Assignment I

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- **Prob 1 :** Consider a two-dimensional hexagonal lattice of lattice spacing $a = 3$ Åand sound velocity $c = 10^3 m/sec$. What is the Debye frequency, ω_D ? (Provide a numerical value in sec^{-1}.)

- **Prob 2 :** Consider a linear chain consists of polarizable molecules which are seperated by lattice spacing a. The molecules are fixed to their position, but they have an internal degree of freedom described by the equation of motion

$$\frac{\partial^2 p}{\partial^2 t} = -{\omega_0}^2 p + E\alpha {\omega_0}^2,$$

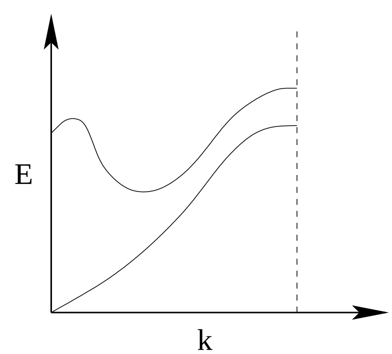
where p is the electric dipole moment of the molecule (assumed to be parallel to the chain), E is the local electric field, and α is the polarizability. Each molecule feels the electric field of the others. The system is at zero temperature, and quantum effects are small. Calculate and plot the dispersion curve $\omega(k)$ for small amplitude polarizatin waves (optical phonons). discuss the behavior of $\omega(k = 0)$ as a function α.

- **Prob 3:** Consider an one-dimensional chain of atoms with a lattice spacing of a. Each atom is represented by the potential $V(x) = aV_0\delta(x)$. Show that the electron energy E and wavenumber k satisfy the relationship

$$cos(ka) = \frac{\kappa}{K} sin(Ka) + cos(Ka)$$

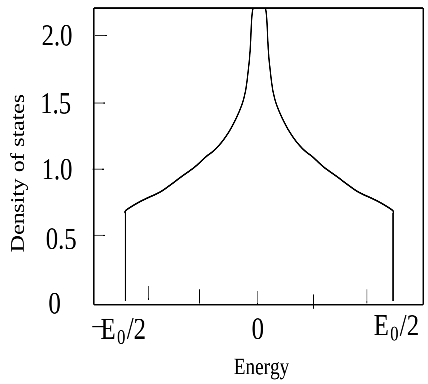
where $K^2 = 2mE/\hbar^2$ and $\kappa = \alpha V_0$. Determine the coefficient α.

- **Prob 4:**



Show that as long as time reversal symmetry or inversion symmetry is preserved a band operlap (see above figure) is not allowed in one dimesion.
(Hint : Note the difference of the Schrödinger equation in 1D compared to higher dimension)

- **Prob 5:**



The density of states for a two-dimesional system of electrons with the "tight-binding" band structure $E(k) = -E_0(cosk_xa + cosk_ya)/4$ is shown in above figure. Investigate the density of states in the neighbourhood of $E = 0$. (In the figure the total area under the curve is normalized to 1.)

(Hint : Note that the density of states is the line integral in **k**-space over an equipotential line i.e $\rho(\epsilon) = \oint \frac{d\mathbf{l}}{|\partial E/\partial \mathbf{k}|}$. However one can use other method of calculating the density of states.)

- **Prob 6:**The Hamiltonian for the anisotropic Heisenberg model is

$$H = -\sum_{i,j}[J'_{ij}S_i^zS_j^z + \frac{1}{2}J_{ij}(S_i^+S_j^- + S_i^-S_j^+)].$$

Now consider a one-dimensional system with nearest-neighbour interactions.

$$H = -\sum_{i}[J'S_i^zS_{i+1}^z + \frac{1}{2}J(S_i^+S_{i+1}^- + S_i^-S_{i+1}^+)].$$

and ferromagnetic coupling, $J > 0$,$J' > 0$.

(a) Show that for $J' > J$ the magnon wavefunctions are similar to the isotropic Heisenberg model solutions, but they have a nonzero energy at $k = 0$.
(b) Show that $J' < J$ the ferromagnetically ordered state (with magnetization aligned in the z direction) is *not* the ground state of the Hamiltonian.

- **Prob 7:** Calculate the leading term in the teperature dependence of the chemical potential for two dimensional free electron gas.