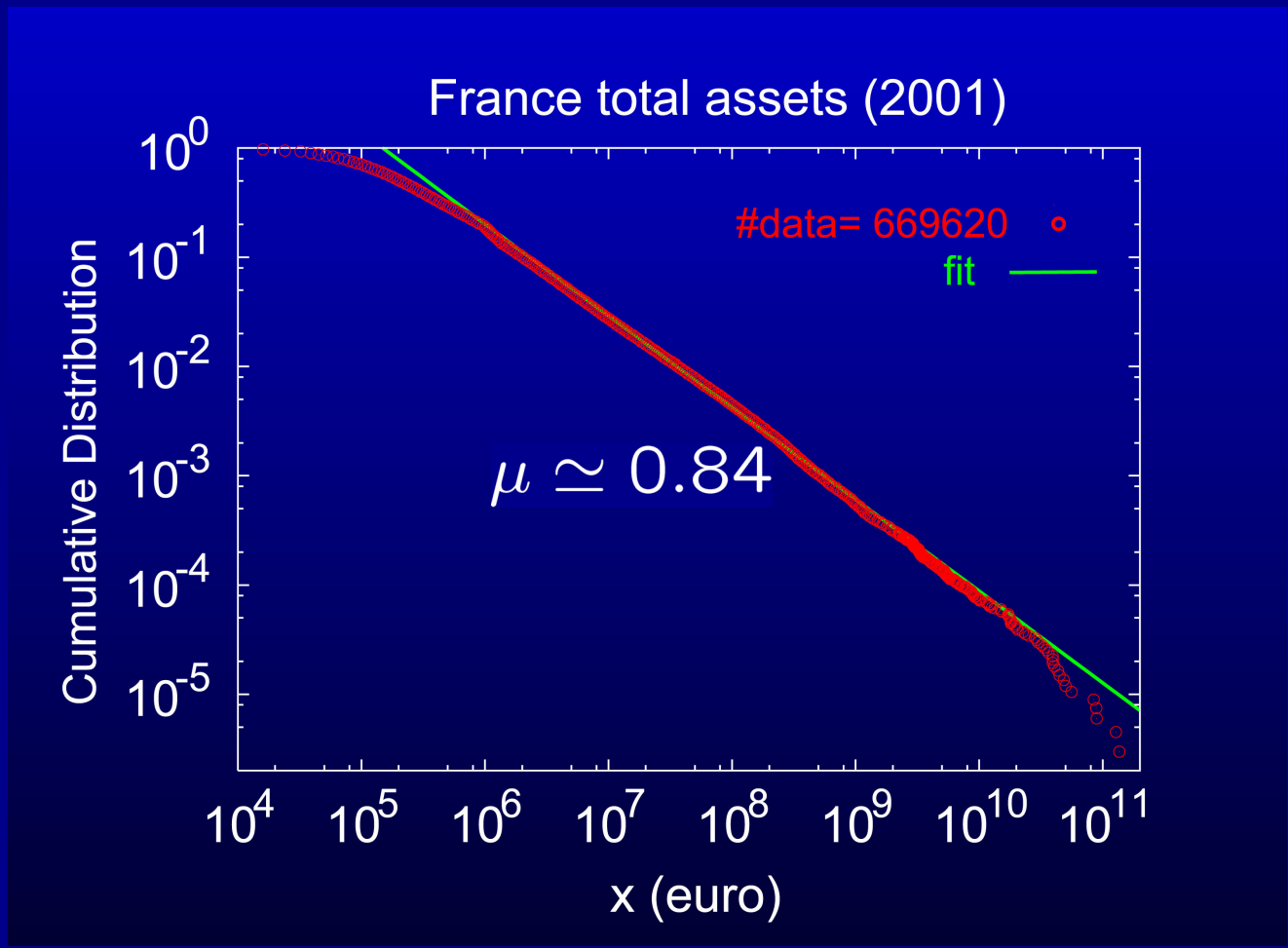
- 1991 rejected
- 1992 not rejected
- 1997 not rejected

Firm size distribution and growth-rate

(1) power-law region

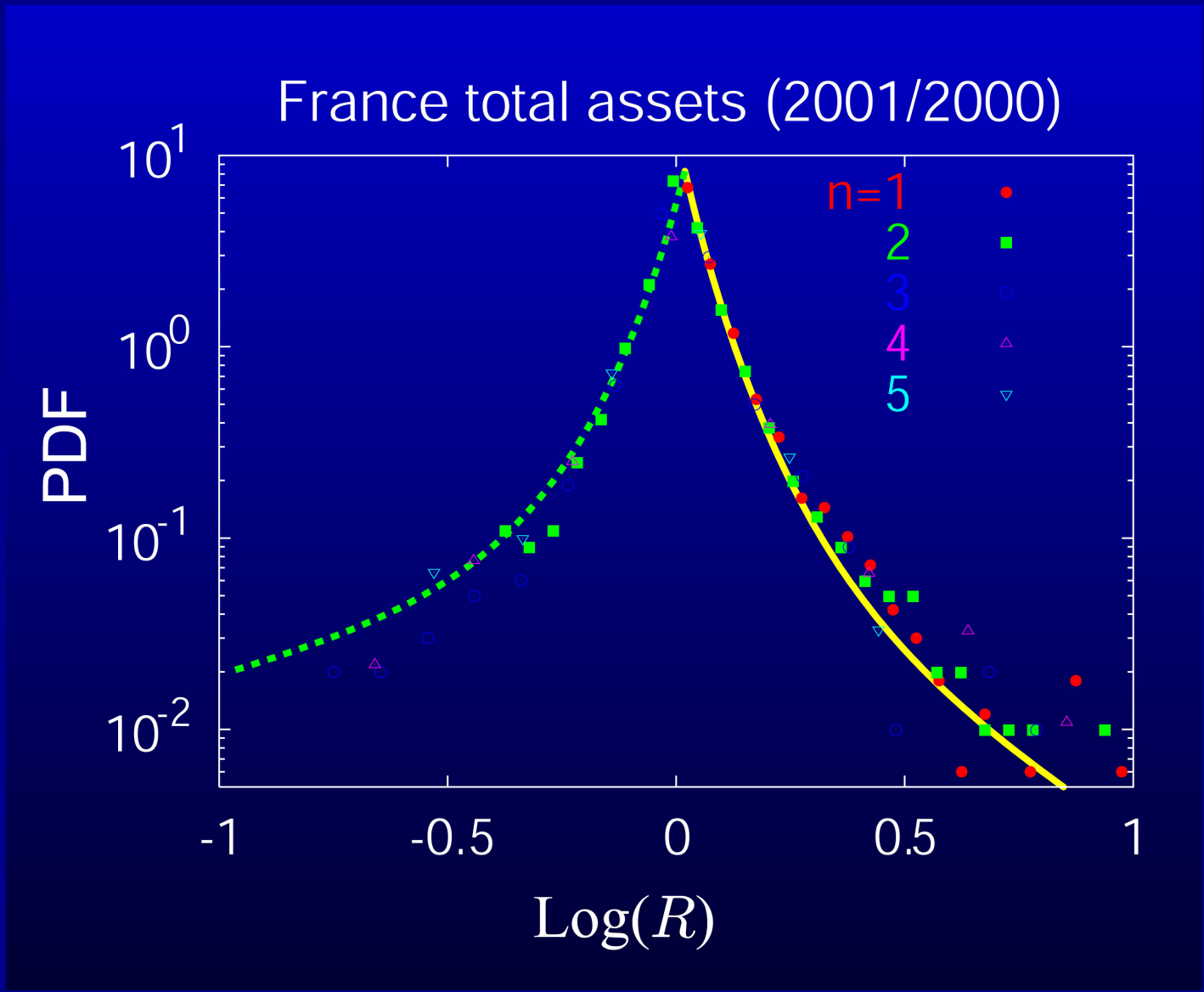
japanese firm profit (top 70,000 firms)

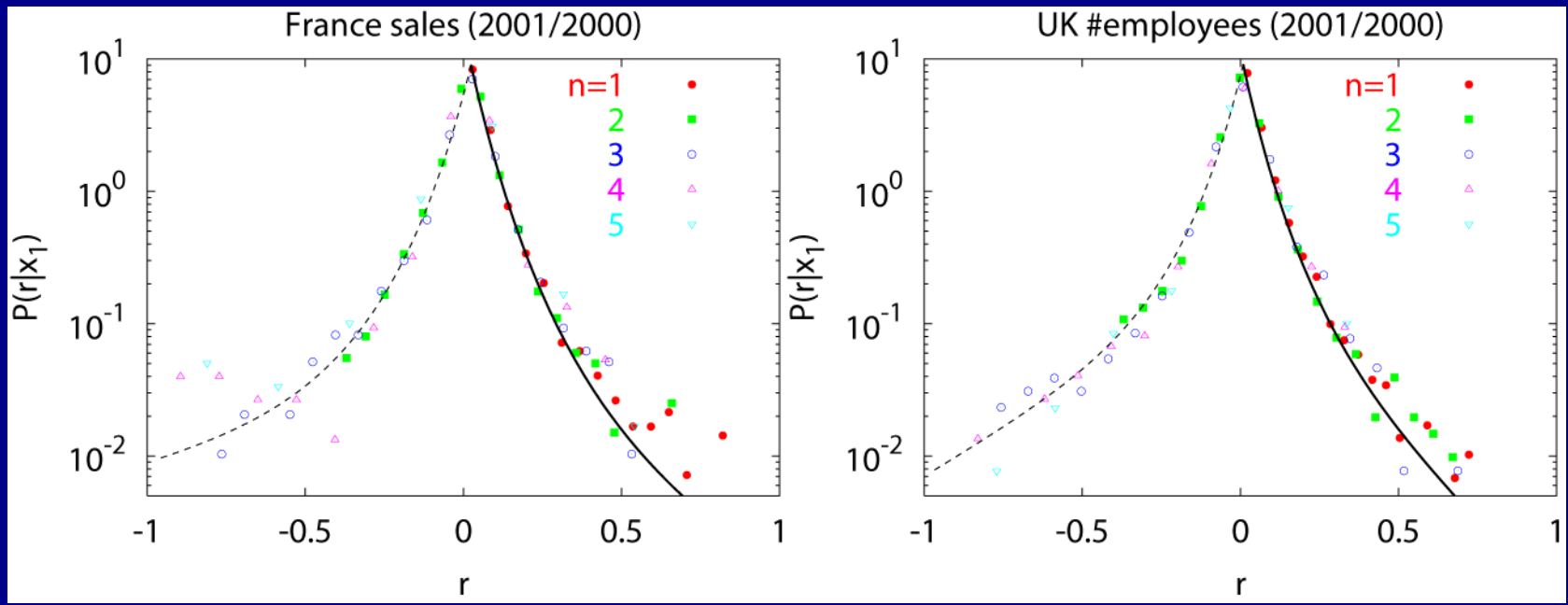




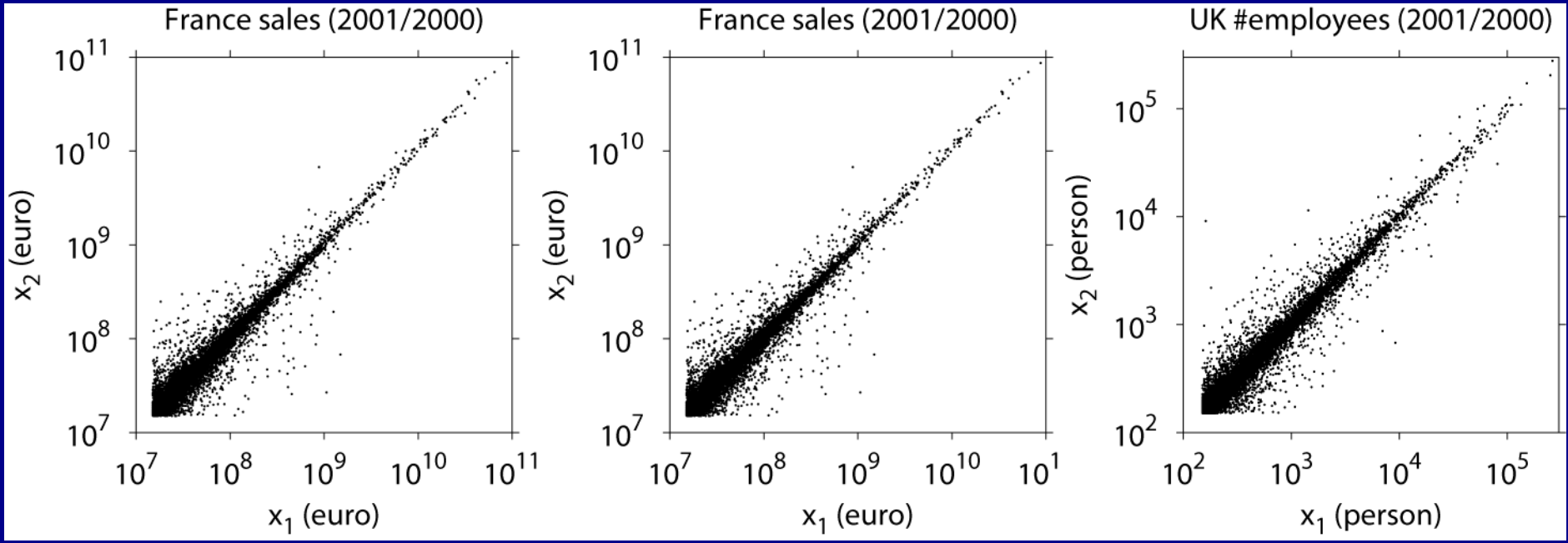
exhaustive list of largest firms in France

(2) Gibrat's law




France sales (2001/2000)
UK #employees (2001/2000)
P(r|x1)
r
n=1
2
3
4
5
10^1
10^0
10^-1
10^-2
-1
-0.5
0
0.5
1

(3) detailed-balance holds



✓ 2-dim Kolmogorov-Smirnov test

Small/midsize firms

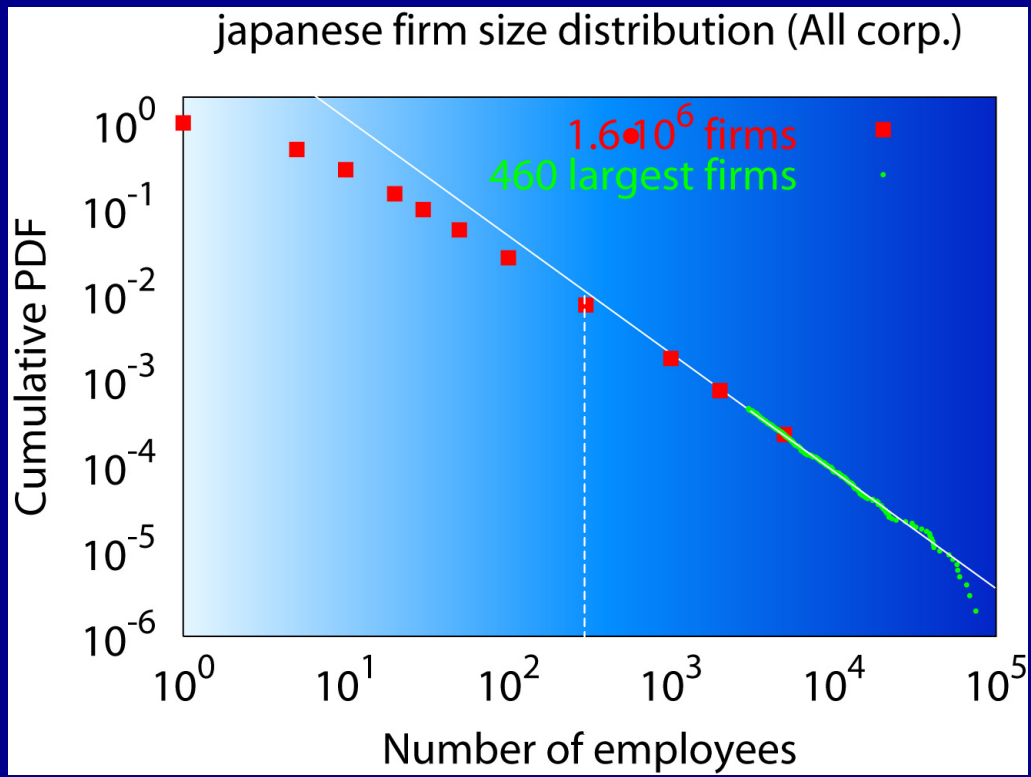
Credit risk database, Japan

- 1 million firms which covers 60% of small/midsize firms
- sampled by credit guarantee association, government-affiliated financial institutes, private-sector financial institutes
- data:
  financial statements, default information, etc.

def. of small/midsize firms

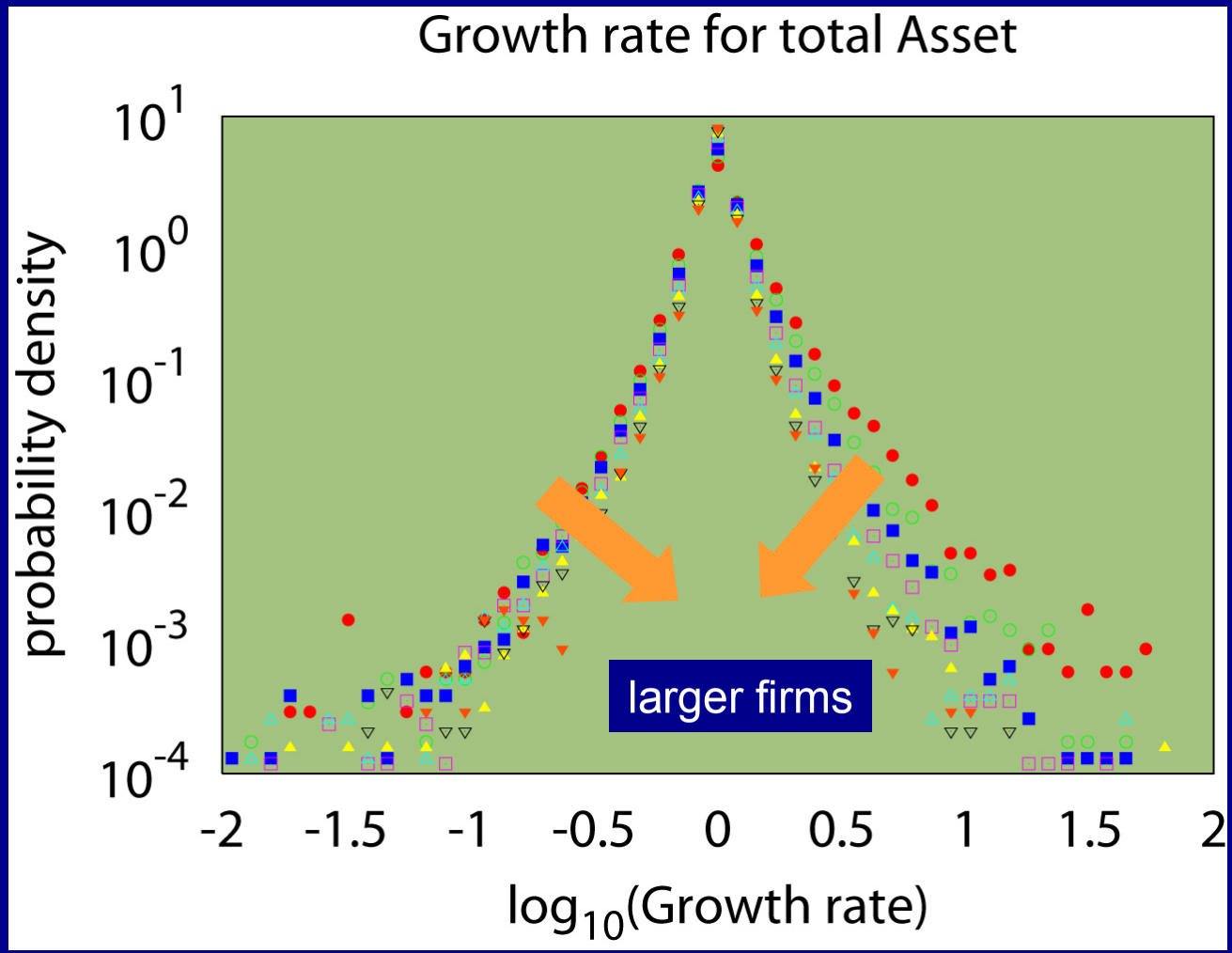
	capital <	or	#employees <
manufacturing etc.	0.3 billion yen		300
wholesales	0.1 billion yen		100
retails	50 million yen		50
service	50 million yen		100

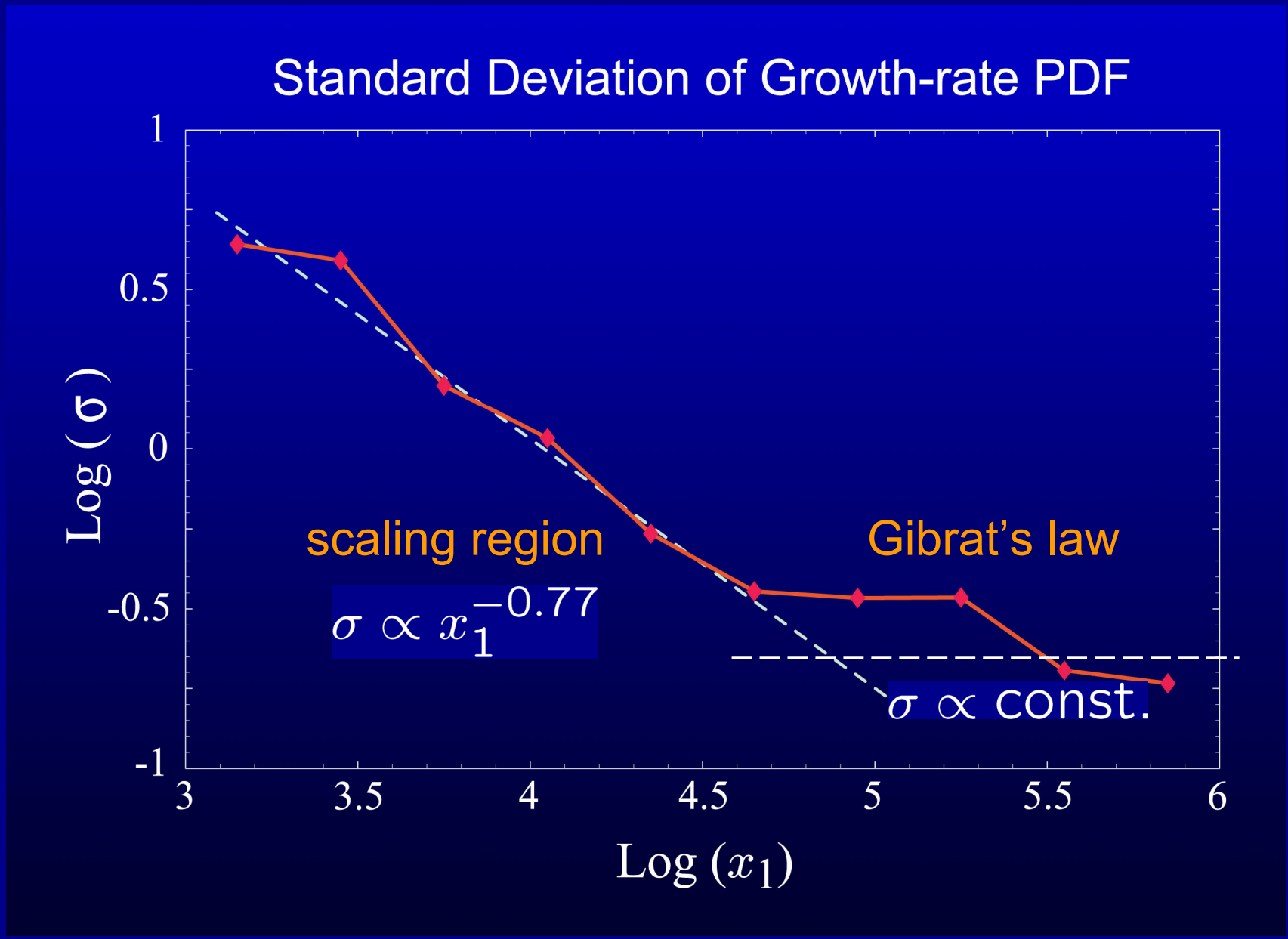
all japanese corporations (2001)=1.6 million firms



	non-power-law regime	power-law regime
# firms	1.6 million (99%)	12,000 (1%)
# employees	60%	40%

Growth-rate PDF for different firm sizes





cf. Stanley's group

Conclusion

- Using japanese personal income data
  (1) detailed-balance
  (2) Gibrat's law
  in Pareto-region
- Under detailed-balance
  Gibrat implies Pareto (not *vice versa*)
- relation between positive and negative growth-rate and cusp in growth-rate distribution
- Breakdown of all laws in "bubble" collapse phase
- Firm size has similar feature with Zipf
- Small/midsize firms have scaling relation in non-power-law region

Thank you

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- Y.Fujiwara, W.Souma, H.Aoyama, T.Kaizoji, M.Aoki
  Physica A 321 (2003) 598 [personal income]
- Y.Fujiwara, C. Di Guilmi, H.Aoyama, M.Gallegati, W.Souma
  Physica A 335 (2004) 197 [firm size]
- Y.Fujiwara
  Physica A 337 (2004) 219 [bankrupted firm size and life-time]

Appendix: proof

detailed-balance: $P_{12}(x_1, x_2) = P_{12}(x_2, x_1)$

$$P_{12}(x_1, x_2) = \frac{1}{x_1} P_{1R}(x_1, R)$$

$$P_{1R}(x_1, R) = \frac{1}{R} P_{1R}(Rx_1, R^{-1})$$

$$P_{1R}(x_1, R) = P_1(x_1) Q(R \,|\, x_1)$$

$$R \frac{Q(R \,|\, x_1)}{Q(R^{-1} \,|\, Rx_1)} = \frac{P_1(Rx_1)}{P_1(x_1)}$$

$$R\frac{Q(R\,|\,x_1)}{Q(R^{-1}\,|\,Rx_1)} = \frac{P_1(Rx_1)}{P_1(x_1)}$$

Gibrat's law states $Q(R\,|\,x_1) = Q(R)$

$$R\frac{Q(R)}{Q(R^{-1})} = \frac{P_1(Rx_1)}{P_1(x_1)}$$

Therefore

$$P_1(x) \propto x^{-\mu-1}$$ Pareto's law

$$Q(R) = R^{-\mu-2}Q(R^{-1})$$